The Efficiency of Patent Litigation*

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Abstract

How efficient is the U.S. patent litigation system? Estimating a novel dynamic model, we characterize the litigation system's role in shaping innovation. In our model, heterogeneous firms innovate and may sue each other for patent infringement. In equilibrium, expected future litigation activity impacts firm innovation incentives. Moreover, some firms create positive innovation externalities while others impose negative externalities. Litigation reform can, therefore, improve or harm welfare, depending on how heterogeneous firms endogenously select into lawsuits. Estimating the model, we evaluate historical and recently proposed litigation reforms. Defendant-friendly reforms promote innovation and boost economic growth, improving welfare by up to 2.98%.

Keywords: patent litigation, innovation, firm value, growth, social welfare.

JEL Classification: G30, E22, O30, O40, K40.

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Innovation is essential for economic growth, but inventors' incentives to innovate depend on their ability to profit from costly research and development (R&D). While the U.S. patent system offers inventors protection, the system relies on the courts to enforce patent-holder rights through patent-infringement litigation.

But is this litigation efficient, and does it benefit consumers or spur growth? These questions are inherently quantitative, so to answer them, we develop and estimate a dynamic model of innovation and patent lawsuits. Our model embeds a realistic patent-litigation system in a dynamic general equilibrium model of corporate innovation and economic growth. These features allow us to evaluate the effects of counterfactual reforms to patent litigation. In particular, we find that weakening plaintiff rights by granting fewer injunctions against defendants raises social welfare by 2.98%. Similarly, we show that a reform that makes it twice as costly for a patent-holder plaintiff to file a lawsuit increases social welfare by 1.85%.

These types of quantitative results are important given the intense and enduring debate around patent-litigation reform, which centers around whether plaintiff (patent holder) rights are too strong or too weak. For example, fourteen patent-reform bills were proposed in the 113$^{\text{th}}$ Congress alone, with goals such as increasing plaintiff pleading requirements to strengthen defendant rights (Gugliuzza, 2015). Similarly, a Senate bill introduced in 2023 provided a compromise between tech companies, who believe excessive patent granting has led to frivolous patent lawsuits, and pharmaceutical companies, who believe it is too difficult to protect their innovation with patents.[1]

To flesh out the intuition behind our results, we begin with a simple conceptual framework that illustrates how the patent litigation system impacts innovation and welfare. At a high level, firms choose their level of innovation to maximize profits. This activity leads to better products, creates spillovers for firms with similar technologies, and allows the innovating firm to steal market share from competitors. Some firms underinvest in innovation, relative to the socially efficient benchmark, because they do not internalize spillovers. Other firms overinvest because they inefficiently internalize the transfer they extract from competitors by stealing customers. In this standard setting (Bloom et al., 2013), we consider the role of the litigation system. When a firm innovates, an incumbent firm might sue to block the innovation. If the court grants an injunction, the innovating firm cannot use its novel

[1] See https://www.reuters.com/legal/litigation/tech-pharma-companies-divided-pto-patent-eligibility-comments-2021-10-19/.

technology. We discuss how a high injunction rate, which is friendly to incumbent firms, could lead to more or less innovation. Similarly, litigation reform (e.g., raising the injunction rate) has ambiguous effects on welfare. If most litigation occurs in technological fields where firms inefficiently overinvest in innovation, then a reform-induced reduction in innovation improves welfare. Conversely, a litigation-reform-induced drop in innovation harms welfare if most litigators are inefficiently underinvesting.

Our conceptual framework demonstrates that a defendant-friendly litigation reform can raise or reduce welfare, depending on which type of innovative firm responds to the reform. We partially resolve this ambiguity through a motivating reduced-form exercise. Using a standard instrument for innovation based on R&D tax credits (Bloom et al., 2013), we show that litigation-intensive firms appear to underinvest in innovation: a quasi-exogenous increase in R&D spending increases firm value. Moreover, these same litigation-intensive firms produce positive knowledge spillovers: they invest more in R&D after a quasi-exogenous increase in innovation by technology-class peers. Both underinvestment and knowledge spillovers are more pronounced for firms frequently involved in litigation, suggesting that litigation reform should aim to increase innovation. Replicating Mezzanotti (2021), we show that a defendant-friendly reform—reducing injunction rates—increases innovation.

Combining our conceptual framework with these empirical facts, our findings suggest that defendant-friendly reforms improve welfare by boosting innovation for the type of firm that underinvests in innovation, relative to a socially efficient benchmark. However, the simple framework overlooks important dynamic incentives and general-equilibrium considerations. Moreover, while our reduced-form findings provide suggestive evidence of underinvestment, our estimates cannot directly speak to welfare without a quantitative model. To overcome these limitations and provide policy counterfactuals, we build a dynamic equilibrium model of innovation and litigation. In the model, heterogeneous firms compete in product markets. Incumbent firms and new potential entrants spend resources on research and development to innovate a better version of an existing product. After a successful innovation, the owner of the newly improved product enjoys a monopoly on that particular product until a competitor innovates a better version. Firms choose innovation levels and production policies to maximize profits, taking prices as given.

Within this dynamic equilibrium setting, we introduce a patent litigation system. Whenever a firm innovates a better version of a product, there is a chance that the new product infringes on a patent of an existing firm. If infringement occurs, the patent holder observes a random cost of filing a lawsuit. The patent holder sues the innovator if the expected lawsuit payoff exceeds the cost of filing. We allow infringement probabilities, defendant win probabilities, litigation costs, and injunction rates to vary across two important dimensions of heterogeneity in our model: firms differ in their technology classes and industries. Infringement probabilities depend on profitability and R&D intensity, capturing the empirical pattern that cash-rich firms face higher litigation risk while R&D-intensive firms invest in stronger patent protection.

In a lawsuit, the defendant (the innovating firm) privately observes its probability of winning a lawsuit. The plaintiff (the patent holder) makes a take-it-or-leave-it offer to settle. The defendant accepts if its continuation value from going to trial is worse than the cost of the proposed settlement payment. Due to the defendant's private information, both settlements and trials occur in equilibrium. If the defendant declines the settlement offer, the lawsuit goes to trial. In the trial, the defendant has an idiosyncratic random probability of winning. If the defendant loses, there is a chance that the plaintiff obtains an injunction, which prevents the defendant from selling its new product. Otherwise, the defendant patents and sells its newly innovated product.

In equilibrium, firms have rational expectations about how the patent system shapes the returns to innovating, which are generated by the following tradeoff. On the one hand, firms recognize that a plaintiff-friendly system makes it likely that an incumbent's patent will block their innovation. Plaintiff-friendly litigation reforms can thus discourage innovation. On the other hand, firms also recognize that a plaintiff-friendly system increases the returns to successful innovation. Conditional on an innovation not getting blocked, the innovating firm enjoys a longer period of monopolist profits because it can sue to block new entrants. Thus, plaintiff-friendly reforms can also encourage innovation.

We introduce firm heterogeneity in the model to allow innovation to have differing social values across firms. As discussed above, innovation creates a positive externality through knowledge spillovers for firms using similar technologies. Firms do not internalize this externality, so some underinvest in innovation relative to a socially efficient benchmark. Other firms innovate products that are barely better than existing products. For these firms, innovation has little social value but a large private value because these firms inefficiently internalize the value they extract from incumbents through creative destruction, that is, by stealing their customers. Thus, these firms overinvest in innovation. By combining this heterogeneity with an endogenous litigation process, we can model which types of firms select into using the patent litigation system.

We estimate the model parameters and use the estimated model to quantify the impact of R&D subsidies and litigation reforms. First, doubling an existing R&D subsidy raises social welfare by 1.91%, implying that aggregate investment in innovation falls below a socially efficient benchmark. However, the degree of over- or under-investment varies widely across firms due to differences in research efficiency. R&D subsidies targeting only high research-efficiency firms increase welfare, while those targeting only low research-efficiency firms reduce it.

Second, we examine how changes to the litigation system affect the impact of R&D subsidies. We find that defendant-friendly reforms boost R&D subsidy efficacy, as potential innovators facing fewer lawsuits respond more to subsidies. We conclude that model-based approaches ignoring patent litigation can overestimate R&D subsidy efficacy because of the endogenous linkages between the incentives that shape innovation and litigation.

Third, we examine the impact of historical and recently proposed litigation reforms. In our model, a defendant-friendly reform could encourage or discourage innovation, as firms can both sue and be sued. Our model estimates resolve this tension, as our counterfactuals show that defendant-friendly reforms promote innovation. Moreover, this rise in innovation is concentrated among firms with high research efficiency, so welfare rises. Specifically, we show that a reform modeled after the 2006 "eBay" Supreme Court ruling, which lowered injunction rates, improves welfare by 2.98%. Similarly, a reform modeled after a recent proposal to increase plaintiff filing costs improves welfare by 1.85%. In each exercise, we show that welfare improvements coincide with increased economic growth. Like R&D subsidies, litigation reforms are most effective when they target high-research-efficiency firms.

Fourth, we study targeted policies. Impeding lawsuits against high-efficiency firms raises welfare; impeding lawsuits against low-efficiency firms lowers it. Reforms also have larger effects when targeted at cases in which the plaintiff seeks to protect its market share than at cases in which the plaintiff seeks to extract rents, because the former unlock more innovation and product-line creation. Similarly, we also examine infringement-specific reforms, finding that policies that impede litigation and that are targeted at market-share cases improve incumbent welfare more than policies targeted at rent-extraction cases.

Finally, we explore two additional scenarios. We find that reducing the risk of infringement raises both innovation and welfare. When we simulate increased patent troll activity, we find less innovation, especially among potential industry entrants, with resources shifting to litigation rather than R&D, ultimately decreasing welfare.

To ensure the reliability of our quantitative findings, we conduct extensive validation and robustness exercises. We validate the model against moments not targeted in estimation. We extend the model to include defendant legal fees and show that our welfare conclusions remain robust across a wide range of values. We also reestimate the model for subsamples with different financial constraints and find that constrained firms exhibit stronger welfare gains from defendant-friendly reforms, consistent with greater sensitivity to litigation risk. Additionally, we discuss how our heterogeneous litigation parameters likely allow our model to match the empirical predictions of an asymmetric-information reputation model, though we leave a formal model of dynamic reputation building to future work.

Our paper lies in a body of work in finance and economics that studies the equilibrium consequences of corporate policies by extending classic endogenous growth models such as Grossman and Helpman (1991), Aghion and Howitt (1992), or Klette and Kortum (2004). Examples include Lentz and Mortensen (2008), Kung and Schmid (2015), Bena et al. (2016), Akcigit and Kerr (2018), Opp (2019), Cavenaile et al. (2019), Malamud and Zucchi (2019), Bena and Garlappi (2020), Kogan et al. (2020), Acemoglu et al. (2022), and Geelen et al. (2022). Our work extends this literature by addressing a novel topic. We examine how litigation systems shape the externalities that dynamically innovating firms impose on each other. Our framework can be used to answer many questions about the effects of innovation and litigation policies on firm and aggregate outcomes.

Specifically, our work lies in a literature that models innovation, litigation, and their interaction. Innovation models include Lin (2012), Acemoglu et al. (2018), Levine and Warusawitharana (2021), Liu and Ma (2021), Akcigit et al. (2022), Celik (2023), and Celik and Tian (2023). Examples of litigation models include Bessen and Meurer (2006), Marco (2006), Choi and Gerlach (2017), and Antill and Grenadier (2023). Models of the interaction between innovation and litigation include Abrams, Akcigit, Oz, and Pearce (2020) and Rempel (2023). Unlike Abrams, Akcigit, Oz, and Pearce (2020), which focuses on the role of patent trolls, our work addresses the efficiency of potential reforms to this system. This focus also differs from that of Rempel (2023), which studies how patents shape industry characteristics.

Our model also includes two features that separate it from all of this work. First, we model heterogeneous firms that impose both positive and negative externalities through their innovation. Second, firms endogenously choose to litigate based on their heterogeneous types, thereby shaping equilibrium innovation incentives. This novel combination of model features is important for our main contribution: quantifying the welfare impact of litigation reforms.

Additionally, we contribute to the empirical literature that studies how changes in plaintiffs' rights affect innovation activity (Sakakibara and Branstetter, 2001; Lerner, 2002; Moser, 2005; Lerner, 2009; Murray and Stern, 2007; Galasso and Schankerman, 2015; Williams, 2013; Cohen, Gurun, and Kominers, 2019; Mezzanotti, 2021; Kempf and Spalt, 2023; Lin, Liu, and Manso, 2021). While these reduced-form studies inform the policy debate around patent litigation reform, a model-based approach provides insights that reduced-form methods cannot. In particular, we quantify the impact of potential counterfactual reforms on both socially beneficial and socially harmful innovation.

1. Institutional Details

1.1. Patents

The U.S. patent system is designed to encourage innovation by giving patent holders the exclusive right to use their patented technology. Following a new discovery, inventors can apply for a patent with the United States Patent and Trademark Office (USPTO). Before granting the patent, the USPTO verifies that the invention is (i) novel; (ii) useful and operable; (iii) a non-obvious improvement relative to prior technology; and (iv) related to a patentable subject matter.[2] To ensure these criteria are met, a patent examiner verifies that the invention is not an obvious extension of an existing patented technology. Once the USPTO grants a patent, it expires 20 years after the application date (35 U.S.C §154).[3] During that period, the patent holder has the right to exclude others from making, using, or selling their patented inventions.

1.2. Patent litigation and injunctions

Patent holders can enforce their patents through patent infringement lawsuits. These lawsuits are typically filed in federal district courts. A patent holder can sue anyone who

[2] See https://www.justia.com/intellectual-property/patents

[3] See https://www.law.cornell.edu/uscode/text/35/154.

"makes, uses, offers to sell, or sells any patented invention."[4] If the lawsuit proceeds to trial, the plaintiff (the patent holder) and the defendant (the alleged infringer) present evidence to a jury. The infringer's product is compared to the plaintiff's patented invention. To establish infringement, the plaintiff must satisfy the "all elements rule,"[5] that is, show that the infringing product includes every element of the patented product. In some instances, the "doctrine of equivalents" allows a plaintiff to show infringement if some element of the patented product is missing in the infringing product, but the differences are insubstantial.[6]

If the plaintiff wins the lawsuit, the judge typically grants a permanent injunction against the infringing defendant. The permanent injunction is an order forcing the defendant to stop all activity that infringes on the patent. If the injunction covers any step in the production of the defendant's product, the defendant must entirely shut down that product until a non-infringing process is developed. While patent infringement itself is a tort and not a criminal offense, violating a permanent injunction can lead to criminal penalties.[7]

Before 2006, permanent injunctions were nearly always granted after plaintiff victories. However, in 2006, the U.S. Supreme Court clarified the criteria for granting a permanent injunction. In *eBay Inc v MercExchange L.L.C.* ("eBay"), the Supreme Court ruled that courts must apply a four-factor test to determine whether a permanent injunction is appropriate.[8] In this test, the plaintiff must show (i) it suffered irreparable harm; (ii) other remedies (e.g., monetary damages) are inadequate to compensate the plaintiff; (iii) comparing the resulting hardships for the plaintiff and defendant, equitable relief (e.g., enforcing the patent) is warranted; and (iv) an injunction would not harm the public interest.[9] The rate at which successful plaintiffs obtained injunctions fell from 95% before eBay to 75% (Chien and Lemley, 2012; Seaman, 2015).

Anticipating the trial process described above, many plaintiffs and defendants settle patent infringements out of court. Often, a plaintiff files a formal lawsuit that is ultimately settled before trial. In other instances, the plaintiff sends a "demand letter," asking the defendant to pay a license fee to use the patented technology. If the defendant agrees, this

[4] See https://www.law.cornell.edu/uscode/text/35/271.

[5] See https://definitions.uslegal.com/a/all-elements-rule/.

[6] See https://www.law.cornell.edu/wex/doctrine_of_equivalents.

[7] See https://www.mandourlaw.com/patent-injunction/.

[8] See https://www.law.cornell.edu/supct/cert/05-130.

[9] See https://content.next.westlaw.com/practical-law/intellectual-property-technology/patent-litigation.

process is a form of out-of-court settlement. In our quantitative framework, we assume that a plaintiff must hire a legal team to reach a settlement. This assumption is realistic because defendants often view demand letters as non-credible threats when they are not accompanied by formal lawsuits. Similarly, our assumption that a plaintiff must pay the cost of a trial to reach a settlement captures the view that defendants do not take plaintiffs seriously until they hire substantive legal counsel. This is consistent with the empirical observation that many settlements occur immediately before trial (Antill and Grenadier, 2023).

Following the law literature, we assume that only the defendant has private information (P'ng, 1983; Bebchuk, 1984; Spier, 2007). One-sided information avoids intractable signaling problems and multiple equilibria that arise with two-sided asymmetry in settlement offers. It is also realistic: as Spier (2007) explains, "the defendant may have first-hand knowledge about his degree of involvement in (or liability for) the [tort]" and defendants "may know better the credibility of their own witnesses and the quality and work ethics of their lawyers."

2. Conceptual Framework and Empirical Motivation

Before presenting our dynamic quantitative model, we summarize our conceptual framework. This stylized framework illustrates how litigation reform can encourage or discourage innovation, and how reform-induced innovation may improve or reduce welfare. We then provide empirical evidence that is consistent with our main quantitative result: defendant-friendly reforms boost innovation among litigation-intensive firms, and this type of innovation appears to boost welfare.

2.1. Conceptual framework

Privately optimizing firms may choose innovation policies that differ from those of a welfare-optimizing social planner. Bloom et al. (2013) attribute this gap to two mechanisms. First, innovation can produce positive technology spillovers. One firm's innovation may help other firms innovate. These spillovers are stronger among firms using similar technologies (Bloom et al., 2013). Because firms do not internalize these positive externalities, they inefficiently underinvest in R&D relative to a social planner, who values spillover benefits.

While some firms underinvest in innovation, Bloom et al. (2013) identify a second mechanism that drives overinvestment: business stealing. Innovation improves product quality, allowing innovators to steal market share from competitors. A social planner does not value this transfer of revenue from one firm to a competitor, but privately optimizing firms do. As a result, firms with weak technology spillovers may overinvest in R&D.

Through these mechanisms, a reform that encourages innovation may increase or reduce welfare, depending on whether affected firms have initially over- or under-invested. For example, the welfare effects of a patent-litigation reform depend on which firms' innovation incentives are most sensitive to litigation dynamics. Adding to this ambiguity, it is unclear a priori how litigation reform affects innovation incentives. A defendant-friendly reform may deter innovation if firms fear weak protection against future infringers or boost it if new technologies face less immediate resistance from incumbent patent holders.

As these arguments demonstrate, the welfare effects of a defendant-friendly reform are theoretically ambiguous along two dimensions: whether innovation rises or falls, and whether the responding firms are high- or low-spillover. Our quantitative model resolves both ambiguities, showing that such reforms boost innovation among high-spillover firms and thereby improve welfare. Before turning to the model, we present motivating empirical evidence consistent with this result.

2.2. Data

Before presenting our motivating empirical evidence, we briefly summarize the relevant datasets. For both this motivating exercise and our structural estimation in Section 5, we use the following datasets: (i) Compustat North America Fundamentals Annual, which contains annual data from firm financial statements; (ii) the Global Corporate Patent Dataset (GCPD) (Bena, Ferreira, Matos, and Pires, 2017), which contains detailed information on patents as well as a link between patents and the patent-holding firms in Compustat; (iii) the USPTO litigation database, which provides lawsuit dockets and lists the litigated patents for each patent lawsuit filed over the period 2003 to 2020, (iv) the Audit Analytics corporate litigation database, which provides further information about lawsuit outcomes and a link between litigation parties and Compustat, and (v) the Federal Judicial Center (FJC) civil-lawsuit database, which includes detailed information on lawsuit outcomes for all patent lawsuits filed in federal courts.

We use these data to construct a firm-year panel from 2003 to 2020. Based on the data, we categorize firms into nine technology classes and four primary industry groups, with details regarding the classification procedure in Section A.1 of Internet Appendix A.

For this motivating empirical exercise, we utilize one additional dataset. We obtain the firm-year panel of R&D tax credits constructed by Lucking et al. (2019).[10] This panel contains two variables, lstate and lfirm, which capture the logarithms of state and federal tax credits received by each firm in each year. Following Bloom et al. (2013); Lucking et al. (2019), we use these tax credits as instruments for R&D spending to provide suggestive evidence on over- and under-investment.

2.3. Motivating empirical evidence

Litigation reform primarily affects firms that frequently litigate patents. To identify this type of firm, we construct a firm-level measure of litigation intensity. Following Mezzanotti (2021), we begin with a technology-class-level measure of litigation activity: the number of lawsuits protecting patents in technology class c divided by the number of patents.[11] We then define the firm-level Litigation Intensity as the weighted average of litigation activity across all classes c in which the firm patents, with weights equal to the fraction of the firm's patents that belong to class c. We normalize Litigation Intensity to have a zero mean and unit standard deviation. We split our sample into firm-year observations above and below the median litigation intensity.

Next, we study how the spillovers documented in Bloom et al. (2013) vary across high- and low-litigation-intensity firms. This exercise requires a measure of a firm's exposure to spillovers from its peers' innovation. We construct such a measure following Bloom et al. (2013). We regress the logarithm of R&D spending, Log(R&D), on R&D tax credits (lstate and lfirm), including firm and year fixed effects. The fitted values capture changes in R&D driven by plausibly exogenous variation in tax credits. For each firm-year, we calculate the aggregate tax-credit-driven R&D spending of the firm's technology-class peers, excluding the firm itself. In the spirit of Bloom et al. (2013), we then construct a "stock" of credit-driven R&D as a rolling three-year sum of this leave-one-out aggregate. Finally, we define Instrumented Peer R&D Spending by normalizing this lagged three-year sum to have zero mean and unit

[10] We download this dataset from Nicholas Bloom's website: `https://nbloom.people.stanford.edu/research`. We are grateful to the authors for sharing this dataset.

[11] This exercise counts lawsuits filed *by* patent holders in tech class c because the USPTO data provides a cleaner mapping of patent-litigation lawsuits to plaintiffs than to defendants. However, because patent litigation typically occurs between innovators in similar technology classes, a technology class with many patent-litigation plaintiffs also has many patent-litigation defendants. As such, we likely attribute each lawsuit to the plaintiff's technology class rather than the defendant's.

standard deviation. In sum, Instrumented Peer R&D Spending measures a firm's exposure to credit-driven R&D by its technology-class peers over the prior three years, leaving out its own exposure to the same credits.

Splitting the sample by Litigation Intensity (below versus above the median), we regress Log(R&D) on Instrumented Peer R&D Spending.[12] We include firm and year fixed effects. We present the results in Table 1. As in Bloom et al. (2013); Lucking et al. (2019), we find evidence of positive technology spillovers: firms increase R&D when tax credits induce their peers to do so, even after accounting for the firm's own exposure to the same credits. Moreover, our sample split reveals a new result. Spillovers are stronger for high-litigation-intensity firms—those patenting in frequently litigated technology classes. For these firms, a one-standard-deviation increase in peer R&D increases Log(R&D) by 0.082. In contrast, the effect for low-litigation-intensity firms is smaller and statistically insignificant.

Following Bloom et al. (2013); Lucking et al. (2019), we repeat the analysis using the logarithm of Tobin's Q, Log(Tobin's Q), as the dependent variable.[13] Again, Table 1 shows evidence of technology spillovers. Firm values rise after the innovation of technology-class peers, with spillovers that are stronger for high-litigation-intensity firms. These results suggest that firms in litigation-intensive fields are more likely to underinvest relative to a social planner, because of stronger technology-class spillovers.

As further evidence of underinvestment, we estimate two-stage-least-square (2SLS) regressions of Log(Tobin's Q) on Log(R&D). We instrument for Log(R&D) using each firm's own exposure to tax credits. Table 1 shows that Tobin's Q increases after a tax credit induces a firm to innovate. The improvement in firm value is larger for high-litigation-intensity firms. This finding suggests that these firms have high marginal returns from R&D, consistent with their underinvestment in innovation.

In sum, the above evidence suggests that high-litigation-intensity firms produce socially valuable innovation. Thus, a reform that boosts innovation for this type of firm likely improves welfare. To conclude this section, we replicate the result of Mezzanotti (2021) that defendant-friendly reforms boost innovation for precisely this type of high-litigation-intensity firm. Specifically, Mezzanotti (2021) showed that the pivotal 2006 *eBay v. MercExchange*

[12]Following Bloom et al. (2013); Lucking et al. (2019), we use standard errors that are robust to arbitrary heteroskedasticity and first-order serial correlation using the Newey-West correction.

[13]We define Tobin's Q as in Erickson and Whited (2012).

Supreme Court ruling, which raised the standard for granting an injunction, led high-litigation-intensity firms to innovate more. We replicate this result with the same difference-in-difference approach of Mezzanotti (2021). We regress Log(R&D) on the interaction between Litigation Intensity and an indicator for the period after 2005. We include firm and year fixed effects. Table 1 shows that high-litigation intensity firms increased R&D after this defendant-friendly reform. Our prior regressions suggest that this increase in R&D likely improves welfare, given the large positive technology spillovers of these high-litigation-intensity firms. In the rest of the paper, we confirm this suggestive empirical evidence by estimating our quantitative model.

2.4. Limitations

Our conceptual framework demonstrates the key intuition behind the equilibrium quantitative model that follows. The tradeoff between positive technology spillovers and negative business stealing incentives implies that firms can innovate too much or too little relative to the socially efficient benchmark. Changes in the litigation system can encourage or discourage innovation, and the welfare effects of such changes depend on whether litigating firms are over-innovators or under-innovators. Moreover, our reduced-form evidence suggests that defendant-friendly reforms boost innovation, and the additional innovation by this type of firm improves welfare.

However, this stylized approach has many limitations. First, this conceptual framework does not consider how trial outcomes such as injunctions shape the incentives of firms to settle out of court. This feature is critical because most lawsuits are settled out of court. Second, the framework ignores how incumbent innovation may crowd out new entrants. Because patents create temporary monopolies, to quantify welfare, we must model the effects of these barriers to entry. Third, the framework ignores dynamic considerations. In practice, firms know when they innovate, they might end up using the litigation system to protect their patents in the future. Capturing the potential for an innovator to be a current defendant and a future plaintiff is essential for understanding how litigation shapes innovation. Similarly, while our reduced-form evidence suggests that litigation-exposed firms are efficient innovators, a formal model is necessary for welfare analysis.

To overcome these limitations, we develop a dynamic general equilibrium model with a realistic litigation system featuring asymmetric information and endogenous settlements.

The model captures firms' dual roles as potential future defendants and plaintiffs, along with incumbents' and entrants' incentives. This realism allows us to match key features of our data, so we can map empirical evidence into welfare implications and quantify how changing the litigation system shapes innovation.

3. Model Setup

3.1. Environment and preferences

Time is continuous and denoted by $t \geq 0$. An infinitely-lived representative household has lifetime preferences given by

$$\int_0^\infty e^{-\rho t} \ln C_t dt, \tag{1}$$

where $\rho > 0$ is the discount rate, and C_t denotes consumption of the final good at time t. The household owns all assets A_t in the economy, which deliver a rate of return equal to r_t. It supplies labor $L = 1$ inelastically to firms at the real wage rate w_t.

3.2. Final good production

The final consumption good Y_t is produced competitively using differentiated goods from different industries indexed by $j \in \{1, \ldots, J\}$. The production function is expressed as

$$\ln Y_t = \sum_{j=1}^{J} \omega_j \ln Y_{jt}, \tag{2}$$

where $\omega_j \in (0,1)$ denotes the Cobb-Douglas weight of industry j's output Y_{jt} in production, with $\sum_{j=1}^{J} \omega_j = 1$. The output of each industry j, in turn, is produced by combining a continuum of differentiated goods in said industry according to the production function

$$\ln Y_{jt} = \int_0^1 \ln y_{ijt} di, \tag{3}$$

where y_{ijt} denotes the quantity of differentiated good $i \in [0,1]$ in industry j at time t. The price of the final consumption good is set as the numeraire, and the price of good i in industry j at time t is denoted as p_{ijt}.

3.3. Differentiated good production

As in Klette and Kortum (2004), in an industry, j, each firm owns a portfolio of blueprints to produce various differentiated goods, and multiple firms own blueprints for each differentiated good, i. A blueprint gives a firm the potential to produce. If it produces, it uses labor as an input, with productivity q_{ijt}. Following the Schumpeterian growth literature, we assume Bertrand competition between firms, so only the productivity leader produces any single good in equilibrium. We refer to each good a firm produces as a "product line," which it produces using the production function

$$y_{ijt} = q_{ijt} l_{ijt}, \tag{4}$$

where $l_{ijt} \geq 0$ is the labor that the leader hires for production.

3.4. Firms, technology classes, and product markets

Ignoring the effects of litigation, a firm can become the leader in a new product line by innovating to discover a better technology than the incumbent's. Likewise, a firm can lose its status as the leader if a competitor discovers a better technology. Without legal intervention, this creative destruction leads the prior leader to cede its product line to the innovating competitor, and a firm with no product lines exits.

Departing from the existing Schumpeterian growth literature, we introduce two further dimensions of firm heterogeneity. First, firms fundamentally differ from each other in terms of their innovation process, which we call a technology class. Specifically, each firm has a technology class $c \in \{1, \ldots, C\}$ that determines the productivity improvement from its successful innovations. As discussed in more detail below, the class c affects the knowledge base for developing new blueprints, and this knowledge base shapes the spillovers that enhance the firm's innovation activities. The technology class also determines whether new innovations can infringe upon the intellectual property of other firms.

Although an industry can contain firms with multiple technology classes, firms compete only within their own industry, so they can only obtain the product lines of firms in the same industry. As such, a firm's industry determines the returns to successful innovation from taking over new product lines and the risk of creative destruction from competitors in the same product market.

3.5. Incumbent innovation

Incumbent firms can engage in risky innovation to improve upon existing blueprints and thus potentially expand into new product lines. Each owned product line provides the firm with a lab to generate a Poisson arrival rate of successful innovation $x_{ijt} \geq 0$. Conditional on success, the firm improves upon one of the existing technology leaders' blueprints to produce a differentiated good, chosen randomly among all possible goods in the innovating firm's product market. The product line might or might not be in the same technology class. The productivity of the improved blueprint is given by

$$q_{ijt}^{new} = (1+\lambda_c) q_{ijt}^{old}, \tag{5}$$

where q_{ijt}^{old} is the productivity of the existing leader, and $\lambda_c > 0$ is the step size by which the new innovation improves upon the previous one. The size of λ_c is determined by the technology class c of the innovating firm.

The innovation process is costly. To generate the arrival rate, x_{ijt}, the firm must spend on R&D according to the cost function

$$C_c(x_{ijt}) = \frac{(1-s_{cj})\chi_{cj} x_{ijt}^{\psi} Y_t}{1+\sigma M_{ct}}, \tag{6}$$

where $\chi_{cj} > 0$ is a scale parameter, $\psi > 1$ introduces convexity, $s_{cj} \in [0,1]$ is an industry- and technology-class specific incumbent R&D subsidy rate, and Y_t ensures the R&D costs scale up with aggregate output along a balanced growth path (BGP) equilibrium. The last term, σM_{ct}, $\sigma \geq 0$, captures the knowledge spillovers from other firms in the same technology class c. To define M_{ct}, we let I_{cjt} denote the set of goods i in industry j for which the leader has technology class c, and $\mu_{cjt} \in [0,1]$ denote the measure of the set I_{cjt}. Then $M_{ct} \in [0,1]$ is

$$M_{ct} = \sum_{j=1}^{J} \omega_j \mu_{cjt}. \tag{7}$$

This expression is the fraction of all product lines in the economy currently owned by firms with technology class c, where different industries receive weight in proportion to their Cobb-Douglas share in final good production. The higher the value of M_{ct} is, the cheaper it is for all firms in technology class c to discover new ideas, so past successful innovation by other firms in the same technology class increases a firm's research efficiency. The strength of this technology-class-specific knowledge spillover is governed by the parameter $\sigma \geq 0$, with a higher value of σ indicating stronger knowledge spillovers within the same technology class.

This technology-class-specific knowledge spillover complements the inherent Schumpeterian knowledge spillovers, which arise from enhancing the productivity of the current leader, as in equation (5). As such, our model includes *within-industry* knowledge spillovers that occur both *within* and *across* technology classes, as well as *within-technology-class* spillovers that span both *within* and *across* industries.

3.6. Entrant innovation

There is a measure-one continuum of identical entrepreneurs that can found new businesses through successful innovation. We use "entrepreneur" and "entrant" interchangeably. Entrants spend on R&D, which allows them to generate a Poisson arrival rate of successful innovation $z_t \geq 0$. The R&D cost function is

$$C_e(z_t) = (1 - s_e)\nu z_t^{\psi} Y_t, \tag{8}$$

where $\nu > 0$ is a scale parameter and $s_e \in [0, 1]$ is the entrant R&D subsidy rate. As is the case for incumbent innovation, $\psi > 1$ is the convexity parameter, and the term Y_t ensures that R&D costs scale up with aggregate output along a BGP equilibrium.

If the entrepreneur's innovation is successful, it forms a new firm. With probability $\eta_{cj} \in [0, 1]$, the new firm is in technology class c and industry j. For all c and j, the probabilities η_{cj} are exogenous parameters that satisfy $\sum_{c=1}^{C} \sum_{j=1}^{J} \eta_{cj} = 1$. The new firm with a single product line is immediately sold off at fair market value by the successful entrepreneur, who remains an entrepreneur and continues to found new businesses.

We treat η_{cj} as exogenous parameters, a reduced-form approach that captures entry barriers, specialized human capital requirements, and regulatory constraints that vary across technology-industry pairs and are difficult for entrepreneurs to circumvent regardless of profit opportunities. Empirically, entry patterns exhibit substantial dispersion across (c, j) cells that profit differences alone cannot explain, suggesting that factors beyond expected returns constrain where entrepreneurs can effectively enter. Conceptually, our approach is consistent with a richer model in which entrepreneurs select (c, j) to maximize expected profits plus idiosyncratic preference or cost shocks; under Type-I extreme value shocks, the resulting entry probabilities take a logit form that can be calibrated to match any observed entry distribution, yielding equilibrium outcomes identical to ours. Our specification has two practical advantages: it transparently reveals which parameters are identified from entry data, and it avoids solving a discrete choice problem at each estimation iteration.

3.7. Patent infringement and litigation

When either an existing firm or an entrant successfully innovates, it creates litigation risk because its new innovation might infringe upon the intellectual property of existing firms in the same technology class. We consider two types of potential patent infringement.

Type-1 infringement occurs when an innovator (either an entrepreneur or an existing firm) attempts to take a product line from an incumbent firm and infringes on the incumbent's patent in the process. We assume that the incumbent must have (i) the same technology class as the innovator, so that the patent overlaps with the newly innovated technology, and (ii) the same industry as the innovator, because innovating firms can only take product lines from firms in the same industry. If the incumbent successfully sues the innovating firm, the incumbent can avoid losing its product line to the innovating firm. Conditional on successful innovation and the incumbent and innovator sharing a technology class, type-1 infringement occurs with exogenous probability $\kappa_{1cj} \in [0,1]$ where c and j refer to the innovating firm's technology class and industry, respectively.

An example of type-1 infringement occurred in 2010 when Motorola filed several patent-infringement lawsuits against Apple. The patents related to technologies used in smartphones, which both Motorola and Apple produced. Motorola sought injunctions to prevent Apple from using these technologies to produce the iPhone and similar products. This example is type-1 because Motorola and Apple directly competed in the market for smartphones.[14]

Type-2 infringement occurs when an innovator tries to take a product line from an incumbent firm and infringes on a *third party*'s patent in the process. The patent holder shares a technology class with the innovator, so the patent overlaps with the new invention. The exact infringed patent is randomly chosen from all product lines in the innovator's technology class, including those in other industries. In contrast to type-1 infringement, the plaintiff in a type-2 lawsuit is not the incumbent who owns the product line being taken. The type-2 plaintiff has no direct stake in whether the innovator or the incumbent owns the product line. However, the plaintiff can extract rents by suing to obtain a possible settlement, as a successful lawsuit can block the innovator's product line capture. Conditional on successful innovation and the incumbent and innovator having different technology classes, type-2 infringement occurs with exogenous probability $\kappa_{2cj} \in [0,1]$ where c and j refer to the innovating firm's technology class and industry, respectively.

An example of type-2 infringement occurred in 2003, when AT&T sued eBay for patent infringement. AT&T, a telecommunications company, had patented a system for secure online payment. eBay, an e-commerce company, owned the PayPal payment system that it used

[14] See, for example, 1:10-cv-23580 in Florida southern district court and https://www.wsj.com/articles/SB10001424052748703735804575536230822496028.

for processing payments. AT&T sought an injunction, claiming that eBay's PayPal system infringed on AT&T's patent. This example is type-2 because eBay and AT&T operated in distinct industries but had sufficient technological overlap for patent infringement to occur.[15]

After either type of infringement, litigation potentially ensues. We model the litigation subgame as follows, with its timeline illustrated in Figure 1. First, the plaintiff decides whether to hire a legal team. The cost of hiring a legal team is γY_t, where $\gamma > 0$ is drawn randomly from the distribution $\Gamma_{cj}(\gamma)$, where c and j refer to the defendant's technology class and industry, respectively, and Y_t ensures that litigation costs grow at the same rate as output in a BGP equilibrium. If the plaintiff chooses not to hire a legal team, the lawsuit is dropped, and the defendant gets to take over the product line. However, if the plaintiff hires a legal team, it makes a take-it-or-leave-it out-of-court settlement offer to the defendant.

Defendants also incur legal costs conditional on going to a trial. Regardless of whether they win or lose, the defendant must pay a fraction $\Lambda_1 \in [0,1]$ or $\Lambda_2 \in [0,1]$ of the expected payoff from taking over the product line as legal fees, for type 1 and type 2 infringements, respectively.

The defendant has private information about its probability of winning the trial, τ, which is drawn from a uniform distribution with endpoints $(\tau^l_{1cj}, \tau^h_{1cj})$ or $(\tau^l_{2cj}, \tau^h_{2cj})$, with $\tau^l_{1cj}, \tau^h_{1cj}, \tau^l_{2cj}, \tau^h_{2cj} \in [0,1]$, for type-1 and type-2 infringements by defendants with technology class c and industry j, respectively. Based on its private information τ, the defendant can accept the settlement or refuse. Refusal leads to a trial.

With probability τ, the defendant wins the trial and takes over the product line. With probability $1-\tau$, the defendant loses and the court decides whether to grant an injunction. With probability $\zeta_{1cj} \in [0,1]$, an injunction is granted for a type-1 infringement by a defendant with technology class c and industry j, thus blocking the product line takeover. With probability $1-\zeta_{1cj}$, no injunction occurs, and the defendant takes over the product line. The equivalent probability is denoted $\zeta_{2cj} \in [0,1]$ for type-2 infringements. Below, we examine changes to ζ_{1cj} and ζ_{2cj}, as these parameters capture the inclination of a court to grant an injunction in the case of a proven patent infringement.

[15] See 1:03-cv-01051 in Delaware district court and https://www.wired.com/2003/11/att-sues-ebay-in-patent-dispute/.

4. Model Solution

To solve the model, we calculate a BGP equilibrium with the following features:

1. All agents act optimally given the equilibrium behavior of other agents, the constant real interest rate r, and the constant rate g at which the aggregate economy grows.
2. The solution to the household's consumption-saving problem determines r.
3. Each firm's policies solve a dynamic optimization in which the key state variable is the number of product lines that the firm produces.
4. Whenever patent infringement occurs, a subgame perfect equilibrium of a litigation subgame determines the outcome of the infringement.
5. The equilibrium policies of incumbents, entrants, and the household determine the growth rate g.

Features 3 and 4 imply that each firm chooses its innovation policy based on its rational expectation of future litigation activity. We now summarize each piece of the equilibrium. For ease of exposition, we delegate formal derivations to Internet Appendix B.

4.1. Household's problem

The representative household solves a consumption-savings problem. Specifically, given initial assets A_0, the representative household chooses its consumption C_t in each instant to maximize its lifetime utility

$$\max_{[C_t, A_t]_{t=0}^{\infty}} \left\{ \int_0^{\infty} e^{-\rho t} \ln C_t dt \right\}, \quad \text{subject to} \tag{9}$$

$$\dot{A}_t = r_t A_t + w_t - C_t, \quad \forall t \geq 0, \tag{10}$$

where $\dot{A}_t$ denotes the asset growth rate dA_t/dt. The household takes the wage rate, w_t, and the real interest rate, r_t, as given and faces a standard tradeoff. It is impatient and prefers to consume early, but doing so hinders the growth of its assets A_t. Given this tradeoff, the household chooses a consumption process. The household's condition for optimality delivers the Euler equation $\frac{\dot{C}_t}{C_t} = r_t - \rho$, which implies that the growth rate of C_t equals $r_t - \rho$. In a BGP equilibrium, C_t must grow at the constant equilibrium rate g, so $r_t = r = \rho + g$.

4.2. Final good producer's problem

Given equations (2) and (3), a competitive final good producer solves a static profit maximization problem at each instant t:

$$\max_{\left\{[y_{ijt}]_{i=0}^{1}\right\}_{j=1}^{J}} \left\{ \exp\left(\sum_{j=1}^{J} \omega_j \left(\int_0^1 \ln y_{ijt} di \right) \right) - \sum_{j=1}^{J} \left(\int_0^1 p_{ijt} y_{ijt} di \right) \right\}. \tag{11}$$

The final good producer chooses the quantity of each input y_{ijt} trading off (i) the marginal output it can produce by using another unit of y_{ijt} against (ii) the marginal cost p_{ijt} of purchasing the additional unit. For any good i in industry j, the first-order condition delivers $y_{ijt} = \left(\omega_j Y_t / p_{ijt}\right)$. This expression pins down the demand for y_{ijt} as a function of the price p_{ijt} charged by the owner of product line i.

4.3. Product-line owner's static pricing problem

Each incumbent firm solves a dynamic optimization described in the next section, but its pricing decisions are static and can be solved independently at each instant t. Under Bertrand competition, only the technology leader produces a positive quantity of good i. This leader has productivity q_{ijt}, while the second-most-productive firm has productivity $q^{old} = q_{ijt}/\left(1+\lambda_c\right)$, where c is the leader's technology class. This productivity gap exists because the leader improved upon the second-most-productive firm's technology by a factor of $1+\lambda_c$. In Bertrand competition, the product-line leader charges a price, p_{ijt}, that would leave the second-most-productive firm with zero profit if it charged p_{ijt}. This price makes the second-most-productive firm (and all other firms) forgo production of good i because they cannot profitably compete. The product-line owner can nonetheless charge p_{ijt} and make a profit due to its unparalleled productivity.

Formally, in Internet Appendix B, we show that the product-line leader optimally charges $p_{ijt} = w_t(1+\lambda_c)/q_{ijt}$. This "limit price" is the highest price that discourages less productive competitors from paying employees the wage rate w_t to produce good i. We also show that the product-line leader makes profit flow $\pi_{ijt}dt$ by charging this price, where

$$\pi_{ijt} = \frac{\lambda_c}{1+\lambda_c} \omega_j Y_t. \tag{12}$$

From this expression, we see that π_{ijt}: (i) grows at the same rate as aggregate output, Y_t, (ii) is linearly related to the industry j's share ω_j, and (iii) is increasing in the technology class-specific productivity step size λ_c, which is also the net markup. Note that π_{ijt} is independent of the productivity q_{ijt}. Intuitively, if q_{ijt} is high, then the product-line owner has a highly productive competitor because $q^{old} = q_{ijt}/(1+\lambda_c)$ is also high, so the owner must charge a low price that cancels out the potential benefits of high productivity. This property of π_{ijt} implies that the relevant state variable for an incumbent firm's dynamic problem is not the set of productivities of owned product lines, but simply the number of them, as in Klette and Kortum (2004).

4.4. Incumbent's dynamic optimization problem

We now summarize the incumbent firm's dynamic optimization problem. Let $V_{cjt}(n)$ be the value of an incumbent firm in technology class c and industry j that owns n product lines at time t. It is the net present value of future cash flows associated with product-line profits, R&D expenses, and litigation activity. Formally, the Hamilton-Jacobi-Bellman (HJB) equation characterizing the incumbent's value function and optimization problem is

$$
\begin{aligned}
r_t V_{cjt}(n) - \dot{V}_{cjt}(n) \quad = \quad & \max_{\{x_{mcjt}\}_{m=1}^{n}} \Bigg\{ \underbrace{\sum_{m=1}^{n} \frac{\lambda_c}{1+\lambda_c}\omega_j Y_t}_{\text{static profit flow}} - \underbrace{\sum_{m=1}^{n} \frac{(1-s_{cj})\chi_{cj} x_{mcjt}^{\psi} Y_t}{1+\sigma M_{ct}}}_{\text{R\&D expenses}} + \underbrace{n \sum_{j'=1}^{J} R_{cj't}}_{\text{litigation rent}} \\
& + \underbrace{\left(\sum_{m=1}^{n} x_{mcjt}\right)}_{\text{successful innov.}} \times \underbrace{\left(V_{cjt}^{+}(n) - V_{cjt}(n)\right)}_{\mathbb{E}(\Delta V|\text{successful innov.})} + \underbrace{n d_{jt}}_{\text{creative destruc.}} \times \underbrace{\left(V_{cjt}^{-}(n) - V_{cjt}(n)\right)}_{\mathbb{E}(\Delta V|\text{other innov.})} \\
& + \underbrace{\delta\left(0 - V_{cjt}(n)\right)}_{\text{exogenous exit}\times\Delta V} \Bigg\}. \qquad (13)
\end{aligned}
$$

Equation (13) offers a great deal of intuition. Because each product line m owned by the incumbent comes with a lab for innovation, at each instant t, the incumbent chooses a level of innovation x_{mcjt} for each lab to maximize its net present value. The first line of equation (13) captures the portion of this value that comes from immediate flow profits, which contain three components. The first term on the right side captures the profits from owning n product lines, from equation (12). The second term is R&D expenses, from equation (6). The third term captures the rents that the incumbent firm can extract through type-2 litigation, which we calculate in Section B.6 in Internet Appendix B. When the incumbent firm holds the patent in a type-2 infringement case, it can sue the innovating firm to extract rents. Because the innovating firm can be in any industry, these potential rents are aggregated across all industries. We denote by R_{cjt} the total rent flows from infringements by firms in industry j for a single product line, which we then multiply by the number of product lines n.

The second line in equation (13) captures changes in value when the firm gains or loses product lines. The first term represents the expected gain from new product lines as the Poisson arrival rate of a successful innovation multiplied by the expected value improvement from innovation. To understand this gain, we must account for patent litigation. Absent litigation, a successful innovation would increase the firm's net present value from $V_{cjt}(n)$ to $V_{cjt}(n+1)$. However, patent litigation implies that successful innovation does not necessarily result in a product line. Therefore, we define $V^+_{cjt}(n)$ as the expected value of a firm conditional on successful innovation, but before potential patent infringement and litigation outcomes are realized. Therefore, successful innovation increases the firm's expected value by $V^+_{cjt}(n) - V_{cjt}(n)$. We describe the litigation subgame that determines $V^+_{cjt}(n)$ in the following section.

The second term captures creative destruction, that is, the loss of product lines when competitors innovate. This term represents this loss as the number of product lines, n, times the rate of creative destruction, d_{jt}, times the incumbent's expected loss in value from a potential product line takeover, $V_{cjt}(n)$ to $V^-_{cjt}(n)$, where $V^-_{cjt}(n)$ is the incumbent's expected value immediately before potential litigation to block the new innovation. We describe the litigation subgame that determines $V^-_{cjt}(n)$ in the following section. In Internet Appendix B, we calculate the "creative destruction rate" d_{jt}, which measures the rate at which competing firms innovate on one of the incumbent firm's product lines. Each of the n product lines owned by the firm faces this displacement risk.

Finally, the third line includes the risk of exogenous firm exit at a rate $\delta \geq 0$, which captures firm exit events for reasons other than losing all product lines. When a firm exogenously exits, it is replaced by an identical firm that inherits its product lines.

To characterize the value function, $V_{cjt}(n)$, fully, we must find the equilibrium objects M_{ct} (equation (7)), $R_{cjt}, d_{jt}, V^+_{cjt}(n)$, and $V^-_{cjt}(n)$, which we calculate in Section B.6 and Section B.7 of Internet Appendix B. However, we can make a few observations without these calculations. First, each product line has the same flow profit, so the total flow profit is linear in the number of product lines n. Second, the first-order condition with respect to the innovation rate x_{mcjt} for any lab m implies the following optimal innovation policy

$$x_{mcjt} = \left(\frac{\left(V^+_{cjt}(n) - V_{cjt}(n)\right)(1+\sigma M_{ct})}{(1-s_{cj})\chi_{cj}\psi Y_t} \right)^{\frac{1}{\psi-1}} \equiv x_{cjt}(n). \tag{14}$$

This expression implies that the firm chooses the same innovation rate, $x_{cjt}(n)$, for each of its n labs. Accordingly, the total R&D expense and the firm-level arrival rate of successful innovation are also linear in the number of product lines n. As we prove in Internet Appendix B, these properties imply that the firm value function $V_{cjt}(n)$ itself is linear in n. Formally, $V_{cjt}(n) = v_{cj} n Y_t$ for coefficients v_{cj} that we calculate in closed form.

Equation (14) also provides helpful intuition about firm innovation incentives. As technology spillovers σM_{ct} grow, R&D becomes cheaper, so firms innovate more. Similarly, subsidies, s_{cj}, lead to more innovation. If a defendant-friendly litigation system makes it unlikely that an innovating firm's invention will be blocked, then the return to innovating, $V^+_{cjt}(n) - V_{cjt}(n)$ is high, and firms innovate more. As such, defendant-friendly reforms can potentially boost innovation. However, a plaintiff-friendly system could also boost innovation in two ways. It raises the rents, R_{cjt}, that a firm can extract by suing other firms for patent infringement, and it helps the firm use patent protection to fend off competitors and retain ownership of its own product lines. Put differently, a defendant-friendly system reduces the probability of forfeiting a successful innovation but also lowers the value of the product line because of reduced intellectual property protection. Our estimation allows us to determine quantitatively which of these countervailing forces dominates.

4.5. Entrepreneur's problem

Next, we characterize the entrant's optimization problem, which is a simplified version of the incumbent's problem. It is static because, conditional on success, the entrepreneur sells the new firm immediately at fair market value. Specifically, in each instant t, the entrepreneur solves

$$\max_{z_t \geq 0} \left\{ -(1-s_e)\nu z_t^{\psi} Y_t + z_t \sum_{c=1}^{C} \sum_{j=1}^{J} \eta_{cj} V^+_{cjt}(0) \right\}. \tag{15}$$

The first term in the maximization is the R&D cost incurred by the entrepreneur, where $s_e \in [0,1]$ is the entrant R&D subsidy rate. The second term is the expected return from entrant innovation. The Poisson arrival rate of successful innovation is z_t. Conditional on success, the new firm has technology class c and industry j with probability η_{cj}.

If there were no litigation, the value of the new firm would be $V_{cjt}(1)$. However, due to litigation risk, the new firm's value equals the expected value of an incumbent firm with zero existing product lines that succeeded in innovation, but before potential patent infringement and consequent litigation outcomes are realized, denoted $V^{+}_{cjt}(0)$.[16]

The first-order condition with respect to entrant innovation z_t in equation (15) pins down its optimal value as

$$z_t \quad = \quad \left(\frac{\sum_{c=1}^{C} \sum_{j=1}^{J} \eta_{cj} V^{+}_{cjt}(0)}{(1-s_e)\nu\psi Y_t} \right)^{\frac{1}{\psi-1}} . \tag{16}$$

As with optimal incumbent innovation, both subsidies and defendant-friendly litigation systems raise innovation. However, entrants can still benefit from plaintiff-friendly systems through changes in product-line value, which, as shown in equation (20) below, depends on the litigation system.

4.6. Litigation and settlements

We now describe the solution to the litigation subgame that ensues after successful innovation. In a BGP equilibrium, a subgame perfect equilibrium (SPE) of the litigation subgame determines the litigation outcome. In Internet Appendix B, we calculate the SPE strategies in closed form and use them to calculate the pre-litigation value functions $V^{+}_{cjt}(n)$ and $V^{-}_{cjt}(n)$. Here, we outline the solution and provide intuition.

Defendant's decision to go to trial: We start with the defendant's decision to go to trial. Its expected payoff from a trial is given by

$$[(\tau + (1-\tau)(1-\zeta)) - \Lambda]\left(V_{cjt}(n+1) - V_{cjt}(n)\right), \tag{17}$$

where $\zeta \in \{\zeta_{1cj}, \zeta_{2cj}\}$ and $\Lambda \in \{\Lambda_1, \Lambda_2\}$ depend on the infringement type. $V_{cjt}(n)$ is the value function for a firm with n product lines, technology class c, and industry j. The first term in the first factor, $(\tau + (1-\tau)(1-\zeta))$, is the defendant's probability of taking over the product line, either by winning the trial or having the court decide not to grant an injunction despite recognizing the infringement. The second term in the first factor is the fraction $\Lambda \in [0,1]$ of the defendant's payoff paid to their legal team, conditional on going to court. The second factor is the defendant's value improvement from gaining an additional product line.

The defendant's alternative to a trial is to accept the plaintiff's settlement offer, s, in which case, the defendant must pay s to the plaintiff. This choice yields a payoff of $V_{cjt}(n+1) - V_{cjt}(n) - s$. The defendant accepts the offer if this payoff is higher than the trial payoff in equation (17). In Internet Appendix B, we calculate a cutoff, $\bar{s}_{cjt}(n,\tau)$, such that

[16]Note that $V^{+}_{cjt}(0) = V^{+}_{cjt}(0) - V_{cjt}(0)$ since $V_{cjt}(0) = 0$.

the defendant accepts if and only if $s \leq \bar{s}_{cjt}(n,\tau)$. This cutoff decreases with the defendant's probability of trial victory, τ, which the plaintiff does not observe.

Plaintiff's choice of settlement offer: We now move backward and examine the plaintiff's choice of a take-it-or-leave-it settlement offer s. The plaintiff's choice of s varies across infringement types.

We first consider settlement offers in the simpler case of type-2 patent infringement. Recall that in type-2 infringement, the plaintiff is not at risk of losing a product line. The plaintiff's payoff is, therefore, equal to the settlement offer of s if the defendant accepts or 0 if the defendant rejects and goes to trial. The plaintiff's problem is thus

$$\max_{s \geq 0} \left\{ s \times \mathbb{P}(s \leq \bar{s}_{cjt}(n,\tau)) \right\}, \tag{18}$$

where the second component is the probability that the offer is accepted. The plaintiff faces a tradeoff: a higher settlement offer of s leads to a higher profit if the defendant accepts. However, a higher settlement offer is less likely to be accepted. Importantly, the plaintiff does not know the defendant's probability τ of winning a trial, so the plaintiff does not know whether a settlement offer will be accepted. In Internet Appendix B, we calculate the optimal settlement offer s^* that solves this problem.

To ensure a solution s^* such that $\mathbb{P}(s \leq \bar{s}_{cjt}(n,\tau)) \in (0,1)$, for type-2 infringements we assume that the endpoints of the uniform distribution of τ satisfy $1+\tau^l_{2cj}+\Lambda_2/\zeta_{2cj} \leq 2\tau^h_{2cj}$. This assumption implies that settlement offers are sometimes rejected and sometimes accepted in equilibrium.

Next, we consider settlement offers in type-1 patent infringement. This problem differs from type-2 infringement because the plaintiff owns the product line facing possible creative destruction. As such, because settling out of court implies losing its product line, the plaintiff requires a higher value for the settlement than it would in the case of a type-2 infringement. Thus, the plaintiff chooses a settlement offer, s, to solve the following problem

$$\begin{aligned}\max_{s\geq 0} \quad \Bigg\{ & \int_{\tau^l_{1cj}}^{1-s/(\zeta_{1cj}(V_{cjt}(n^d+1)-V_{cjt}(n^d)))+\Lambda_1/\zeta_{1cj}} \left(V_{cjt}(n^p-1)-V_{cjt}(n^p)+s\right)\frac{1}{\tau^h_{1cj}-\tau^l_{1cj}}d\tau \\ & + \int_{1-s/(\zeta_{1cj}(V_{cjt}(n^d+1)-V_{cjt}(n^d)))+\Lambda_1/\zeta_{1cj}}^{\tau^h_{1cj}} (\tau+(1-\tau)(1-\zeta_{1cj}))(V_{cjt}(n^p-1)-V_{cjt}(n^p))\frac{1}{\tau^h_{1cj}-\tau^l_{1cj}}d\tau\Bigg\},\end{aligned} \tag{19}$$

where n^d and n^p are the numbers of product lines held by the defendant and plaintiff, respectively. The term $V_{cjt}(n^p - 1) - V_{cjt}(n^p)$ is negative and reflects the plaintiff's cost of losing the product line. The defendant's value function $V_{cjt}(n^d)$ appears in the integral bounds because of the defendant's subsequent choice of whether to accept the settlement offer. Thus, the first integral represents the value to the plaintiff (the settlement amount minus lost product line value) over the range of defendants who accept. The second integral gives the expected value when settlement is rejected: the probability of losing at trial or lacking injunction protection, multiplied by the loss in value from giving up the product line.

Plaintiff's choice of whether to hire a legal team: Finally, we move backward in time and characterize the plaintiff's decision to hire a legal team. Recall that the plaintiff observes a stochastic cost γY_t of hiring a legal team to pursue a lawsuit. The plaintiff hires a legal team if its expected value from proceeding to the next stage (making a settlement offer) exceeds γY_t. In Internet Appendix B, we calculate equilibrium thresholds for type-1 and type-2 infringement such that the plaintiff hires a legal team if and only if γ is below the respective threshold. Given these thresholds, we can calculate the equilibrium probability that the plaintiff files a lawsuit after an infringement.

In Internet Appendix B, we solve for the SPE and calculate all equilibrium objects in closed form. Proposition 1 in Section B.2 of the Internet Appendix B describes the SPE of the type-2-infringement game. Proposition 2 in Section B.3 of the Internet Appendix B describes the SPE of the type-1-infringement game when $\Lambda_1 = 0$. Notably, when $\Lambda_1 = 0$, we prove that the model solution depends on the average probability of defendant victory in type-1 infringement, $\bar{\tau}_{1cj} \equiv (\tau^h_{1cj} + \tau^l_{1cj})/2$, rather than the bounds $\tau^h_{1cj}, \tau^l_{1cj}$. Motivated by this result, for the remainder of the text, we refer only to $\bar{\tau}_{1cj}$ rather than the bounds when discussing parameters related to type-1 infringement. Finally, Proposition 3 in Section B.4 of Appendix B describes the SPE of the type-1-infringement game when $\Lambda_1 > 0$.

The SPE continuation values are influenced by the firm value function $V_{cjt}(n)$ (which establishes the stakes for all parties involved). The continuation values in turn determine the values R_{cjt}, $V^+_{cjt}(n)$, and $V^-_{cjt}(n)$ that appear in the HJB equation. In this way, the litigation equilibrium influences innovation incentives.

4.7. Completing the equilibrium

The previous sections summarized each agent's optimization problem, the SPE of the litigation subgame, and the determination of the real interest rate, r, based on the household's problem. We now summarize how we verify our conjectures and complete the model solution. We focus on the intuition and relegate the formal definition of the BGP equilibrium and proofs to Section B.6 of Internet Appendix B.

Firm value and innovation: We first confirm that the firm's value function $V_{cjt}(n)$ depends linearly on the number of product lines n, the value of a product line v_{cj}, and scales with aggregate output Y_t. Formally, $V_{cjt}(n) = v_{cj} n Y_t$. In equilibrium, the value of product line v_{cj} is given by

$$v_{cj} = \frac{\frac{\lambda_c}{1+\lambda_c}\omega_j + \sum_{j'=1}^{J} \hat{R}_{cj'} - \frac{(1-s_{cj})\chi_{cj} x_{cj}^{\psi}}{1+\sigma M_c}}{\rho + \delta - x_{cj} L_{cj}^{\text{def}} + d_j L_{cj}^{\text{plain}}}, \tag{20}$$

where L_{cj}^{def} and L_{cj}^{plain}, defined precisely below, represent the share of a new product line's value the firm retains from a successful innovation (after accounting for the risk of infringing on another firm's patent), and the share of an existing product line's value the firm loses to a competitor's innovation (after accounting for the protection it has as a plaintiff).

Intuitively, equation (20) resembles the value of a growing perpetuity. The numerator captures the cash flow that the firm receives from owning a product line. The first term, $\frac{\lambda_c}{1+\lambda_c}\omega_j$, is the firm's profits from selling its product. The second term is the rents that the firm extracts by suing other firms for type-2 patent infringement, where $\hat{R}_{cj} \equiv R_{cjt}/Y_t$ is the descaled flow of litigation rents. The third term is the R&D costs for the lab associated with the product line. Thus, holding all else fixed, a product line becomes more valuable when a change in parameters either increases profits, increases litigation rents, or reduces R&D expenses.

The denominator in equation (20) captures the *effective discount rate*. A higher discount rate, $\rho = r - g$, or exogenous exit rate, δ, reduces the present value of future cash flows. As a departure from the standard growing perpetuity formula, in our model, innovation and litigation not only influence the cash flows in the numerator but also affect the likelihood of the firm securing these cash flows, thereby altering the effective discount rate.

First, higher innovation, x_{cj}, increases the likelihood of securing the product line, thus reducing the effective discount rate. This effect is amplified by lower infringement losses, which are captured by L_{cj}^{def}. Formally, we define L_{cj}^{def} as

$$V_{cjt}^{+}(n) - V_{cjt}(n) \equiv L_{cj}^{\text{def}}\big(V_{cjt}(n+1) - V_{cjt}(n)\big), \tag{21}$$

where we calculate L_{cj}^{def} in closed form in equation (B.60) in Internet Appendix B. The left-hand side captures the change in the expected value of the firm after its own successful innovation. In the absence of patent infringement, a successful innovation allows the firm to gain a product line with certainty ($L_{cj}^{\text{def}} = 1$), and the number of product lines it owns rises from n to $n+1$. Thus, $1 - L_{cj}^{\text{def}}$ represents the proportion of innovation value lost because of the risk of infringing on other firms' intellectual property.

The final component in the denominator of equation (20) captures the impact of creative destruction on the firm's effective discount rate. Recall that d_j is the rate at which an incumbent in industry j faces a threat to its product line due to a competitor's innovation. When this creative destruction rate is high, the incumbent has a high risk of its product line being taken, increasing the effective discount rate. This effect is amplified by weaker IP protection, which is captured by L_{cj}^{plain}. Formally, we define L_{cj}^{plain} through the equation

$$V_{cjt}^{-}(n) - V_{cjt}(n) \equiv L_{cj}^{\text{plain}}\big(V_{cjt}(n-1) - V_{cjt}(n)\big), \tag{22}$$

where we calculate L_{cj}^{plain} in closed form in equation (B.63) in Internet Appendix B.

This expression is analogous to equation (21). The left-hand side captures the change in the expected value of the firm after its competitors' successful innovation. Without intellectual property protection, when competitors successfully innovate, the firm loses the product line with certainty ($L_{cj}^{\text{plain}} = 1$), and the number of product lines it owns falls from n to $n-1$. Therefore, $1 - L_{cj}^{\text{plain}}$ reflects the value gained from using patent protection to fend off competitors and retain ownership of the product line.

In summary, the value v_{cj} of a product line depends on cash flows (profits, R&D expense, and litigation rent extraction) and the effective discount rate, which reflects equilibrium outcomes, such as the growth of the economy and firms' litigation and innovation policies. Given this value, we prove that the equilibrium per-product-line incumbent innovation arrival rate is

$$x_{cj} \quad = \quad \left(\frac{L_{cj}^{\text{def}} v_{cj}(1+\sigma M_c)}{(1-s_{cj})\chi_{cj}\psi}\right)^{\frac{1}{\psi-1}}. \tag{23}$$

Equation (23) indicates that incumbent innovation x_{cj} rises with the value of a product line, as higher expected returns incentivize firms to invest more in innovation. In contrast, higher value losses from infringement (lower L_{cj}^{def}) reduce innovation. Further details are in Internet Appendix B, which includes the derivation of the entrant's optimal innovation decision.

Legal reforms affect firm value and innovation through multiple terms in equations (20) and (23), which capture effects on both cash flows and discount rates. Because these channels can work in opposing directions, a reform's impact on firm and aggregate outcomes is *ex ante* ambiguous.

Product line and firm distribution: Next, we calculate the stationary equilibrium distribution of product lines across industries and technology classes. Formally, we calculate the stationary measure μ_{cj} of all product lines in industry j for which the leader has technology class c. These measures sum to one for each industry, $\sum_{c=1}^{C} \mu_{cj} = 1$, and they depend on the relative equilibrium levels of innovation across technology classes in equation (14).

While not needed to compute the BGP equilibrium, we can also compute the stationary firm size distributions $\varphi_{cj}(n)$ for firms of type (c, j). The details of their derivation are relegated to Section B.8 of Internet Appendix B.

Growth rate: For each technology class c and industry j, we calculate the contribution to aggregate growth made by product lines in that class and industry. We let f_{cj} denote this contribution, which depends on three factors: the probabilities of different types of innovations landing on product lines in class c and industry j, the patent infringement probabilities for these product lines, and the outcomes of the ensuing litigation subgames. We provide details in Internet Appendix B (equation (B.77)). We use these equilibrium objects to calculate the equilibrium growth rate g as

$$g = \sum_{j=1}^{J} \omega_j \sum_{c=1}^{C} \mu_{cj} f_{cj}. \tag{24}$$

Given these closed-form solutions, it is straightforward to solve the model numerically. We simply iterate between calculating the value function, the equilibrium growth rate, and other model objects until all model objects are mutually consistent. The closed-form solutions make this process remarkably fast, aiding our estimation.

4.8. Output and welfare

Finally, we calculate social welfare, which is useful for comparing our estimated equilibrium to counterfactual economies. First, we need to compute the consumption stream of the representative household. From the utility function of the representative household in equation (1), we have

$$W = \int_0^\infty e^{-\rho t} \ln C_t dt = \int_0^\infty e^{-\rho t} \ln(e^{gt} C_0) dt = \frac{\ln C_0}{\rho} + \frac{g}{\rho^2}. \quad (25)$$

This expression shows how welfare depends on the initial level of consumption C_0 and the growth rate of the economy g. Intuitively, welfare tends to be higher when the growth rate of the economy is higher. However, if a much higher fraction of output goes to R&D or litigation expenses in the high-growth economy, then C_0 is lower, so welfare can be lower in the high-growth economy. To compute C_0, we need to calculate the initial output level, Y_0, and the fraction of output spent on R&D and litigation by all firms in the economy. The details are relegated to Section B.9 in Internet Appendix B.

Finally, for two economies A and B, we can define a consumption equivalent welfare change (CEWC) measure ϖ, which is the percentage increase in lifetime consumption that an agent in economy A would need to be indifferent between being in economy A or B

$$W_B = \frac{\ln(C_0^A(1+\varpi))}{\rho} + \frac{g^A}{\rho^2}. \quad (26)$$

Solving for ϖ, we get

$$\varpi = \exp\left(\left(W_B - \frac{g^A}{\rho^2}\right)\rho - \ln(C_0^A)\right) - 1. \quad (27)$$

5. Estimation and Identification

5.1. Parameterizing the model

To map the model to the data, we add some parametric assumptions. First, we assume the litigation cost γ is drawn from an exponential distribution with parameter ξ_{cj}, with CDF $1 - e^{-\xi_{cj}\gamma}$. Second, we use a combination of model parameters and empirical patterns to determine the distribution of $\lambda_c, \{\bar{\tau}_{1cj}, \tau^l_{2cj}, \tau^h_{2cj}\}$, and χ_{cj} across technology classes and industries. While we use data to calibrate the shape of each distribution across $c-j$ bins, the mean and standard deviation of each distribution are ultimately determined by our generalized method of moments (GMM) estimation described below. We provide details in Internet Appendix A.

Additionally, we set a few parameters based on literature conventions. The discount rate ρ is set to 0.04, which implies a real interest rate of 6% when the growth rate is 2%. The R&D subsidy s_{cj} is set to 8%, the implied tax subsidy rate on R&D expenditures in the US from the OECD database. The exit rate δ is set to 3% following Acemoglu et al. (2018). For the baseline estimation, we set $\Lambda_1 = \Lambda_2 = 0$, and investigate defendant litigation costs in a dedicated section in Section C.2 of the Internet Appendix.

We also use our firm-year panel to estimate some parameters based on observable measures. We directly estimate $\{\omega_j\}$ as the observed share of all sales attributable to each industry j. We estimate $\{\eta_{cj}\}$ as the share of all new-entrant sales attributable to each industry j and technology class c. Additionally, we obtain injunction rate data from Seaman (2015). For missing technology classes and industries, we obtain comprehensive docket data from the USPTO covering virtually all patent lawsuits and identify entries containing "injunction" or "enjoin." We then employ GPT-o3 API[17] to classify whether each case resulted in a permanent injunction being granted, using carefully designed prompts that extract both the classification and supporting text to verify accuracy. This approach yields injunction rates for each (c, j) pair that we directly incorporate into our model as ζ_{cj}. We provide details in Internet Appendix A.

5.2. Parameter identification and estimation by GMM

Using GMM, we estimate the remaining 21 parameters: (i) the mean μ_λ and standard deviation σ_λ of λ_c; (ii) the mean μ_χ and standard deviation σ_χ of R&D cost scale χ_{cj}; (iii) the R&D cost convexity ψ; (iv) the entrant R&D cost scale ν; (v) the knowledge spillovers σ; (vi) the infringement probability parameters $\beta_{\kappa_1,0}, \beta_{\kappa_1,1}, \beta_{\kappa_1,2}$ for type-1 infringement and $\beta_{\kappa_2,0}, \beta_{\kappa_2,1}, \beta_{\kappa_2,2}$ for type-2 infringement, which determine κ_{1cj} and κ_{2cj} through equations described below; (vii) the mean $\mu_{\bar{\tau}_1}$ and standard deviation $\sigma_{\bar{\tau}_1}$ of defendant win rates in type-1 infringements; (viii) the mean and standard deviation parameters $\mu_{\tau_2^l}, \sigma_{\tau_2^l}, \mu_{\tau_2^h}, \sigma_{\tau_2^h}$ determining the distribution of defendant win rates in type-2 infringements; and (ix) the litigation cost parameters $\beta_{\xi,0}$ and $\beta_{\xi,1}$ determining ξ_{cj}, as described below. We jointly estimate these parameters by minimizing the distance between 24 model-implied moments in Table 3 and their empirical counterparts, whose detailed construction is in Section A.2 of Internet Appendix A.

We now explain how our empirical moments help us infer the values of these parameters. Technically, each model-implied moment depends jointly on all parameters through the model solution. However, certain moments are more sensitive to particular parameters, aiding

[17] We tested across multiple large language models, and verified GPT-o3 API does far better than other models for this type of task.

identification. Table IA.1 in the Internet Appendix presents a sensitivity matrix, where each cell reports the elasticity ε_{jk}: the percent change in moment j when parameter k changes by one percent. Highlighted cells indicate economically intuitive identifying relationships between parameters and moments within each category. Importantly, the elasticities along the diagonal are economically meaningful in magnitude. While the model's equilibrium structure creates interdependencies across categories, the sensitivity matrix confirms that each parameter responds meaningfully to variation in its primary identifying moments.

The mean and standard deviation of the innovation step size, μ_λ and σ_λ: Growth in our model depends on the parameters μ_λ and σ_λ, which govern the distribution of λ_c across technology classes. Specifically, the distribution of λ_c across technology classes is characterized by mean μ_λ and standard deviation σ_λ, generating technology-class-level heterogeneity in innovation step sizes. These two parameters are primarily identified by two growth-rate moments: the average output growth rate and the standard deviation of sales growth across technology classes. We follow a literature convention by targeting the growth rate of average output rather than average sales because our model captures a balanced growth path for the entire economy, not just the growth of large publicly traded firms.[18] As shown in Table IA.1, the output growth rate exhibits a strong elasticity with respect to μ_λ (1.043), confirming that average output growth increases with the mean innovation step size. Similarly, the standard deviation of sales growth is sensitive to σ_λ (0.342), as greater dispersion in λ_c across technology classes generates more variation in growth rates. Among the productivity growth parameters, these moments provide the strongest identifying variation for μ_λ and σ_λ, respectively.

The mean and standard deviation of R&D cost scale, μ_χ and σ_χ: The R&D cost scale χ_{cj} varies across technology class-industry (c, j) pairs. The distribution of χ_{cj} across (c, j) cells is characterized by mean μ_χ and standard deviation σ_χ. Two moments related to R&D intensity (R&D spending divided by sales) help identify these parameters. As shown in Table IA.1, average R&D intensity has a strong negative elasticity with respect to μ_χ (-0.650). Intuitively, higher values of χ_{cj} raise the marginal cost of R&D spending, reducing firms' innovation rates. While a higher value of χ_{cj} mechanically raises R&D spending, the reduction in innovation dominates, allowing the mean R&D cost μ_χ to be identified from total

[18] See Akcigit et al. (2016), Celik and Tian (2023), and Terry (2023), among others.

R&D spending.

Second, the standard deviation of R&D intensity across technology classes helps identify σ_χ, with a positive elasticity (0.063). Firms in technology classes with high λ_c typically spend more on R&D due to higher expected benefits. Rising σ_χ increases dispersion in χ_{cj}, generating wider variation in average R&D spending across technology classes.

The R&D cost convexity parameter, ψ: The R&D cost convexity ψ is primarily identified by the skewness of R&D intensity across technology classes. Table IA.1 confirms that R&D intensity skewness has a positive elasticity with respect to ψ (0.327). This positive relationship arises because higher convexity makes R&D spending increase disproportionately for high-innovation firms, generating greater right skewness in R&D intensity.

The entrant R&D cost scale parameter, ν: The parameter ν governs R&D costs for potential entrants, determining their innovation intensity. Higher values of ν make R&D more costly, reducing both innovation and firm entry. Table IA.1 shows that the entry rate has a strong negative elasticity with respect to ν (-0.330), confirming this monotonic relationship.

Knowledge spillover parameter, σ: We identify the technology-spillover parameter σ using the slope coefficient from a regression of firm R&D spending on the share of total sales from the firm's technology class. This regressor is correlated with M_{ct}, the fraction of all product lines owned by firms in technology class c at time t. As shown in Equation (6), the R&D cost function includes M_{ct} in the denominator, so an increase in M_{ct} reduces the marginal cost of R&D through knowledge spillovers, and firms optimally innovate more. As the coefficient σ rises, two countervailing forces affect total R&D spending. Because σ is in the denominator, total R&D spending mechanically falls. However, more optimal innovation implies more R&D spending. The convexity of the cost function limits the strength of this second effect, so overall, σ has a negative effect on the relationship between R&D spending and M_{ct}. Table IA.1 confirms this pattern, showing that this regression coefficient has a negative elasticity with respect to σ (-0.035). Importantly, while μ_χ also affects this regression coefficient, it moves mean R&D intensity in the opposite direction from σ, allowing the joint estimation to distinguish between these parameters.

The infringement probability parameters, β_{κ_1} and β_{κ_2}: The infringement probabilities κ_{1cj} and κ_{2cj} represent the probability of infringement conditional on successful innovation, and vary across technology class-industry (c, j) pairs based on the innovating firm's characteristics. We parameterize this heterogeneity as linear functions of observable characteristics:

$$\kappa_{1cj} = \beta_{\kappa_1,0} + \beta_{\kappa_1,1} \cdot \pi_{cj} + \beta_{\kappa_1,2} \cdot RD_{cj}, \tag{28}$$

$$\kappa_{2cj} = \beta_{\kappa_2,0} + \beta_{\kappa_2,1} \cdot \pi_{cj} + \beta_{\kappa_2,2} \cdot RD_{cj}, \tag{29}$$

where $\pi_{cj} = \frac{\lambda_c}{1+\lambda_c}\omega_j$ captures average profitability and RD_{cj} captures R&D intensity in each (c, j) cell. The intercepts $\beta_{\kappa_1,0}$ and $\beta_{\kappa_2,0}$ govern the baseline infringement rates conditional on innovation, while the slope coefficients allow infringement to vary systematically with cell characteristics.

In type-1 infringement, the innovator attempts to take over a product line within the same industry and technology class, and the incumbent (plaintiff) litigates to retain ownership. In type-2 infringement, a firm with the same technology class as the innovator (but potentially in a different industry) sues to extract rents by threatening an injunction. Consequently, type-1 infringement generates only same-industry lawsuits, while type-2 infringement can generate both same-industry and different-industry lawsuits.

Table IA.1 shows that the average probability of being a plaintiff has a positive elasticity with respect to $\beta_{\kappa_1,0}$ (0.349): higher baseline infringement rates generate more litigation overall, increasing the number of plaintiffs. The slope coefficients $\beta_{\kappa_1,1}$ and $\beta_{\kappa_1,2}$ are identified by the regression coefficients of litigation probability on profitability and R&D intensity, with elasticities of −0.118 and 0.068, respectively. While $\beta_{\kappa_1,0}$ affects the overall level of litigation, the slope coefficients affect how litigation varies across (c, j) cells with different characteristics.

The elasticities for $\beta_{\kappa_1,1}$ and $\beta_{\kappa_1,2}$ have opposite signs due to an important asymmetry in how the covariates respond to infringement risk. For $\beta_{\kappa_1,2}$, the effect operates primarily through a direct channel: An increase in $\beta_{\kappa_1,2}$ increases litigation risk for high R&D firms, causing more lawsuits for this type of firm and thus a higher correlation between litigation activity and R&D spending. Thus, an increase in $\beta_{\kappa_1,2}$ increases the correlation, implying a positive elasticity. In contrast, for $\beta_{\kappa_1,1}$ the effect operates primarily through an indirect channel: an increase in $\beta_{\kappa_1,1}$ increases litigation risk for profitable firms, which endogenously disincentivizes innovation. As these profitable firms innovate less, the overall level of litigation falls despite a higher conditional likelihood of infringement. Thus, an increase in $\beta_{\kappa_1,1}$ reduces the correlation between profitability and litigation activity, implying a negative elasticity. We find that $\beta_{\kappa_1,1} > 0$, implying that profitable innovators have higher conditional infringement probabilities. Also, $\beta_{\kappa_1,2} < 0$, implying that R&D-intensive innovators have lower conditional infringement probability, consistent with stronger patent protection through higher-quality claims and prior art searches.

For type-2 infringement, the parameters govern the composition of same-industry versus different-industry litigation. The fraction of same-industry lawsuits has a negative elasticity with respect to $\beta_{\kappa_2,0}$ (-0.044): higher baseline type-2 infringement generates more cross-industry cases, reducing the same-industry share. The slope coefficients $\beta_{\kappa_2,1}$ and $\beta_{\kappa_2,2}$ are identified by the regression coefficients of same-industry fraction on profitability and R&D intensity, with elasticities of -0.071 and 0.061, respectively. The same asymmetry between the exogenous profitability covariate and endogenous R&D covariate explains the opposite signs. Our estimates yield $\beta_{\kappa_2,1} > 0$ and $\beta_{\kappa_2,2} < 0$, consistent with the type-1 estimates.

The litigation cost parameters, $\beta_{\xi,0}$ and $\beta_{\xi,1}$: In practice, litigation costs may vary systematically with firm characteristics: firms in more profitable markets often face higher-stakes cases and may invest more in legal resources. To capture this heterogeneity, we allow the litigation cost scale ξ_{cj} to vary across technology class-industry (c, j) pairs as a linear function of profitability:

$$\xi_{cj} = \beta_{\xi,0} + \beta_{\xi,1} \cdot \pi_{cj}. \tag{30}$$

The intercept $\beta_{\xi,0}$ governs the baseline litigation cost scale, while the slope $\beta_{\xi,1}$ allows litigation costs to vary with firm characteristics. Since ξ is the rate parameter of an exponential distribution (with mean $1/\xi$), higher values of ξ_{cj} imply lower expected litigation costs. We focus on profitability rather than R&D intensity because the cross-sectional relationship between litigation costs and R&D intensity is statistically insignificant in our data.

The average ratio of litigation spending to revenue has a negative elasticity with respect to $\beta_{\xi,0}$ (-0.165): higher $\beta_{\xi,0}$ implies higher ξ_{cj} and thus lower expected litigation costs, which reduces total litigation spending relative to revenue. The regression coefficient of litigation costs on profitability has a negative elasticity with respect to $\beta_{\xi,1}$ (-0.166): higher $\beta_{\xi,1}$ implies higher ξ_{cj} for profitable firms, reducing their litigation costs relative to less profitable firms and thus lowering the regression coefficient. Our estimate of $\beta_{\xi,1} < 0$ indicates that profitable firms face higher litigation costs, consistent with higher stakes in more profitable markets.

The defendant win rate parameters in type-1 infringements, $\mu_{\bar{\tau}_1}$ and $\sigma_{\bar{\tau}_1}$:

The defendant win rate in type-1 infringements $\bar{\tau}_{1cj}$ varies across technology class-industry (c,j) pairs. The distribution of $\bar{\tau}_{1cj}$ across (c,j) cells is characterized by mean $\mu_{\bar{\tau}_1}$ and standard deviation $\sigma_{\bar{\tau}_1}$. The mean $\mu_{\bar{\tau}_1}$ is identified by the plaintiff's win rate in same-industry lawsuits. Table IA.1 shows this moment has a strong negative elasticity with respect to $\mu_{\bar{\tau}_1}$ (-0.966), confirming that higher defendant win rates mechanically reduce plaintiff victories. The standard deviation $\sigma_{\bar{\tau}_1}$ is identified by the dispersion of plaintiff win rates across same-industry lawsuits, which has a positive elasticity (0.520): greater variation in $\bar{\tau}_{1cj}$ across (c,j) pairs generates more dispersion in observed plaintiff win rates.

The defendant win rate parameters in type-2 infringements, $\mu_{\tau_2^l}$, $\sigma_{\tau_2^l}$, $\mu_{\tau_2^h}$, and $\sigma_{\tau_2^h}$: In type-2 infringements, the defendant's win rate τ_{2cj} is drawn uniformly from the interval $[\tau_{2cj}^l, \tau_{2cj}^h]$, where both the lower and upper bounds vary across technology class-industry (c,j) pairs. The distribution of the lower bound τ_{2cj}^l across (c,j) cells is characterized by mean $\mu_{\tau_2^l}$ and standard deviation $\sigma_{\tau_2^l}$, and similarly the upper bound τ_{2cj}^h is characterized by mean $\mu_{\tau_2^h}$ and standard deviation $\sigma_{\tau_2^h}$.

The lower bound parameters are identified using plaintiff win rates in different-industry lawsuits. The plaintiff's win rate in different-industry cases has a negative elasticity with respect to $\mu_{\tau_2^l}$ (-0.084): shifting the mean lower bound of defendant win rates upward reduces plaintiff victories on average. The dispersion of plaintiff win rates in different-industry cases has a strong positive elasticity with respect to $\sigma_{\tau_2^l}$ (0.871): greater variation in the lower bound τ_{2cj}^l across (c,j) pairs generates more dispersion in observed plaintiff win rates.

The upper bound parameters are identified using settlement behavior. The settlement rate has a negative elasticity with respect to $\mu_{\tau_2^h}$ (-0.198): as the mean upper bound of defendant win probability rises, defendants become more likely to win at trial and thus less willing to settle. The standard deviation of settlement rates has a positive elasticity with respect to $\sigma_{\tau_2^h}$ (0.029): greater variation in the upper bound τ_{2cj}^h across (c,j) pairs generates more dispersion in settlement behavior.

Overidentifying restrictions: Our estimation includes three additional moments that serve as overidentifying restrictions: the standard deviation of plaintiff probabilities, the correlation between litigation rates and sales growth, and the correlation between litigation rates and R&D intensity. As shown in the bottom panel of Table IA.1, these moments have sensitivities to multiple parameters rather than loading primarily on a single parameter, providing external validation of the model's structure. The standard deviation of plaintiff probabilities responds strongly to parameters governing heterogeneity in innovation (σ_λ, with elasticity 0.822) and R&D costs (μ_χ, with elasticity −2.030). The correlation between litigation and growth responds to innovation parameters (μ_λ with elasticity −0.362, ψ with elasticity 0.828) and infringement parameters. The correlation between litigation and R&D intensity similarly responds to multiple parameter groups, with notable elasticities for ψ (0.959) and $\beta_{\kappa_2,0}$ (−0.423). These patterns are consistent with our identifying assumptions, confirming that the model's mechanisms interact in economically sensible ways.

5.3. Parameter estimates and model fit

Parameter estimates: Panel A of Table 2 reports the estimated parameter values and the associated standard errors. All of the parameters are precisely estimated because, as shown in Table IA.1 in the Internet Appendix, our model moments are highly sensitive to the parameters that they identify.

Several of our parameters are easily interpretable. First, the estimates of $\sigma_\chi = 2.303$ and $\sigma_\lambda = 0.148$ imply that both R&D costs and innovation efficacy have meaningful variation across technology class-industry pairs. This variation validates our focus on how heterogeneous firms select into lawsuits. Second, consistent with Bloom et al. (2013), we estimate positive and statistically significant knowledge spillovers ($\sigma = 0.053$). Third, we estimate $\psi = 2.218$, close to the value of 2 that is frequently assumed in the literature (Bloom et al., 2002). Fourth, our estimate of entrants' R&D cost scale parameter, $\nu = 2.035$, is lower than the mean for incumbents, $\mu_\chi = 6.022$, suggesting that entrants face lower R&D costs. Finally, our litigation cost scale parameters ($\beta_{\xi,0} = 16.997$, $\beta_{\xi,1} = -14.293$) indicate that more profitable firms face higher average litigation costs (lower ξ_{cj}), potentially reflecting higher-stakes cases that require greater legal expenditure.

The parameters describing litigation require further interpretation. The infringement probability parameters $\beta_{\kappa_1,0} = 0.562$ and $\beta_{\kappa_2,0} = 0.901$ govern the baseline rates of type-1 and type-2 infringement. The model matches the empirical fact that most lawsuits are filed between firms in the same industry (86.18% in the data, 93.63% in the model), which reflects both type-1 infringement (which occurs exclusively within industries) and the within-industry component of type-2 infringement. The infringement probability parameters also vary with firm characteristics: both are higher for more profitable firms ($\beta_{\kappa_1,1} = 1.509$, $\beta_{\kappa_2,1} = 0.906$) and lower for more R&D-intensive firms ($\beta_{\kappa_1,2} = -0.657$, $\beta_{\kappa_2,2} = -1.869$). The positive profitability coefficients are consistent with higher-value markets attracting more litigation. The negative R&D coefficients suggest that R&D-intensive innovators face lower infringement risk, potentially reflecting stronger patent protection through higher-quality claims and more thorough prior art searches. Note that these coefficients govern the infringement probability conditional on innovation (the probability that an innovator triggers a lawsuit as a defendant) rather than the probability of being a plaintiff. As discussed in Section 5.2, this distinction explains why the coefficient signs differ from the regression coefficients relating plaintiff probability to firm characteristics.

The defendant win probability parameters govern trial outcomes. For type-1 cases, the distribution of defendant win probabilities across technology class-industry pairs is characterized by $\mu_{\bar{\tau}_1} = 0.499$ and $\sigma_{\bar{\tau}_1} = 0.113$. For type-2 cases, the defendant win probability for each cell ranges between a lower bound τ^l_{2cj} and upper bound τ^h_{2cj}, with the distributions of these bounds governed by $(\mu_{\tau^l_2}, \sigma_{\tau^l_2}) = (0.087, 0.479)$ and $(\mu_{\tau^h_2}, \sigma_{\tau^h_2}) = (0.479, 0.166)$. All standard deviation parameters are precisely estimated, indicating sizable heterogeneity across cells.

Model fit: Table 3 reports the targeted moments in the data and the model. The model closely matches the data moments on several important dimensions.

First, our model matches observed innovation activity and the resultant economic growth. For example, the average output growth rate is 2.19% in our model, compared to 1.91% in the data. The average R&D intensity is 4.46% in the model, compared to 6.97% in the data. Our model similarly matches the distribution of innovation across technology classes. The standard deviation of R&D intensity across technology classes is 2.12% in our model, compared to 3.67% in the data, and the skewness is 0.449 in the model versus 0.481 in the data. The model also captures knowledge spillovers, with the coefficient of R&D spending on technology-class share at 0.042 in the model versus 0.027 in the data.

Second, the model matches observed litigation activity. In our model, a patent-holding firm has an annual 8.28% chance of filing a patent lawsuit, which is reasonably close to 12.67% in the data. The settlement rate in our model is 50.43%, close to the 58.83% rate in the data. The model also matches trial outcomes: plaintiff win rates are 49.50% for same-industry cases (47.17% in the data) and 33.04% for different-industry cases (41.18% in the data). The mean litigation cost-to-revenue ratio is 0.81% in the model versus 0.58% in the data.

Third, our model closely matches the interaction between innovation and litigation. The across-technology-class correlation between litigation rates and sales growth is 0.727 in our model, quite close to the empirical correlation of 0.764. The correlation between litigation probability and R&D intensity is 0.739 in the model versus 0.703 in the data. The model also matches the regression coefficients relating litigation probability and case composition to firm profitability and R&D intensity. These correlations and regression coefficients are particularly important for our welfare implications: they indicate that high-λ_c technology classes are more frequently involved in litigation. As a result, reducing the injunction rate benefits the most innovative firms, improving welfare. The following section demonstrates this prediction quantitatively.

6. Quantitative Analysis

6.1. Increasing R&D subsidies

We begin by using our model to quantify the effects of one of the most fundamental innovation policies: increasing R&D subsidies. In three separate exercises, we double the R&D subsidy s_{cj} from its baseline value for three groups: (i) the whole sample, (ii) firms in the lowest tercile of research efficiency (measured by λ_c/χ_{cj}), and (iii) firms in the highest tercile of research efficiency. In each exercise, we use the parameters from Table 2 to solve the model and compute several model moments of interest: social welfare, the output growth rate, the distribution of firm value, and firms' innovation and litigation decisions.

Table 4 presents the results. As expected, subsidizing R&D reduces its marginal R&D cost, thereby increasing R&D intensity. Column 2 of Table 4 indicates that doubling the R&D subsidy for all firms in the economy would significantly boost innovation, which would raise the growth rate for the economy, increasing social welfare by 1.91%. This result implies that aggregate investment in innovation is too low relative to a socially efficient benchmark.

However, the degree of over- or under-investment varies across firms because of differences in research efficiency. First, we assume that only the least efficient technology classes receive the subsidy. Column 3 of Table 4 shows that subsidizing only inefficient firms reduces welfare by 0.09%. The subsidy induces a slight shift in the composition of R&D activity toward less efficient firms, even though aggregate innovation remains essentially unchanged. This compositional shift reduces average product line value, as the marginal product lines created by inefficient firms are less valuable. Because low-efficiency firms have smaller innovation step sizes, they are less effective at creating and accumulating product lines. Consequently, the average number of product lines per firm also falls slightly. Together, these effects reduce incumbent firm value and dampen the output growth rate. Consequently, welfare falls modestly. The decline in average product line value and fewer product lines reduce litigation overall, as measured by the average probability of becoming a plaintiff.

Next, we consider a scenario where only firms in the most efficient technology classes receive the subsidy. Column 4 of Table 4 shows that this policy improves welfare substantially by 1.63%. The improvement occurs because the reduced cost of R&D leads incumbents to innovate more. There are more product lines in equilibrium, and they are more valuable because R&D activity moves to these highly efficient firms, so growth and welfare rise.

Table 4 also reveals a redistribution effect. The welfare improvement coincides with a transfer from entrants to incumbents. Greater incumbent innovation means entrepreneurs are more likely to lose their product lines, dampening their incentive to innovate, so entrant innovation and entrepreneurial value fall. Product-line owners also file more lawsuits to protect their increasingly valuable products from innovators obtaining them.

Overall, these two exercises underscore the importance of firm heterogeneity in our model. High-efficiency firms underinvest in innovation relative to a socially efficient benchmark, so subsidizing their R&D improves welfare. Conversely, low-efficiency firms overinvest in innovation, so subsidizing their R&D amplifies inefficient activity, thereby reducing welfare. This heterogeneity implies that the effectiveness of any litigation reform depends on the types of firms it affects. Our model, therefore, needs to capture both the impact of the litigation system on innovation and the selection into lawsuits by heterogeneous firms.

6.2. Litigation reforms and the efficacy of R&D subsidies

Table 4 illustrates that R&D subsidies affect both innovation and litigation because the two are endogenously linked in our model. So, a natural question is whether changes to the litigation system alter the effectiveness of R&D subsidies. To explore this connection, we modify four aspects of the legal system: the injunction rate, the cost of filing lawsuits, the likelihood of patent infringement, and the ability to litigate at all, modeled through prohibitively high filing costs. For each modification, we reevaluate the three R&D subsidy scenarios from Section 6.1.

We present the results in Table 5, focusing on welfare for brevity. The first row replicates the result of Section 6.1. Next, starting with the parameters from Table 2, we change one litigation parameter at a time and repeat the same analysis of R&D subsidies. Row 2 shows the results from reducing the injunction rate parameter ζ_{cj} to 85% of its baseline value. Row 3 shows the results from reducing ξ_{cj} (the litigation cost scaling parameter) to 85% of its baseline value, which increases the average cost of hiring a legal team. Row 4 shows the results from reducing the infringement probability parameters κ_{1cj} and κ_{2cj} to 85% of their baseline values. Row 5 shows the results when ξ_{cj} is set low enough to make litigation prohibitively expensive.

Several patterns emerge from Table 5. First, defendant-friendly litigation reforms consistently amplify the welfare effects of R&D subsidies: across all four reforms, welfare gains from doubling R&D subsidies range from 1.9% to 2.4% for all firms and from 1.6% to 2.6% for high-efficiency firms, consistently exceeding their baseline values, because firms invest more in innovation when they face lower litigation risk. Subsidizing inefficient firms remains welfare-reducing across all legal environments, with losses ranging from 0.09% to 0.11%. Second, this amplification is strongest for high-research-efficiency firms, suggesting that litigation reform and targeted subsidies reinforce each other. Third, among the three reforms that feature positive litigation rates, reducing the injunction rate generates the largest amplification of subsidy effectiveness for high-efficiency firms. These results carry a broader implication. Models that omit litigation can overstate the impact of R&D subsidies by ignoring how litigation risk dampens firms' responses to subsidies, as the returns to R&D subsidies depend importantly on the legal environment in which firms operate.

In summary, litigation reforms can significantly enhance the effectiveness of R&D subsidies. However, a litigation reform will have its own baseline impact on the economy that must be considered in addition to the interaction between the reform and R&D subsidies. In the following sections, we examine the impact of litigation reforms in the absence of increased R&D subsidies. For each reform, we study its impact on litigation, innovation, firm dynamics, firm value, economic growth, and social welfare. We quantify these reforms' average effects while highlighting outcome heterogeneity across firms and infringement types.

6.3. The impact of the *eBay v. MercExchange* ruling

While litigation reforms can enhance the effectiveness of R&D subsidies, they can have important effects in their own right, even without subsidies. We now turn to quantifying both the average effects of possible reforms, while also highlighting the substantial heterogeneity in outcomes across firms and types of patent infringement.

First, we use the model to study a historical policy. Before 2006, a plaintiff victory in a patent litigation trial almost always led to an injunction. The pivotal 2006 *eBay v. MercExchange* Supreme Court ruling raised the standard for granting an injunction. According to Chien and Lemley (2012), injunctions were granted in 95% of plaintiff victories before the eBay ruling and 75% after. Therefore, using the estimated model, we simulate the eBay ruling by adjusting the ζ_{cj} parameter from pre-eBay to post-eBay levels and assess its impact on innovation, firm values, litigation, growth, and social welfare.

The consequences of reducing the injunction rate are ex-ante ambiguous. Potential innovators might be deterred by the fear that their future patents will not be adequately protected from infringers. Alternatively, potential innovators might be encouraged by the reduced threat of an injunction blocking their innovation. Columns 1 and 2 of Table 6 show that reducing the injunction rate increases innovation among incumbents and entrants by 2.98% and 2.74%, respectively. Innovation increases because firms worry less that a patent lawsuit will block their innovation. Incumbent firms exhibit a greater increase in innovation compared to entrants, resulting in a slight decline in the contribution of entrants to growth.

The effect of lowering the injunction rate on firm value is nuanced. We find that our simulation of the eBay ruling lowers the average value of a product line by 3.62%. Firms value a product line less after eBay because they are less able to protect it. Even if they win a lawsuit against an infringer, they are less likely to save their monopoly with an injunction. However, the increased innovation activity described above leads to the creation of new product lines. The average number of product lines per firm increases by 2.15%. This increase partially offsets the decreased value of each product line, but incumbent firm value still declines by 1.55%. The reduced injunction threat, however, offsets the decrease in average product line value, leading to an increase in entrepreneur value of 6.18%.

This rise in innovation activity also widens the dispersion in the number of product lines held by different types of firms. While all firms innovate more after eBay, high-research-efficiency firms are more likely to become product-line leaders and, thus, hold more product lines. Conversely, low-research-efficiency firms hold fewer product lines. This redistribution raises the standard deviation of the number of product lines per firm by 2.07%.

Moreover, the eBay ruling affects the potential gain from litigation, influencing firms' legal strategies. The probability of hiring a legal team after an infringement falls by 9.76% because of the lower potential gain from litigation. However, the increase in overall innovation leads to more infringements and thus more creative destruction. The decline in legal team hiring dominates, leading to a decrease in the average plaintiff probability by 6.69%.

Regarding aggregate implications, the increase in incumbent and entrant innovation boosts the output growth rate. Higher R&D costs crowd out consumption, which falls slightly. However, for welfare, higher economic growth dominates lower consumption, so social welfare rises by 2.98%. To provide context for this welfare magnitude, we simulate the 2017 Tax Cuts and Jobs Act (TCJA), which reduced the implied R&D subsidy by approximately 4% after 2022. This experiment generates a 0.86% welfare loss in our model. The eBay ruling's 2.98% welfare gain is thus sizable—roughly three times the magnitude of a major recent tax policy change. These effects are also comparable to estimates from related literature: managerial short-termism at 1% (Terry, 2023), trade frictions at 2.5% (Melitz and Redding, 2015), and agency frictions at 7.3% (Celik and Tian, 2023). Our welfare magnitudes align well with the endogenous growth literature, which typically finds large welfare effects from innovation policies due to compounding growth effects. Even small changes in long-run growth rates generate substantial welfare gains.

Heterogeneous injunction rates for different types of firms: Next, we examine the effects of different standards for granting injunctions for different types of patents. Specifically, we consider a counterfactual in which we reduce the injunction rate ζ_{cj} from pre-eBay to post-eBay levels for only the most efficient innovators (the highest tercile of technology classes by λ_c/χ_c) while maintaining the pre-eBay rate for all other patent lawsuits. Column 4 of Table 6 shows that this targeted policy improves welfare by 2.49%, which is close to eBay's 2.98% improvement. In contrast, column 3 shows that reducing injunction rates only for the least efficient innovators reduces welfare by 0.03%. The stark difference in outcomes demonstrates the importance of targeting litigation reforms based on firm innovation efficiency.

Heterogeneous injunction rates for different types of infringement: We examine how reducing injunction rates differentially affects outcomes when targeting type-1 versus type-2 infringement. Table IA.2 in the Internet Appendix presents four scenarios. We repeat the baseline results and counterfactuals in columns 1 and 2. Column 3 presents results from reducing the injunction rate from pre-eBay to post-eBay levels for type-1 only, and Column 4 presents results for type-2 only. Reducing injunction rates for type-1 infringement (column 3) increases both incumbent and entrant innovation, with a somewhat larger effect on incumbents (1.73% versus 1.48%). Reducing injunction rates for type-2 infringement (column 4) also increases both, with a somewhat larger effect on entrants (1.34% versus 1.14%). The smaller overall effect of the type-2 reform reflects that it generates less total innovative activity and product line creation than the type-1 reform.

Creative destruction rises in both scenarios, reducing average product-line value and the incentive to hire legal teams. The rise in creative destruction increases litigation opportunities, but this effect is dominated by reduced incentives to hire legal teams, lowering the per-product-line probability of becoming a plaintiff. When only type-1 injunction rates fall, the probability of hiring a legal team falls substantially by 6.69%, and the per-product-line probability of becoming a plaintiff falls by 6.04%. When only type-2 rates fall, a similar pattern emerges with a 3.24% decline in the probability of hiring a legal team and a 1.88% decline in per-product-line plaintiff probability.

The average number of product lines increases when only type-1 injunction rates fall, driven by stronger incumbent innovation. In contrast, the increase in the average number of product lines is more modest when only type-2 rates fall. These differences in the number of product lines affect firm values. When type-1 rates fall, more product lines outweigh their lower average value, raising incumbent value. When type-2 rates fall, the modest change in product lines cannot offset lower product-line value, reducing incumbent value.

When only type-1 injunction rates fall, growth improves more because of a larger rise in creative destruction, which is driven by incumbent innovation. The welfare gains are also larger, reflecting the broader expansion of innovative activity and product-line creation driven by the type-1 reform.

6.4. Increasing plaintiff filing costs

Our model allows us to study other potential reforms. For example, one recent proposed reform suggests increasing plaintiff pleading requirements (Gugliuzza, 2015). In our model, this proposal is analogous to increasing plaintiffs' filing costs. To examine its impact, we vary the litigation cost parameter, ξ_{cj}. A lower ξ_{cj} value corresponds to higher average litigation costs for plaintiffs, making it more costly to file lawsuits. We reduce ξ_{cj} to half of its baseline value, thereby effectively doubling the average plaintiff filing costs. The values of all other parameters are kept unchanged at their baseline levels in Table 2. Columns 1 and 2 of Table 7 report the impact of increasing filing costs for plaintiffs.

In response to the higher litigation costs, the probability of hiring a legal team declines. Both incumbents and entrants innovate more because they are less concerned about being sued if they infringe on other firms' intellectual property. Entrants exhibit a larger increase in innovation (2.41%) than incumbents (1.58%), and entrants' contribution to growth rises slightly by 0.44%.

The increase in innovation spurs a rise in creative destruction, which, along with the reduced ability to protect a product line with litigation, lowers average product line value by 2.95%. For entrants, who worry less about protecting product lines, the reduced intellectual property infringement risks outweigh this decline, improving entrepreneur value.

As with reduced injunction rates, the increase in innovation activity due to higher plaintiff filing costs boosts both the mean and the standard deviation of the number of product lines per firm. The average number of product lines per firm rises only by 0.72% and fails to offset the 2.95% reduction in average product line value. The result is a slight decrease in incumbent firm value by 2.25%.

Higher filing costs significantly influence litigation behavior by reducing the average probability of potential plaintiffs filing lawsuits. This reduction is primarily due to a decline in the per-product-line probability of becoming a plaintiff. The lower likelihood of engaging legal teams in the event of infringement is directly attributed to the increased filing costs.

The rise in filing costs also leads to aggregate implications comparable to reduced injunction rates. The rise in innovation by both incumbents and entrants propels the output growth rate. While increased R&D expenditure tempers consumption levels, the improved growth outweighs the lower consumption level. Consequently, social welfare rises by 1.85%.

Reducing injunction rates and increasing plaintiff filing costs generate similar welfare gains but operate through different margins. Reducing injunction rates works on the intensive margin by reducing the expected payoff per case, while increasing filing costs works on the extensive margin by deterring lawsuits. The two reforms also have slightly different distributional effects on the values of entrepreneurs and incumbents.

Heterogeneous filing costs for different types of firms: Next, we consider the impact of imposing different filing costs for different firms. Starting with the parameters in Table 2, we halve the ξ_{cj} parameter for all firms, only lawsuits targeting firms in the lowest tercile of research efficiency (measured by λ_c/χ_{cj}), and only lawsuits targeting firms in the highest tercile of research efficiency. We report the results in columns 3 and 4 of Table 7. We report the results of halving the ξ parameter for lawsuits targeting low-efficiency firms in column 3. In column 4, we report the analogous results from changing only the costs of filing lawsuits against high-efficiency firms.

The results are similar to the analogous heterogeneity exercise for reduced injunction rates. We find that raising the filing costs for only lawsuits targeting low-efficiency firms reduces overall welfare. In contrast, increasing the filing costs for lawsuits targeting high-efficiency firms boosts innovation and growth, thereby improving welfare. This finding underscores the importance of tailoring litigation reforms to the specific types of innovative firms that they affect.

Heterogeneous filing costs for different types of infringements: We also examine the effects of applying different filing costs to different types of infringement. We present the results in Table IA.3 in the Internet Appendix. We repeat the baseline results and counterfactuals in columns 1 and 2. To produce column 3, we halve the ξ_{cj} parameter for only type-1 infringement, and for column 4, we halve the ξ_{cj} parameter for only type-2 infringement.

Increasing filing costs substantially reduces the incentive to hire a legal team, thereby lowering average litigation probabilities. The probability of hiring a legal team declines across columns 2 to 4. Comparing columns 3 and 4, the type-1-only reduction in filing costs reduces incumbent value by much less than the type-2-only reduction, while generating a larger improvement in welfare. This pattern reflects the broader expansion in innovative activity and product line creation driven by the type-1 reform, consistent with the corresponding results for injunction rates. Overall, this exercise highlights the importance of tailoring litigation reforms to specific types of patent infringement.

6.5. Simulating the effects of reducing patent infringement risk

The tractability and realism of our model make it a useful laboratory for evaluating other patenting trends. In recent years, two key developments have shaped the U.S. patent litigation environment by curbing opportunistic patent enforcement. First, the Patent Trial and Appeal Board (PTAB), introduced under the America Invents Act of 2012, provides an administrative venue for re-examining the validity of granted patents. By offering a more efficient and technically informed review process, especially through Inter Partes Review (IPR), the PTAB improves patent clarity and limits the extent to which a patent holder can pursue litigation. Second, the 2014 Supreme Court decision in *Alice Corp. v. CLS Bank International* tightened the criteria for patent-eligible subject matter, particularly in software and business-method patents. This shift discourages broad, abstract claims and lowers the likelihood that weak patents will survive legal scrutiny, thereby reducing litigation risk.

We use our model to illustrate the effects of policies that reduce patent-infringement risk, such as the introduction of PTAB and the *Alice* decision. We report the results in Table IA.4. In column 1, we present baseline values for our model objects. We reduce the infringement parameters κ_{1cj} and κ_{2cj} to 75% of their baseline values: both types in column 2, type-1 only in column 3, and type-2 only in column 4.

Reducing both types of patent-infringement risk to 75% of baseline raises incumbent innovation by 2.38% and entrant innovation by 3.12%. When only type-1 risk falls, incumbent innovation rises by 1.07% while entrant innovation rises by 1.19%. In contrast, reducing only type-2 risk raises entrant innovation by 1.97% with a 1.39% rise in incumbent innovation. The creative destruction rate rises in all cases, leading to welfare gains of 2.74%, 1.76%, and 1.06%. We conclude that reforms to the patent-approval process can achieve welfare improvements similar to those from patent litigation reform.

6.6. Simulating the effects of patent trolls

In recent decades, patent litigation filed by entities commonly known as "patent trolls" has surged. These plaintiffs are often non-practicing entities (NPEs) that accumulate broad or weak patents primarily to engage in rent extraction through infringement lawsuits. Their growing presence has raised significant policy concerns. Critics argue that these patent trolls increase uncertainty, discourage genuine innovation, and divert resources away from productive activities.

In our model, a rise in patent trolls can be modeled through two channels: an increased probability of type-2 infringement, where plaintiffs file lawsuits purely for rent extraction, and a higher entry rate for low-research-efficiency firms that may serve as vehicles for opportunistic litigation rather than innovation. We simulate the potential effects of a rise in patent-troll activity on firm and aggregate outcomes. Specifically, we double both the type-2 infringement risk, κ_{2cj}, and the entry rate of firms in the lowest tercile of research efficiency.

Table IA.5 shows that patent trolls reduce entrant innovation by 4.01% and average R&D intensity by 4.27%. As more low-R&D-efficiency firms enter, R&D activities shift toward less capable innovators, with creative destruction falling by 2.62%. The average number of product lines rises slightly by 0.33%, and incumbent value falls by 1.74%, while entrant value falls substantially by 8.74%. The average probability of being a plaintiff rises by 3.26%, and the per-product-line plaintiff probability rises by 2.71%, so more resources are devoted to litigation. Combined with a 5.72% decline in economic growth from reduced innovation and creative destruction, social welfare falls by 2.82%.

7. Model Validation and Robustness

We conduct a battery of robustness checks and extensions. First, we validate the model externally by examining untargeted moments, finding that the model matches Mezzanotti (2021)'s estimated 3% increase in patenting following the *eBay* ruling replicates the skewness of the litigation cost distribution and reproduces cross-sectional patterns documented by Lanjouw and Schankerman (1997) relating litigation rates to firm value and technology class crowding. Second, we show that our main welfare conclusions are robust to allowing defendants to face positive litigation costs, though the relative welfare gains from different policies shift in intuitive ways as defendant costs rise. Third, we show that our baseline model captures the first-order effects of financial constraints through heterogeneous litigation behavior, and subsample estimates confirm that constrained firms benefit slightly more from defendant-friendly reforms. Fourth, we discuss how our structural parameters capture the equilibrium implications of reputation-building incentives in litigation. Full details are provided in Internet Appendix C.

8. Conclusion

We develop a novel dynamic general equilibrium model with endogenous growth to quantify the influence of the litigation system on innovation, firm value, growth, and social welfare. Heterogeneous firms innovate while facing potential patent lawsuits. Innovation can lead to the theft of market share from competitors, so firms can inefficiently internalize this transfer when they innovate, leading to inefficient overinvestment in innovation. However, innovation also creates positive externalities through technology spillovers that firms do not internalize. Thus, firms can also underinvest in innovation in equilibrium.

We embed a realistic patent litigation model in this dynamic general equilibrium framework. When firms innovate to steal a competitor's product line, they can infringe on an existing patent. The patent holder's decision to file a lawsuit and the joint decision of whether to go to trial are both determined endogenously. In a trial, the court may grant an injunction that stops the innovating firm from taking over the product line.

We estimate the model to evaluate the effectiveness of R&D subsidies, historical patent-litigation reforms, and proposed reforms. We find that low-research-efficiency firms tend to overinvest in innovation, while high-research-efficiency firms underinvest. The effectiveness of R&D subsidies is significantly amplified in alternative legal environments with weaker plaintiff rights.

We then study two legal reforms. Both are defendant-friendly, and both raise innovation and welfare. The 2006 Supreme Court *eBay* ruling, which lowered injunction rates, raised welfare by 2.98%. Similarly, a proposed reform that raises plaintiff pleading requirements, making it more difficult to file lawsuits, also raises welfare.

We also assess how the effects of these reforms vary across firms with different research efficiency and across types of infringement. We find that tailoring injunction criteria and pleading requirements to firm type or infringement type addresses overinvestment and underinvestment more effectively than a uniform rule.

Our research contributes to the discourse on patent litigation reform, offering policy-makers and stakeholders guidance on crafting reforms that align with the patent system's original purpose of promoting technological progress and economic well-being. Our rich yet highly tractable dynamic framework can be applied to study a wide range of patent-related issues, providing a versatile tool for analyzing the interplay between legal reforms and innovation dynamics.

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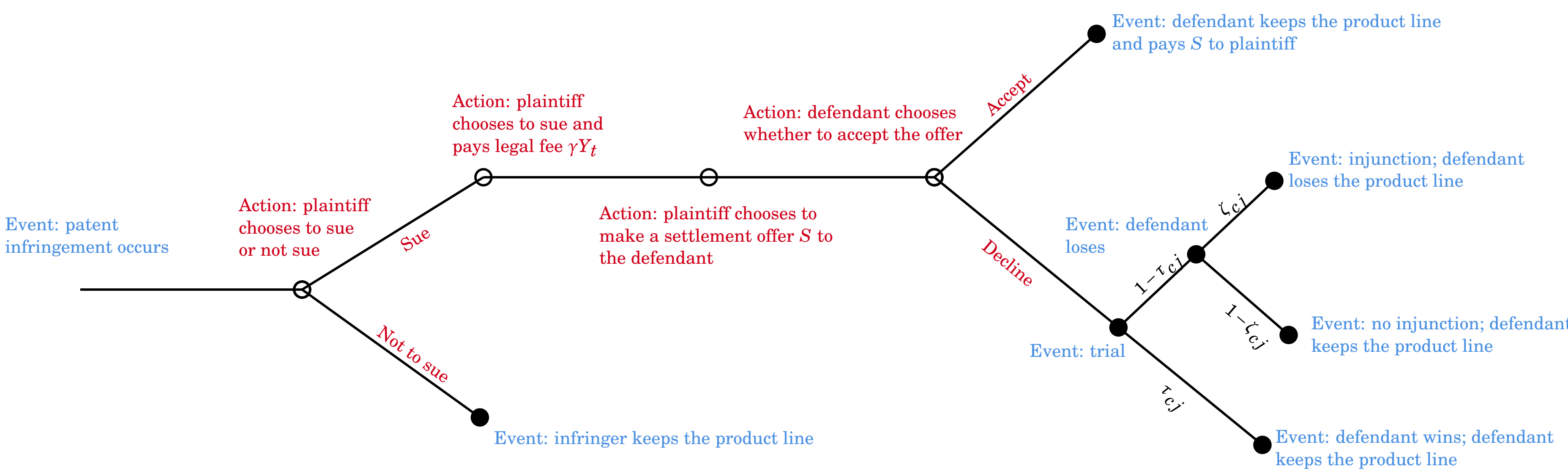

Figure 1: Litigation Subgame Timeline

Notes: This figure illustrates the timeline of the litigation subgame. Conditional on patent infringement, the plaintiff has to make a decision of whether to hire a legal team. A legal team is necessary for making a settlement offer or going to court. For firms in technology class c and industry j, the cost of hiring a legal team is γY_t, where $\gamma > 0$ is a random variable drawn from the distribution $\Gamma_{cj}(\gamma)$, and the Y_t term ensures that litigation costs grow at the same rate as output in a BGP equilibrium. If the plaintiff chooses to pay γY_t and hire the legal team, it then makes a take-it-or-leave-it settlement offer to the defendant. The defendant has private information about its chances of winning the trial. Let $\tau_{cj} \in [0,1]$ denote the probability that the defendant wins the trial. Given its private information τ_{cj}, the defendant can accept the settlement or refuse. Refusal leads to a trial. With probability τ_{cj}, the defendant wins the trial and the product line takeover is realized. With the complementary probability $1-\tau_{cj}$, the defendant loses. If the defendant loses, then the court decides on whether to grant an injunction or not. With probability $\zeta_{1cj} \in [0,1]$, an injunction is granted for a type-1 infringement, thus blocking the product line takeover. With probability $1-\zeta_{1cj}$, there is no injunction, so the defendant can still take over the product line. The same probability is denoted $\zeta_{2cj} \in [0,1]$ in the case of type-2 infringements. The parameters ζ_{1cj} and ζ_{2cj} are policy parameters that capture the inclination of a court to grant an injunction in the case of a proven patent infringement.

Table 1: Motivating Reduced-Form Evidence

Notes: This table presents reduced-form evidence consistent with the results of our quantitative model. For each technology class c, we calculate litigation activity as the number of lawsuits defending patents in class c divided by the number of patents. We calculate a firm-level measure of litigation intensity for firm i by averaging across the litigation activity in each technology class c and weighting by the fraction of firm i's patents in class c. We split our firm-year sample into two subsamples: above and below the median litigation intensity. In each subsample, we estimate the following regressions. In columns 1 and 2, we regress the logarithm of firm i's R&D spending in year t on Instrumented Peer R&D Spending, which is a leave-one-out average of the tax-credit-induced R&D spending in firm i's technology class in year t. In columns 3 and 4, we regress the logarithm of Tobin's Q on the same independent variable. In columns 5 and 6, we regress the logarithm of Tobin's Q on the logarithm of R&D spending in a 2SLS regression, where the independent variable is instrumented using the same tax credit instruments used to calculate Instrumented Peer R&D Spending. Finally, in column 7, we regress the logarithm of R&D spending on the interaction between litigation intensity and an indicator for the post eBay (post 2005) period. All regressions include year and firm fixed effects. Following Bloom et al. (2013), we report standard errors that account for heteroskedasticity and first-order serial correlation using the Newey-West correction. *,**,*** indicate statistical significance at the 10%,5%, and 1% level, respectively.

	Log RD Spending		Log Tobin Q				Log RD Spending
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Instrumented Peer RD Spending	0.082*	0.002	0.477***	0.286***			
	(0.045)	(0.036)	(0.077)	(0.059)			
Log RD Spending					0.651**	-0.177	
					(0.306)	(0.331)	
Post Ebay × Litigation Intensity							0.052***
							(0.015)
Firm FE	Y	Y	Y	Y	Y	Y	Y
Year FE	Y	Y	Y	Y	Y	Y	Y
Estimator	OLS	OLS	OLS	OLS	2SLS	2SLS	OLS
Litigation Intensity	High	Low	High	Low	High	Low	All
Observations	3914	3947	3608	3509	5475	5215	22562

Table 2: Model Estimation: Parameter Estimates

Notes: The table reports structural parameter estimates and standard errors from SMM estimation. The model is estimated by minimizing a weighted distance between 24 empirical and simulated moments. μ_λ and σ_λ govern the distribution of innovation step sizes. μ_χ, σ_χ, ψ, and ν determine incumbent and entrant R&D costs; σ controls knowledge spillovers across technology classes. $\beta_{\xi,0}$ and $\beta_{\xi,1}$ parameterize litigation costs as a function of firm profitability. $\beta_{\kappa_1,\cdot}$ and $\beta_{\kappa_2,\cdot}$ govern the probability of Type-1 and Type-2 infringement as functions of profitability and R&D intensity, respectively. $\mu_{\bar{\tau}_1}$, $\sigma_{\bar{\tau}_1}$ parameterize the distribution of defendant win probabilities in Type-1 cases; $\mu_{\tau_2^l}$, $\sigma_{\tau_2^l}$, $\mu_{\tau_2^h}$, $\sigma_{\tau_2^h}$ parameterize the lower and upper bounds of defendant win probabilities in Type-2 cases. See Section 5 and Appendix A for details.

Estimated Parameters	**Description**	**Values**	**Std. Err.**
μ_λ	Mean of innov. step size	0.181	0.0003
σ_λ	Stdv. of innov. step size	0.148	0.0010
μ_χ	Mean of incumbent R&D cost scale	6.022	0.3335
σ_χ	Stdv. of incumbent R&D cost scale	2.303	0.2186
ψ	R&D cost convexity	2.218	0.0216
ν	Entrant R&D cost scale	2.035	0.0901
σ	Knowledge spillover strength	0.053	0.0156
$\beta_{\xi,0}$	Litigation cost scale	16.997	0.5670
$\beta_{\xi,1}$	Litigation cost, profitability coef.	−14.293	7.0772
$\beta_{\kappa_1,0}$	Type-1 infringement prob., constant	0.562	0.0131
$\beta_{\kappa_1,1}$	Type-1 infringement prob., profitability coef.	1.509	0.2797
$\beta_{\kappa_1,2}$	Type-1 infringement prob., R&D coef.	−0.657	0.1644
$\beta_{\kappa_2,0}$	Type-2 infringement prob., constant	0.901	0.0198
$\beta_{\kappa_2,1}$	Type-2 infringement prob., profitability coef.	0.906	0.3715
$\beta_{\kappa_2,2}$	Type-2 infringement prob., R&D coef.	−1.869	0.2057
$\mu_{\bar{\tau}_1}$	Mean of type-1 def. win prob.	0.499	0.0002
$\sigma_{\bar{\tau}_1}$	Stdv. of type-1 def. win prob.	0.113	0.0005
$\mu_{\tau_2^l}$	Mean of type-2 def. win prob. (lb)	0.087	0.0063
$\sigma_{\tau_2^l}$	Stdv. of type-2 def. win prob. (lb)	0.479	0.0039
$\mu_{\tau_2^h}$	Mean of type-2 def. win prob. (ub)	0.479	0.0027
$\sigma_{\tau_2^h}$	Stdv. of type-2 def. win prob. (ub)	0.166	0.0074

Table 3: Model Estimation: Moment Conditions

Notes: This table reports the empirical (Data) and simulated (Model) moments used in our SMM estimation. Moments (1)–(7) capture innovation, growth, and industry dynamics. Moments (8)–(17) correspond to litigation outcomes including plaintiff probabilities, case outcomes, and litigation costs. Moments (18)–(24) are correlations and regression coefficients relating litigation activity to firm profitability, innovation, and growth. Standard errors of the moment conditions are reported in the last column. See Section 5 and Appendix A for the definition and construction of data moments.

Moment	**Data**	**Model**	**Std. Err.**
Innovation, Growth, and Industry dynamics			
(1) Output growth rate	1.91%	2.19%	0.0023
(2) Stdv. of sales growth	1.86%	1.46%	0.0032
(3) Mean of R&D intensity	6.97%	4.46%	0.0005
(4) Stdv. of R&D intensity	3.67%	2.12%	0.0005
(5) Skewness of R&D intensity	0.481	0.449	0.0246
(6) Entry rate	3.55%	5.44%	0.0029
(7) β(R&D spending, tech-class share)	0.027	0.042	0.0012
Litigation Outcomes			
(8) Mean prob. of being a plaintiff	12.67%	8.28%	0.0020
(9) Stdv. prob. of being a plaintiff	8.64%	4.76%	0.0067
(10) Fraction of same-industry lawsuits	86.18%	93.63%	0.0188
(11) P(plaintiff win | same-industry)	47.17%	49.50%	0.0239
(12) P(plaintiff win | diff.-industry)	41.18%	33.04%	0.0592
(13) P(settlement | being a plaintiff)	58.83%	50.43%	0.0025
(14) Stdv. of P(plaintiff win | same-industry)	14.02%	13.67%	0.0317
(15) Stdv. of P(plaintiff win | diff.-industry)	19.09%	15.44%	0.0300
(16) Stdv. of P(settlement | being a plaintiff)	16.21%	20.14%	0.0177
(17) Mean litigation costs/revenue	0.58%	0.81%	0.0001
Correlations and Regressions			
(18) Corr(litigation prob., sales growth)	0.764	0.727	0.1885
(19) Corr(litigation prob., R&D intensity)	0.703	0.739	0.0370
(20) β_1(lit. prob. ~ profit, R&D int.)	0.287	0.375	0.0919
(21) β_2(lit. prob. ~ profit, R&D int.)	0.945	1.278	0.0608
(22) β_1(lit. costs/rev. ~ profit)	0.034	0.025	0.0238
(23) β_1(frac. same-ind. ~ profit, R&D int.)	1.087	1.072	0.5292
(24) β_2(frac. same-ind. ~ profit, R&D int.)	1.088	1.114	0.3547

Table 4: The Impact of Increasing R&D Subsidies

Notes: This table presents the implications of increasing R&D subsidies. To simulate the influence of R&D subsidies, we conduct three exercises. We double the subsidy parameter s_{cj} from its baseline value for one of three groups: (i) the whole sample, (ii) firms in the lowest tercile of research efficiency (measured by λ_c/χ_{cj}), and (iii) firms in the highest tercile of research efficiency, respectively. The values of other parameters are kept unchanged at their baseline values. We evaluate the impact of these R&D subsidies on innovation, firm value, firm dynamics, litigation, growth, and social welfare.

	(1) **Baseline**	(2) **Whole Sample**		(3) **Low λ_c/χ_{cj}**		(4) **High λ_c/χ_{cj}**	
		Counterfactual	**% Change**	**Counterfactual**	**% Change**	**Counterfactual**	**% Change**
incumbent innovation	0.0622	0.0668	7.285%	0.0622	0.025%	0.0662	6.356%
avg. R&D intensity	4.455%	5.154%	15.689%	4.458%	0.057%	5.112%	14.730%
entrant innovation	0.1244	0.1231	−1.030%	0.1244	0.021%	0.1231	−0.992%
avg. entry rate	5.442%	5.580%	2.529%	5.436%	−0.117%	5.557%	2.104%
contribution of entrants to growth	33.134%	31.353%	−5.374%	33.136%	0.006%	31.735%	−4.223%
creative destruction rate	9.332%	9.753%	4.514%	9.334%	0.023%	9.696%	3.907%
avg. incumbent value	0.2158	0.2215	2.643%	0.2149	−0.424%	0.2233	3.504%
avg. entrepreneur value	0.1414	0.1382	−2.263%	0.1414	0.042%	0.1383	−2.178%
avg. product line value	0.1166	0.1153	−1.065%	0.1163	−0.279%	0.1169	0.244%
avg. number of product lines	1.8509	1.9203	3.748%	1.8483	−0.145%	1.9111	3.252%
stdv. number of product lines	2.3066	2.5750	11.633%	2.2775	−1.261%	2.6064	12.998%
avg. plaintiff prob.	8.279%	8.698%	5.068%	8.274%	−0.056%	8.639%	4.347%
per product line plaintiff prob.	4.929%	5.064%	2.742%	4.927%	−0.040%	4.997%	1.371%
avg. prob. of hiring legal team	72.276%	72.106%	−0.235%	72.224%	−0.072%	71.821%	−0.629%
output growth rate	2.188%	2.298%	5.039%	2.185%	−0.168%	2.287%	4.496%
consumption	0.2328	0.2308	−0.863%	0.2328	0.007%	0.2309	−0.842%
output	0.2527	0.2527	0.009%	0.2527	−0.005%	0.2527	0.009%
CEWC	——	1.908%	——	−0.085%	——	1.627%	——

Table 5: Welfare Effects of R&D Subsidies across Different Legal Environments

Notes: This table reports the effects of R&D subsidies in different legal environments on social welfare. To simulate the influence of R&D subsidies, we conduct three exercises by doubling the subsidy parameter s_{cj} from its baseline value for one of three groups: (i) the whole sample, (ii) firms in the lowest tercile of research efficiency (measured by λ_c/χ_{cj}), and (iii) firms in the highest tercile of research efficiency. All other parameters are held constant at their baseline values. Row (1) reports the change in consumption-equivalent welfare in the baseline model. Row (2) provides the welfare change in an economy where the injunction rate parameter ζ_{cj} is set to 85% of its baseline value. Row (3) reports the welfare change in an economy where ξ_{cj} (litigation cost scaling parameter) is set to 85% of its baseline value, which effectively increases the average expenses associated with hiring a legal team. Row (4) shows the welfare change in an economy where κ_{1cj} and κ_{2cj} (the infringement probability parameters) are reduced to 85% of their baseline values. Row (5) presents the welfare change in an alternative economy without litigation risk, where ξ_{cj} is set to a very small value, making it prohibitively expensive for firms to hire legal teams.

	(1)	(2)	(3)
Welfare Changes	**Whole Sample**	**Low λ_c/χ_{cj} Subsample**	**High λ_c/χ_{cj} Subsample**
(1) **CEWC Baseline**	**1.908%**	**−0.085%**	**1.627%**
(2) CEWC lower injunction risk ($\zeta_{cj} = 0.85\zeta^*_{cj}$)	1.964%	−0.095%	1.955%
(3) CEWC higher filing cost ($\xi_{cj} = 0.85\xi^*_{cj}$)	1.926%	−0.088%	1.649%
(4) CEWC lower infringement risk ($\kappa_{1cj,2cj} = 0.85\kappa^*_{1cj,2cj}$)	1.950%	−0.093%	1.925%
(5) CEWC no litigation risk ($\xi_{cj} \to 0$)	2.393%	−0.109%	2.590%

Table 6: Impact of Reducing the Injunction Rate

Notes: This table reports the impact of the 2006 eBay ruling. To model the effects of the eBay ruling, we adjust the ζ_{cj} parameter from pre-eBay levels to post-eBay levels while keeping other parameters unchanged at their baseline values. We evaluate its effects on innovation, firm values, firm dynamics, litigation, growth, and social welfare. Column (1) reports the values of the variables of interest for the baseline model. Column (2) reports the results of reducing the injunction rate for all firms from pre-eBay levels to post-eBay levels. In column (3), we reduce the injunction rate parameter for firms in the lowest tercile of research efficiency (measured by λ_c/χ_{cj}) from pre-eBay levels to post-eBay levels while keeping the rate for other firms unchanged at pre-eBay levels. In column (4), we reduce the injunction rate parameter for firms in the highest tercile of research efficiency (measured by λ_c/χ_{cj}) from pre-eBay levels to post-eBay levels while keeping the rate for other firms unchanged at pre-eBay levels.

	(1) Baseline	(2) Whole Sample		(3) Low λ_c/χ_{cj}		(4) High λ_c/χ_{cj}	
		Counterfactual	**% Change**	**Counterfactual**	**% Change**	**Counterfactual**	**% Change**
incumbent innovation	0.0609	0.0627	2.984%	0.0609	−0.023%	0.0622	2.080%
avg. R&D intensity	4.278%	4.504%	5.282%	4.276%	−0.029%	4.475%	4.614%
entrant innovation	0.1217	0.1250	2.739%	0.1220	0.273%	0.1241	1.948%
avg. entry rate	5.258%	5.504%	4.680%	5.262%	0.072%	5.443%	3.511%
contribution of entrants to growth	33.377%	33.004%	−1.117%	33.448%	0.214%	33.196%	−0.541%
creative destruction rate	9.132%	9.397%	2.902%	9.139%	0.076%	9.318%	2.036%
avg. incumbent value	0.2186	0.2152	−1.547%	0.2173	−0.587%	0.2185	−0.059%
avg. entrepreneur value	0.1347	0.1430	6.179%	0.1355	0.601%	0.1406	4.371%
avg. product line value	0.1199	0.1155	−3.623%	0.1194	−0.391%	0.1178	−1.760%
avg. number of product lines	1.8240	1.8633	2.154%	1.8204	−0.196%	1.8556	1.731%
stdv. number of product lines	2.3488	2.3973	2.065%	2.3180	−1.312%	2.3755	1.137%
avg. plaintiff prob.	8.664%	8.084%	−6.692%	8.586%	−0.908%	8.176%	−5.640%
per product line plaintiff prob.	5.217%	4.812%	−7.779%	5.177%	−0.771%	4.815%	−7.713%
avg. prob. of hiring legal team	77.809%	70.218%	−9.757%	77.181%	−0.807%	70.982%	−8.774%
output growth rate	2.092%	2.217%	5.974%	2.090%	−0.073%	2.197%	5.031%
consumption	0.2332	0.2327	−0.184%	0.2332	0.010%	0.2328	−0.171%
output	0.2527	0.2527	−0.000%	0.2527	−0.005%	0.2527	−0.002%
CEWC	—	2.984%	—	−0.028%	—	2.491%	—

Table 7: The Impact of Increasing Plaintiff Filing Costs

Notes: This table decomposes the impact of increasing plaintiff filing costs while keeping other parameters unchanged at their baseline values. We evaluate its effects on innovation, firm values, firm dynamics, litigation, growth, and social welfare. Column (1) reports the values of the variables of interest for the baseline model. In column (2), we halve the ξ_{cj} parameter from its baseline value for all lawsuits. In column (3), we halve the ξ_{cj} parameter from its baseline value for lawsuits targeting firms in the lowest tercile of research efficiency (measured by λ_c/χ_{cj}), effectively doubling the average expense associated with hiring a legal team for these cases. In column (4), we halve the ξ_{cj} parameter from its baseline value for lawsuits targeting firms in the highest tercile of research efficiency (measured by λ_c/χ_{cj}), effectively doubling the average expense associated with hiring a legal team for these cases.

	(1)	(2)		(3)		(4)	
	Baseline	**Whole Sample**		**Low λ_c/χ_{cj}**		**High λ_c/χ_{cj}**	
		Counterfactual	**% Change**	**Counterfactual**	**% Change**	**Counterfactual**	**% Change**
incumbent innovation	0.0622	0.0632	1.581%	0.0622	−0.036%	0.0629	1.078%
avg. R&D intensity	4.455%	4.609%	3.442%	4.455%	−0.010%	4.593%	3.086%
entrant innovation	0.1244	0.1274	2.410%	0.1246	0.212%	0.1266	1.802%
avg. entry rate	5.442%	5.604%	2.970%	5.444%	0.039%	5.561%	2.183%
contribution of entrants to growth	33.134%	33.278%	0.435%	33.200%	0.199%	33.298%	0.494%
creative destruction rate	9.332%	9.505%	1.857%	9.336%	0.046%	9.455%	1.319%
avg. incumbent value	0.2158	0.2109	−2.251%	0.2148	−0.459%	0.2132	−1.190%
avg. entrepreneur value	0.1414	0.1490	5.422%	0.1420	0.467%	0.1471	4.039%
avg. product line value	0.1166	0.1131	−2.949%	0.1162	−0.288%	0.1146	−1.682%
avg. number of product lines	1.8509	1.8643	0.719%	1.8478	−0.171%	1.8602	0.500%
stdv. number of product lines	2.3066	2.3202	0.587%	2.2794	−1.182%	2.2208	−3.719%
avg. plaintiff prob.	8.279%	6.208%	−25.016%	8.099%	−2.170%	6.677%	−19.353%
per product line plaintiff prob.	4.929%	3.615%	−26.655%	4.829%	−2.031%	3.884%	−21.201%
avg. prob. of hiring legal team	72.276%	52.433%	−27.455%	70.865%	−1.952%	56.225%	−22.208%
output growth rate	2.188%	2.270%	3.720%	2.187%	−0.075%	2.260%	3.300%
consumption	0.2328	0.2324	−0.203%	0.2329	0.009%	0.2324	−0.194%
output	0.2527	0.2527	−0.002%	0.2527	−0.004%	0.2527	−0.001%
CEWC	—	1.849%	—	−0.032%	—	1.625%	—

Internet Appendices:
The Efficiency of Patent Litigation

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A. Data, Moment Construction, and Estimation Procedure

A.1. Data

We use several data sets in our estimation. First, we download the Compustat North America Fundamentals Annual dataset. We obtain a firm-year dataset with the following variables: (i) total sales (revenue); (ii) R&D spending; (iii) profits (oibdp); (iv) industry (SIC code); and (v) firm name.

Second, we download the Global Corporate Patent Dataset (GCPD) (Bena, Ferreira, Matos, and Pires, 2017).[5] For the period 1980 to 2017, the GCPD provides a comprehensive link between patents awarded by the U.S. Patent and Trademark Office (USPTO) and the publicly listed Compustat firms that received those patents.

Third, we download the USPTO patent-litigation database,[6] which covers all patent lawsuits filed in federal courts over the period 2003 to 2020. For each lawsuit, the dataset includes identifiers for all of the infringed patents. We supplement this using the Audit Analytics corporate litigation database, which contains similar information for patent lawsuits involving public firms, and the dataset from Seaman (2015) covering injunction decisions in patent trials.

We merge these data sets together. We construct a firm-year panel covering the period 2003 to 2020. It includes all Compustat observations for all firms that hold at least one patent in the GCPD.[7] We assign each firm a technology class c and an industry j by the following procedure. We use the first digit of the GCPD technology-class classification to construct nine different potential technology classes. We assign each firm a time-invariant technology class using the GCPD classification.[8] We use SIC codes from Compustat to construct the Fama-French twelve industries.[9] We exclude firms in Finance, Utilities, or Other. We then aggregate the remaining nine industries into four industry groups.[10] Thus, we have nine

[5] See `https://patents.darden.virginia.edu/`.

[6] See `https://www.uspto.gov/ip-policy/economic-research/research-datasets/patent-litigation-docket-reports-data`.

[7] We exclude firm years with missing SIC codes, assets, or Compustat identifiers. We exclude firm years with under $50 million in sales.

[8] If a firm has multiple patents, we take the modal technology class across its patents. If there is a tie, we consider the firm's technology class to be missing. If the firm's modal technology class is the "missing" technology class, we consider the firm's technology class to be missing.

[9] See `https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library/det_12_ind_port.html`.

[10] Based on the Fama-French industry classification, our analysis focuses on four primary industry groups:

technology classes and four industry groups in our data. Since we analogously solve our model assuming there are nine technology classes and four industry groups, the reduction in the number of industry groups eases computation.

We also construct a separate lawsuit-level dataset. To construct this dataset, we begin with the Federal Judicial Center (FJC) database.[11] This contains every civil lawsuit filed in federal courts. Using the FJC classification, we isolate patent lawsuits. For each patent lawsuit, the FJC contains an indicator equal to one if a lawsuit is settled out of court. It also contains an indicator equal to one if a lawsuit goes to trial. Further, it contains a variable specifying whether the plaintiff or defendant won in trial. We supplement this with a similar dataset from Audit Analytics, which contains similar information about patent lawsuits involving publicly traded firms. We merge this with the USPTO litigation database, which allows us to count the number of docket entries in each lawsuit's PACER docket and to identify and exclude patent invalidity lawsuits. Thus, our sample of lawsuits does not include any invalidity suits. Using information from all of these datasets, as well as information from the GCPD dataset, we merge this lawsuit dataset to Compustat. For lawsuits that we merge to Compustat, this merge allows us to identify the industry and technology class of the defendant and the plaintiff. Our final lawsuit-level dataset contains the industry and technology class of the defendant and the plaintiff, the number of docket entries in the lawsuit's PACER docket, the outcome of the lawsuit (settlement versus trial), and the trial outcome (plaintiff or defendant victory) for lawsuits ending in trials.

We use the merge between our lawsuit-level dataset and Compustat to construct a firm-year level indicator for a firm filing a lawsuit. This is a key variable in our firm-year panel.

(i) Manufacturing: This encompasses Fama-French industry classifications 1 (Consumer NonDurables – Food, Tobacco, Textiles, Apparel, Leather, Toys), 2 (Consumer Durables – Cars, TVs, Furniture, Household Appliances), and 3 (Manufacturing – Machinery, Trucks, Planes, Office Furniture, Paper, Commercial Printing); (ii) Extraction and Chemicals: This group includes Fama-French industry classifications 4 (Oil, Gas, and Coal Extraction and Products) and 5 (Chemicals and Allied Products); (iii) Information and Communication Technology (ICT): This encompasses Fama-French industry classifications 6 (Business Equipment – Computers, Software, and Electronic Equipment) and 7 (Telephone and Television Transmission); (iv) Services: This group includes Fama-French industry classifications 9 (Wholesale, Retail, and Some Services such as Laundries, Repair Shops) and 10 (Healthcare, Medical Equipment, and Drugs). We exclude firms categorized under the 'other' industry classification. Additionally, we exclude firms operating in the financial and utility sectors due to their heavy regulation and distinct business models compared to other industries.

[11] See https://www.fjc.gov/research/idb/civil-cases-filed-terminated-and-pending-sy-1988-present.

Using permanent injunction data from Seaman (2015) combined with our industry-technology classification system, we construct an industry-technology level dataset containing the average rate at which injunctions are granted in lawsuits filed by firms in a given industry and technology class. We augment this dataset using a dataset from the USPTO, which provides a text summary of each docket entry in each lawsuit. We use this dataset to identify docket entries that mention an injunction, then use GPT-o3 API to classify when a permanent injunction is granted. We combine this with the Seaman (2015) data to measure the average injunction rate ζ_{cj} in each industry and technology class.

Finally, for a separate empirical moment, we download quarterly data on year-over-year US GDP growth from the Federal Reserve Bank of St. Louis.[12]

A.2. Empirical Moments

We use the data described above to calculate 24 empirical moments. We estimate our model by minimizing the distance between these empirical moments and their model-implied counterparts. We now describe the empirical moments.

First, we calculate the average annual GDP growth rate in our sample period: it is 1.91%.

Second, we use our firm-year panel to calculate several moments. We calculate firm-year-level sales growth,[13] take an average for each technology class, then take a standard deviation across technology classes. The corresponding standard deviation, 1.86%, corresponds to the variation in sales growth across technology classes. Next, we calculate R&D intensity as the ratio of R&D spending to sales. We calculate a sample average of 6.97%. We then take the average R&D intensity for each technology class and take a standard deviation across technology classes: we find this standard deviation is 3.67%. We similarly calculate that the skewness of R&D intensity across technology classes is 0.481. Next, we measure whether firms in technologies with a higher share of sales in the economy tend to have higher R&D spending. For each technology class in each year, we calculate the share of all sales attributable to firms in that technology class. We regress the logarithm of R&D spending on this sales share, controlling for year fixed effects, and find a regression coefficient on the sales share that is equal to 0.027.[14]

[12] See https://fred.stlouisfed.org/series/USAGDPRQPSMEI.

[13] We calculate this as the sales for firm i in year t minus the sales for the same firm in the previous year, divided by sales in the previous year. The previous year is the most recent year prior to t that appears in Compustat.

[14] We normalize the dependent variable, the logarithm of R&D spending, by subtracting the sample mean of

Next, we study our indicator for a firm being a plaintiff in a patent lawsuit in a given year. The average for this indicator is 12.67% in our sample. We take an average for each industry j and technology class c. Taking a standard deviation across industries and technology classes, we find the standard deviation is 8.64%. We then explore how high-litigation technology classes differ from low-litigation technology classes. We take an average of our litigation indicator for each technology class. We similarly take the average sales growth and R&D intensity at the technology class level. Taking a correlation across technology classes, we find a correlation between litigation and sales growth equal to 0.764. We find a correlation between litigation and R&D intensity equal to 0.703.

We also use the firm-year panel to calculate the rate at which new firms enter the economy. For each firm-year observation, we define an indicator equal to one if that year is the first time that the firm appears in our panel. Excluding 2003, the first year in our panel, we calculate that the mean of this indicator is equal to 3.55%.

We then turn to our lawsuit-level dataset. Conditional on a plaintiff having the same industry as the defendant and a lawsuit going to trial, we find that 47.17% of plaintiffs win in trial.[15] Conditional on a plaintiff having a different industry than the defendant and a lawsuit going to trial, we find that 41.18% of plaintiffs win in trial. Across all of these cases that end in trial, the plaintiff and defendant share the same industry in 86.18% of cases. Overall, we find that 58.83% of lawsuits end in settlement.[16]

Next, we use a US courts survey to measure an annual time series of corporate litigation spending. Specifically, in each year over the period 2003 to 2008, the survey reports the average ratio of litigation spending to revenue for public US corporations. Calculating an average across years, we find that an average public US firm spends roughly 0.6% of its revenue on litigation costs annually.[17]

Next, we calculate averages of several variables at the industry-by-technology-class-by-year ("$j-c-t$") level. In this $j-c-t$ level dataset, we regress Plaintiff_{cjt} (the fraction of firms

the log R&D spending and dividing by the standard deviation of the log R&D spending.

[15]When we calculate these statistics, we keep only cases in which either the plaintiff wins a trial or the defendant wins a trial. We exclude cases in which the FJC indicates that both parties had a partial victory.

[16]When we calculate the settlement rate, we use our sample of patent lawsuits from the FJC. We exclude cases that are transferred or dismissed for reasons other than settlement. We therefore only keep cases that are settled or conclude with a judgment.

[17]See Figure 7 in https://www.uscourts.gov/sites/default/files/litigation_cost_survey_of_major_companies_0.pdf. We use the time series for consistent reporters over the period 2003 to 2008.

in technology class c and industry j that file a lawsuit in year t) on $j-c-t$ level averages of profitability and R&D intensity.[18] We match the coefficients on average profitability and R&D intensity, which are 0.287 and 0.945, respectively. In this same dataset, we regress Same Industry$_{cjt}$ (the fraction of all lawsuits filed by firms in bin $j-c-t$ that are filed against firms in industry j) on $j-c-t$ level averages of profitability and R&D intensity. We match the coefficient on profitability (1.09) and R&D intensity (1.088). We also regress Litigation Cost$_{cjt}$ on average profitability and match the coefficient 0.034.[19]

In a similar exercise, we average several variables at the industry-by-technology-class level and calculate: (i) the standard deviation across industries j and technology classes c of the fraction of lawsuits in bin $j-c$ that end in settlement (0.16); (ii) the standard deviation across $j-c$ bins of the fraction of lawsuits in which the plaintiff wins in trial, conditional on the lawsuit going to trial and the defendant sharing the *same* industry (0.14); and (iii) the standard deviation across $j-c$ bins of the fraction of lawsuits in which the plaintiff wins in trial, conditional on the lawsuit going to trial and the defendant being in a *different* industry $j' \neq j$ (0.19).

We directly calibrate ω_j to match the share of all sales attributable to industry j.[20] Likewise, we directly calibrate η_{cj} to match the share of all new-entrant sales attributable to industry j and technology class c.[21]

Finally, we use a combination of model parameters and empirical patterns to determine the distribution of $\lambda_c, \{\bar{\tau}_{1cj}, \tau^l_{2cj}, \tau^h_{2cj}\}, \chi_{cj}$ across technology classes and industries. For each model object (e.g., $\bar{\tau}_{1cj}$), we start with an empirical proxy for the object that we can directly measure in the data (e.g., the empirical defendant win rate for same-industry lawsuits that end in trial in bin $j-c$), then rescale the proxy to have a mean and standard deviation consistent with the corresponding model parameters (e.g., $\mu_{\bar{\tau}_1}, \sigma_{\bar{\tau}_1}$), where the model parameters

[18]We define profitability as the ratio of Compustat item oibdp to sales. We include year and industry fixed effects.

[19]Litigation costs are proxied by the ratio of the length of the lawsuit docket to the plaintiff's revenue and normalized to have the same mean as the time-series average of litigation spending over revenue. We calculate this for each lawsuit then average across lawsuits filed by plaintiffs in bin $j-c-t$.

[20]In each industry j and year t, we calculate the fraction of all sales attributable to firms in industry j. For each j, we average across years to calculate ω_j.

[21]We call a firm i a new entrant in year t if it is the first year over the period 2003 to 2020 in which the firm appears in Compustat. In each industry j and technology class c and year t, we calculate the fraction of new-entrant sales attributable to industry j and technology class c. Excluding the first year 2003, in which all firms are new entrants, we average across years to construct a share η_{cj} of new-entrant sales.

are estimated through GMM as described below. The rescaled proxy serves as the value for the model object (e.g., $\bar{\tau}_{1cj}$) in the model.[22] In this sense, the empirical proxies determine the shape of the distribution across $j-c$ bins, but we do not assume that the empirical proxies are identical to the model objects. Thus, for each guess of the model parameters in the GMM algorithm, this parameter-based rescaling of observed statistics determines the distribution of these model objects in our model solution. The GMM estimation then delivers the parameters that determine the rescaling of the distributions.

A.3. Calculating the GMM Weighting Matrix and Objective

We estimate our parameters by GMM. We calculate the covariance matrix Ω for our 24 empirical moments. We calculate the efficient weighting matrix W as the inverse of Ω. We define a vector $\boldsymbol{\theta}$ containing the 21 estimated model parameters. We define a vector M_{emp} containing our 24 empirical moments. We define a vector $M(\boldsymbol{\theta})$ containing the model counterparts, which depend on the vector $\boldsymbol{\theta}$. We estimate $\boldsymbol{\theta}$ by minimizing the GMM objective:

$$\boldsymbol{\theta}_{\text{GMM}} = \underset{\boldsymbol{\theta}}{\arg\min}\,(M_{\text{emp}} - M(\boldsymbol{\theta}))' W (M_{\text{emp}} - M(\boldsymbol{\theta})). \tag{A.1}$$

We calculate GMM asymptotic standard errors using the standard sandwich formula.

We calculate the covariance matrix Ω by bootstrapping. We face a challenge: some moments are calculated in a firm-year panel while others are calculated at a different unit of observation (e.g., the time series of GDP growth or the settlement rate across lawsuits). To overcome this challenge, we must make a simplifying assumption: the covariance between two moments calculated in different samples is zero. This assumption allows us to stack covariance matrices. Specifically, we first bootstrap a covariance matrix for 10 moments calculated in our firm-year panel. We then stack this on top of a covariance matrix for 11 moments calculated in a smaller sample of lawsuits. We impute zeros for the covariances between the first 10 moments and the latter 11 moments. We do the same for the following moments: the average GDP growth rate, the average lawsuit settlement rate, and the average ratio of litigation spending to revenue, which is based on time-series data from an external source. Our approach is similar to the approaches used in Acemoglu et al. (2018) and Bordalo et al. (2020). For each bootstrapping exercise, we bootstrap 500 samples from our original sample.

[22] We impose the constraint regarding the gap between each τ^{l}_{2cj} and τ^{h}_{2cj} after rescaling.

A.4. Distance to Default

For one analysis, we split our sample by the probability of default. We calculate this probability using the "Naive Merton Distance to Default" as in Farre-Mensa and Ljungqvist (2016), following the implementation described in Farre-Mensa and Ljungqvist (2016). Specifically, we merge our Compustat data with monthly equity return data from CRSP. Using monthly equity returns, we calculate annual equity volatility estimates and annual returns as in Farre-Mensa and Ljungqvist (2016). We then use the equations presented in Farre-Mensa and Ljungqvist (2016) to calculate the distance to default and the annualized probability of default.

B. Theory Appendix

B.1. Additional Calculations for the Model

B.1.1. Product-line owner's static pricing problem

Bertrand competition implies that only the productivity leader for any differentiated good produces a positive quantity. Because the monopoly price tends to infinity, the final good production function assures that the leader always chooses to follow a limit pricing strategy. For a leader with productivity q_{ijt} and technology class c, the productivity of the most productive competitor is $q_{ijt}/(1+\lambda_c)$, and the limit price is therefore $w_t(1+\lambda_c)/q_{ijt}$. At this price, if the competitor produced $y > 0$, it would need labor $l = y(1+\lambda_c)/q_{ijt}$ and would therefore make profit

$$py - wl = (w_t(1+\lambda_c)/q_{ijt})y - w(y(1+\lambda_c)/q_{ijt}) = 0.$$

The static profit flow of the leader for good i in industry j at time t is

$$\pi_{ijt} = \left(p_{ijt} - \frac{w_t}{q_{ijt}}\right) y_{ijt} = \left(\frac{w_t(1+\lambda_c)}{q_{ijt}} - \frac{w_t}{q_{ijt}}\right) \frac{q_{ijt}\omega_j Y_t}{w_t(1+\lambda_c)} = \frac{\lambda_c}{1+\lambda_c}\omega_j Y_t, \tag{B.1}$$

where the first equality uses $y = ql$ and the second equality uses the final good producer's demand at the limit price.

B.1.2. The defendant's settlement acceptance cutoff

The defendant accepts a settlement offer if and only if

$$\begin{aligned} V_{cjt}(n+1) - V_{cjt}(n) - s &> [\tau + (1-\tau)(1-\zeta) - \Lambda]\big(V_{cjt}(n+1) - V_{cjt}(n)\big) \\ ((1-\tau)\zeta + \Lambda)\big(V_{cjt}(n+1) - V_{cjt}(n)\big) &> s \end{aligned} \tag{B.2}$$

where $(1-\tau)\zeta$ is the probability of an injunction conditional on a trial, and Λ reflects the value loss due to legal fees. $\zeta \in \{\zeta_{1cj}, \zeta_{2cj}\}$ and $\Lambda \in \{\Lambda_1, \Lambda_2\}$ depend on the infringement type. Equation (B.2) shows that the defendant accepts the settlement offer only if the value of removing the injunction likelihood plus avoiding legal fees is above a threshold value of s, denoted $\bar{s}_{cjt}(n,\tau)$, that leaves the defendant indifferent.

B.2. Proof of Proposition 1

Proposition 1. *When successful innovation by a firm with technology class c in industry j with n product lines leads to a type 2 patent infringement, the following are true in the subgame perfect equilibrium of the litigation game:*

1. *The ex-ante probability that the plaintiff hires a legal team is*

$$p^{LT}_{2,cjt} \equiv \mathbb{P}\left(\gamma \leq \frac{(1-\tau^{l}_{2cj}+\Lambda_2/\zeta_{2cj})^2\zeta_{2cj}(V_{cjt}(n+1)-V_{cjt}(n))}{4(\tau^{h}_{2cj}-\tau^{l}_{2cj})Y_t}\right) \tag{B.3}$$

2. *The optimal settlement offer made by the plaintiff is*

$$s^* \equiv \frac{(1-\tau^{l}_{2cj}+\Lambda_2/\zeta_{2cj})\zeta_{2cj}(V_{cjt}(n+1)-V_{cjt}(n))}{2} \tag{B.4}$$

3. *The defendant accepts the settlement if* $\tau \leq \tau^* \equiv \frac{1+\tau^{l}_{2cj}+\Lambda_2/\zeta_{2cj}}{2}$*, and rejects otherwise. The ex-ante acceptance probability is* $\mathbb{P}(s^* < \bar{s}_{cjt}(n,\tau)) = \frac{1}{2}\frac{1-\tau^{l}_{2cj}+\Lambda_2/\zeta_{2cj}}{\tau^{h}_{2cj}-\tau^{l}_{2cj}}$.

4. *The expected payoff of the plaintiff is*

$$W^{plain}_{2,cjt} \equiv p^{LT}_{2,cjt}\frac{(1-\tau^{l}_{2cj}+\Lambda_2/\zeta_{2cj})^2\zeta_{2cj}(V_{cjt}(n+1)-V_{cjt}(n))}{4(\tau^{h}_{2cj}-\tau^{l}_{2cj})} - Y_t\int_0^{\frac{(1-\tau^{l}_{2cj}+\Lambda_2/\zeta_{2cj})^2\zeta_{2cj}(V_{cjt}(n+1)-V_{cjt}(n))}{4(\tau^{h}_{2cj}-\tau^{l}_{2cj})Y_t}} \gamma d\Gamma_{cj}(\gamma) \tag{B.5}$$

5. *The expected payoff of the defendant is*

$$
\begin{aligned}
W^{def}_{2,cjt} \equiv & \Bigg((1-p^{LT}_{2,cjt}) + p^{LT}_{2,cjt}\Bigg[\left(1-\frac{(1-\tau^l_{2cj}+\Lambda_2/\zeta_{2cj})\zeta_{2cj}}{2}\right)\left(\frac{1}{2}\frac{1-\tau^l_{2cj}+\Lambda_2/\zeta_{2cj}}{\tau^h_{2cj}-\tau^l_{2cj}}\right) \\
& +(1-\zeta_{2cj}-\Lambda_2)\left(1-\frac{1}{2}\frac{1-\tau^l_{2cj}+\Lambda_2/\zeta_{2cj}}{\tau^h_{2cj}-\tau^l_{2cj}}\right) \\
& +\frac{\zeta_{2cj}}{2(\tau^h_{2cj}-\tau^l_{2cj})}\left((\tau^h_{2cj})^2-\frac{(1+\tau^l_{2cj}+\Lambda_2/\zeta_{2cj})^2}{4}\right)\Bigg]\Bigg)(V_{cjt}(n+1)-V_{cjt}(n)) \qquad \text{(B.6)}
\end{aligned}
$$

Proof. We consider the decision problem of a plaintiff facing a type 2 patent infringement in which they don't face any risk of losing product lines. They must choose a take-it-or-leave-it settlement offer s without knowing the realization of τ – the defendant's probability of winning at court. Their problem is written as

$$\max_{s\geq 0}\left\{s\mathbb{P}(s\leq \bar{s}_{cjt}(n,\tau))\right\} \qquad \text{(B.7)}$$

where the second term is the probability that the offer is accepted. We can rewrite this probability as

$$
\begin{aligned}
\mathbb{P}(s\leq \bar{s}_{cjt}(n,\tau)) &= \mathbb{P}\left(s\leq((1-\tau)\zeta_{2cj}+\Lambda_2)\left(V_{cjt}(n+1)-V_{cjt}(n)\right)\right) \\
&= \mathbb{P}\left(\tau\leq 1-\frac{s}{\zeta_{2cj}\left(V_{cjt}(n+1)-V_{cjt}(n)\right)}+\frac{\Lambda_2}{\zeta_{2cj}}\right) \\
&= \int_{-\infty}^{1-s/(\zeta_{2cj}(V_{cjt}(n+1)-V_{cjt}(n)))+\Lambda_2/\zeta_{2cj}} dT_{2cj}(\tau)
\end{aligned} \qquad \text{(B.8)}
$$

which, under the distributional assumption, becomes

$$
\mathbb{P}(s\leq \bar{s}_{cjt}(n,\tau)) = \begin{cases} 0 & \text{if } 1-s/(\zeta_{2cj}(V_{cjt}(n+1)-V_{cjt}(n)))+\Lambda_2/\zeta_{2cj}<\tau^l_{2cj} \\ 1 & \text{if } 1-s/(\zeta_{2cj}(V_{cjt}(n+1)-V_{cjt}(n)))+\Lambda_2/\zeta_{2cj}>\tau^h_{2cj} \\ \frac{1-s/(\zeta_{2cj}(V_{cjt}(n+1)-V_{cjt}(n))+\Lambda_2/\zeta_{2cj}-\tau^l_{2cj}}{\tau^h_{2cj}-\tau^l_{2cj}} & \text{otherwise} \end{cases} \qquad \text{(B.9)}
$$

Note that the optimal s must be such that $\tau^l_{2cj}\leq 1-s/(\zeta_{2cj}(V_{cjt}(n+1)-V_{cjt}(n)))+\Lambda_2/\zeta_{2cj}\leq$

τ^h_{2cj}.[23] Then we can rewrite the objective function over this range as

$$\frac{s(1-\tau^l_{2cj}+\Lambda_2/\zeta_{2cj})-s^2/(\zeta_{2cj}(V_{cjt}(n+1)-V_{cjt}(n))}{\tau^h_{2cj}-\tau^l_{2cj}} \tag{B.10}$$

with the first order condition delivering

$$\begin{aligned} 1-\tau^l_{2cj}+\Lambda_2/\zeta_{2cj} &= \frac{2s}{\zeta_{2cj}(V_{cjt}(n+1)-V_{cjt}(n))} \\ s &= \frac{(1-\tau^l_{2cj}+\Lambda_2/\zeta_{2cj})\zeta_{2cj}(V_{cjt}(n+1)-V_{cjt}(n))}{2} \equiv s^* \end{aligned} \tag{B.11}$$

which pins down the optimal s if the solution is interior. Given this expression, the cut-off τ for which the defendant is indifferent is given as

$$\tau^* = \frac{1+\tau^l_{2cj}+\Lambda_2/\zeta_{2cj}}{2} \tag{B.12}$$

If $\tau^* \leq \tau^h_{2cj}$, then the solution is interior, and the optimal s is given by equation (B.4). If not, then we have a corner solution:

$$s = (1-\tau^h_{2cj}+\Lambda_2/\zeta_{2cj})\zeta_{2cj}(V_{cjt}(n+1)-V_{cjt}(n)) \tag{B.13}$$

In the case of an interior solution, the identity for the acceptance probability becomes

$$\mathbb{P}(s < \bar{s}_{cjt}(n,\tau)) = \frac{1}{2}\frac{1-\tau^l_{2cj}+\Lambda_2/\zeta_{2cj}}{\tau^h_{2cj}-\tau^l_{2cj}} \tag{B.14}$$

which is independent of t and n. The optimal expected rent is then

$$s\mathbb{P}(s < \bar{s}_{cjt}(n,\tau)) = \frac{(1-\tau^l_{2cj}+\Lambda_2/\zeta_{2cj})^2\zeta_{2cj}(V_{cjt}(n+1)-V_{cjt}(n))}{4(\tau^h_{2cj}-\tau^l_{2cj})} \tag{B.15}$$

In the case of a corner solution, the probability of acceptance is unity, and the optimal expected rent is simply equal to equation (B.13). Given our assumption that $1+\tau^l_{2cj}+\Lambda_2/\zeta_{2cj} \leq 2\tau^h_{2cj}$,

[23] Below τ^l_{2cj}, the probability of acceptance is zero, and so are the extracted rents. Above τ^h_{2cj}, the plaintiff is asking for a smaller payment even though it does not increase the probability of acceptance, thus losing out on rents. Both are suboptimal.

the solution is always interior.

Given the optimal expected rent expression, we can now turn to the plaintiff's decision to hire a legal team or not. The plaintiff will choose to hire a legal team if the expected rent is higher than the cost γY_t where γ is drawn from the distribution $\Gamma_{cj}(\gamma)$. The probability of hiring a legal team is given by

$$p^{LT}_{2,cjt} \equiv \mathbb{P}\left(\gamma \leq \frac{(1-\tau^l_{2cj}+\Lambda_2/\zeta_{2cj})^2\zeta_{2cj}(V_{cjt}(n+1)-V_{cjt}(n))}{4(\tau^h_{2cj}-\tau^l_{2cj})Y_t}\right) \tag{B.16}$$

and the expected rents conditional on a type 2 patent infringement minus legal team cost is given as

$$\begin{aligned} W^{plain}_{2,cjt} \quad \equiv \quad & p^{LT}_{2,cjt}\frac{(1-\tau^l_{2cj}+\Lambda_2/\zeta_{2cj})^2\zeta_{2cj}(V_{cjt}(n+1)-V_{cjt}(n))}{4(\tau^h_{2cj}-\tau^l_{2cj})} \\ & -Y_t\int_0^{\frac{(1-\tau^l_{2cj}+\Lambda_2/\zeta_{2cj})^2\zeta_{2cj}(V_{cjt}(n+1)-V_{cjt}(n))}{4(\tau^h_{2cj}-\tau^l_{2cj})Y_t}} \gamma d\Gamma_{cj}(\gamma) \end{aligned} \tag{B.17}$$

Turning to the defendant's side, conditional on a type 2 patent infringement, they will receive a settlement offer only if the plaintiff chooses to hire a legal team, the probability of which is $p^{LT}_{2,cjt}$. Conditional on receiving a settlement offer, their expected payoff is

$$
\begin{aligned}
&\int_{\tau^l_{2cj}}^{\tau^*} \left(V_{cjt}(n+1)-V_{cjt}(n)-s^*\right) dT_{2cj}(\tau) \\
&+\int_{\tau^*}^{\tau^h_{2cj}} [\tau+(1-\tau)(1-\zeta_{2cj})-\Lambda_2]\left(V_{cjt}(n+1)-V_{cjt}(n)\right) dT_{2cj}(\tau) \\
= &\left(1-\frac{(1-\tau^l_{2cj}+\Lambda_2/\zeta_{2cj})\zeta_{2cj}}{2}\right)(V_{cjt}(n+1)-V_{cjt}(n))\left(\frac{1}{2}\frac{1-\tau^l_{2cj}+\Lambda_2/\zeta_{2cj}}{\tau^h_{2cj}-\tau^l_{2cj}}\right) \\
&+(1-\zeta_{2cj}-\Lambda_2)(V_{cjt}(n+1)-V_{cjt}(n))\left(1-\frac{1}{2}\frac{1-\tau^l_{2cj}+\Lambda_2/\zeta_{2cj}}{\tau^h_{2cj}-\tau^l_{2cj}}\right) \\
&+\zeta_{2cj}(V_{cjt}(n+1)-V_{cjt}(n))\int_{\tau^*}^{\tau^h_{2cj}} \frac{\tau}{\tau^h_{2cj}-\tau^l_{2cj}} d\tau \\
= &\Bigg[\left(1-\frac{(1-\tau^l_{2cj}+\Lambda_2/\zeta_{2cj})\zeta_{2cj}}{2}\right)\left(\frac{1}{2}\frac{1-\tau^l_{2cj}+\Lambda_2/\zeta_{2cj}}{\tau^h_{2cj}-\tau^l_{2cj}}\right) \\
&+(1-\zeta_{2cj}-\Lambda_2)\left(1-\frac{1}{2}\frac{1-\tau^l_{2cj}+\Lambda_2/\zeta_{2cj}}{\tau^h_{2cj}-\tau^l_{2cj}}\right) \\
&+\frac{\zeta_{2cj}}{2(\tau^h_{2cj}-\tau^l_{2cj})}\left((\tau^h_{2cj})^2-\frac{(1+\tau^l_{2cj}+\Lambda_2/\zeta_{2cj})^2}{4}\right)\Bigg](V_{cjt}(n+1)-V_{cjt}(n)) \qquad \text{(B.18)}
\end{aligned}
$$

Therefore, the defendant's expected payoff conditional on a type 2 patent infringement and before learning whether they will receive a settlement offer can be written as

$$
\begin{aligned}
W^{def}_{2,cjt} \equiv &\Bigg((1-p^{LT}_{2,cjt})+p^{LT}_{2,cjt}\Bigg[\left(1-\frac{(1-\tau^l_{2cj}+\Lambda_2/\zeta_{2cj})\zeta_{2cj}}{2}\right)\left(\frac{1}{2}\frac{1-\tau^l_{2cj}+\Lambda_2/\zeta_{2cj}}{\tau^h_{2cj}-\tau^l_{2cj}}\right) \\
&+(1-\zeta_{2cj}-\Lambda_2)\left(1-\frac{1}{2}\frac{1-\tau^l_{2cj}+\Lambda_2/\zeta_{2cj}}{\tau^h_{2cj}-\tau^l_{2cj}}\right) \\
&+\frac{\zeta_{2cj}}{2(\tau^h_{2cj}-\tau^l_{2cj})}\left((\tau^h_{2cj})^2-\frac{(1+\tau^l_{2cj}+\Lambda_2/\zeta_{2cj})^2}{4}\right)\Bigg]\Bigg)(V_{cjt}(n+1)-V_{cjt}(n)) \qquad \text{(B.19)}
\end{aligned}
$$

Note that the whole expression is linear in the expected change in firm value if they take over the product line, $V_{cjt}(n+1)-V_{cjt}(n)$, which will be of use in deriving a closed-form expression for the firm value function. □

B.3. Proof of Proposition 2

Proposition 2. *Suppose $V_{cjt}(n)$ is linear in n and $\Lambda_1 = 0$. When successful innovation by a firm with technology class c in industry j with n^d product lines leads to a type 1 patent infringement on the IP of an incumbent with n product lines, the following are true in the subgame perfect equilibrium of the litigation game:*

1. *The ex-ante probability that the plaintiff hires a legal team is*

$$p^{LT}_{1,cjt} \equiv \mathbb{P}\left(\gamma \leq \left(-\zeta_{1cj} + \zeta_{1cj}\frac{\tau^h_{1cj} + \tau^l_{1cj}}{2}\right)\frac{(V_{cjt}(n-1) - V_{cjt}(n))}{Y_t}\right) \tag{B.20}$$

2. *Due to adverse selection, the plaintiff never chooses to settle out of court. That is, the plaintiff makes an offer that the defendant always rejects, independent of the realization of τ.*

3. *The expected payoff of the plaintiff is*

$$\begin{aligned} W^{plain}_{1,cjt} \quad \equiv \quad & p^{LT}_{1,cjt}(V_{cjt}(n-1) - V_{cjt}(n))\left(1 - \zeta_{1cj} + \zeta_{1cj}\frac{\tau^h_{1cj} + \tau^l_{1cj}}{2}\right) \\ & -Y_t\int_0^{\left(-\zeta_{1cj}+\zeta_{1cj}\frac{\tau^h_{1cj}+\tau^l_{1cj}}{2}\right)\frac{(V_{cjt}(n-1)-V_{cjt}(n))}{Y_t}} \gamma d\Gamma_{cj}(\gamma) \\ & +(1-p^{LT}_{1,cjt})(V_{cjt}(n-1) - V_{cjt}(n)) \end{aligned} \tag{B.21}$$

4. *The expected payoff of the defendant is*

$$\begin{aligned} W^{def}_{1,cjt} \quad \equiv \quad & p^{LT}_{1,cjt}(V_{cjt}(n^d+1) - V_{cjt}(n^d))\left(1 - \zeta_{1cj} + \zeta_{1cj}\frac{\tau^h_{1cj} + \tau^l_{1cj}}{2}\right) \\ & +(1-p^{LT}_{1,cjt})(V_{cjt}(n^d+1) - V_{cjt}(n^d)) \end{aligned} \tag{B.22}$$

Proof. We consider the decision problem of a plaintiff facing a type 1 patent infringement which means they are the owner of the product line that is facing the risk of creative destruction. We further know that the plaintiff and the defendant share the same technology class c. Unlike a type 2 infringement, this time the plaintiff cares about more than the potential settlement they can extract from the defendant, since settling out of court also means they lose their product line for sure. Assume the distribution $T_{1cj}(\tau)$ is the continuous uniform distribution $U(\tau^l_{1cj}, \tau^h_{1cj})$ with $0 \le \tau^l_{1cj} < \tau^h_{1cj} \le 1$. Then the plaintiff's problem is written as

$$\max_{s\ge 0} \quad \Bigg\{ \quad \int_{\tau^l_{1cj}}^{1-s/(\zeta_{1cj}(V_{cjt}(n^d+1)-V_{cjt}(n^d)))+\Lambda_1/\zeta_{1cj}} \big(V_{cjt}(n-1) - V_{cjt}(n) + s\big)\, dT_{1cj}(\tau)$$

$$+ \int_{1-s/(\zeta_{1cj}(V_{cjt}(n^d+1)-V_{cjt}(n^d)))+\Lambda_1/\zeta_{1cj}}^{\tau^h_{1cj}} (\tau + (1-\tau)(1-\zeta_{1cj}))(V_{cjt}(n-1) - V_{cjt}(n))\, dT_{1cj}(\tau) \Bigg\} \quad \text{(B.23)}$$

where the first integral is the expected payoff from defendants who accept the settlement and the second integral is the expected payoff from those who reject. The term $V_{cjt}(n-1) - V_{cjt}(n)$ is negative, and reflects the cost of losing the product line. In the cases when the defendant accepts, the plaintiff gives up their $(1-\tau)\zeta_{1cj}$ chance of retaining their product line in exchange for a settlement amount s.

Note that there is an inherent adverse selection problem here: Conditional on a settlement offer s, only firms with the lowest chance of winning the trial τ will accept. From the defendant's problem, we know that a defendant strictly prefers the settlement offer if and only if

$$((1-\tau)\zeta_{1cj} + \Lambda_1)(V_{cjt}(n^d+1) - V_{cjt}(n^d)) \quad > \quad s \qquad \text{(B.24)}$$

where n^d stands for the defendant's number of product lines. On the other hand, the difference in the plaintiff's payoff in the case of acceptance is

$$(1-\tau)\zeta_{1cj}(V_{cjt}(n-1) - V_{cjt}(n)) + s \qquad \text{(B.25)}$$

Consider the defendant with the threshold τ^* who is indifferent, and recall $\Lambda_1 = 0$. Then the abovementioned difference becomes

$$(1-\tau^*)\zeta_{1cj}(V_{cjt}(n-1) - V_{cjt}(n)) + (1-\tau^*)\zeta_{1cj}(V_{cjt}(n^d+1) - V_{cjt}(n^d)) \qquad \text{(B.26)}$$

which is exactly zero if $V_{cjt}(n) - V_{cjt}(n-1) = V_{cjt}(n^d+1) - V_{cjt}(n^d)$, that is, if the value change from having one more product line in industry j for firms with technology class c is the same regardless of how many product lines the company owns, n. We will later on show that this is exactly the case in a stationary equilibrium, since the value function of the firm will turn out to be linear in n. But this highlights the adverse selection problem: Even in the best case scenario, the plaintiff gains exactly zero from the firm with the highest probability of winning the trial among those who accept. For all other firms who accept that have a probability of winning the trial below the threshold firm with $\tau < \tau^*$, the abovementioned difference becomes

$$(1-\tau)\zeta_{1cj}(V_{cjt}(n-1)-V_{cjt}(n))+(1-\tau^*)\zeta_{1cj}(V_{cjt}(n^d+1)-V_{cjt}(n^d)) \tag{B.27}$$

which is strictly negative if $V_{cjt}(n)-V_{cjt}(n-1) = V_{cjt}(n^d+1)-V_{cjt}(n^d)$. This means the plaintiff would be making no extra return from the threshold firm that accepts, and would make an extra loss from every other firm that accepts. As a consequence, it is optimal for a plaintiff to always make settlement offers that will be rejected by every defendant – the adverse selection problem completely undermines any chance of out-of-court settlements for type 1 patent infringements.[24]

Having figured out that plaintiffs always pick a high enough settlement amount s such that every defendant will reject, we can calculate the expected payoffs for the agents. The payoff of the plaintiff from going to court is:

$$\begin{aligned}
& \int_{\tau^l_{1cj}}^{\tau^h_{1cj}} (\tau+(1-\tau)(1-\zeta_{1cj}))(V_{cjt}(n-1)-V_{cjt}(n))dT_{1cj}(\tau) \\
= \quad & (V_{cjt}(n-1)-V_{cjt}(n))\left(1-\zeta_{1cj}+\zeta_{1cj}\int_{\tau^l_{1cj}}^{\tau^h_{1cj}} \tau dT_{1cj}(\tau)\right) \\
= \quad & (V_{cjt}(n-1)-V_{cjt}(n))\left(1-\zeta_{1cj}+\zeta_{1cj}\frac{\tau^h_{1cj}+\tau^l_{1cj}}{2}\right)
\end{aligned} \tag{B.28}$$

Given this expression, we can turn to the plaintiff's decision to hire a legal team or not. If the plaintiff does not hire a legal team, then their payoff is simply $V_{cjt}(n-1)-V_{cjt}(n)$ since they will lose their product line for certain. Therefore, they will strictly prefer to hire a legal team if and only if

$$\begin{aligned}
(V_{cjt}(n-1)-V_{cjt}(n))\left(1-\zeta_{1cj}+\zeta_{1cj}\frac{\tau^h_{1cj}+\tau^l_{1cj}}{2}\right)-\gamma Y_t \quad &> \quad (V_{cjt}(n-1)-V_{cjt}(n)) \\
\left(-\zeta_{1cj}+\zeta_{1cj}\frac{\tau^h_{1cj}+\tau^l_{1cj}}{2}\right)(V_{cjt}(n-1)-V_{cjt}(n)) \quad &> \quad \gamma Y_t
\end{aligned} \tag{B.29}$$

[24] Note that this result owes to two facts: (1) The defendant and the plaintiff have the same technology class c in type 1 patent infringements, and (2) the firm value function is linear in n.

Then the probability of hiring a legal team is given by

$$p^{LT}_{1,cjt} \equiv \mathbb{P}\left(\gamma \leq \left(-\zeta_{1cj} + \zeta_{1cj}\frac{\tau^h_{1cj} + \tau^l_{1cj}}{2}\right)\frac{(V_{cjt}(n-1) - V_{cjt}(n))}{Y_t}\right) \tag{B.30}$$

and the expected payoff of the plaintiff conditional on a type 1 patent infringement minus legal team cost is given as

$$\begin{aligned} W^{plain}_{1,cjt} \quad \equiv \quad & p^{LT}_{1,cjt}(V_{cjt}(n-1) - V_{cjt}(n))\left(1 - \zeta_{1cj} + \zeta_{1cj}\frac{\tau^h_{1cj} + \tau^l_{1cj}}{2}\right) \\ & - Y_t \int_0^{\left(-\zeta_{1cj}+\zeta_{1cj}\frac{\tau^h_{1cj}+\tau^l_{1cj}}{2}\right)\frac{(V_{cjt}(n-1)-V_{cjt}(n))}{Y_t}} \gamma d\Gamma_{cj}(\gamma) \\ & + (1 - p^{LT}_{1,cjt})(V_{cjt}(n-1) - V_{cjt}(n)) \end{aligned} \tag{B.31}$$

where the first term is the probability to hire a legal team times the expected returns to the plaintiff not including legal team costs, the second term is the expected legal team costs, and the third term is the probability not to hire a legal team times the expected returns, which is simply the value change from losing a product line for certain.

Now, let's turn to the payoff of the defendant. We know the settlement will always be sufficiently high such that every defendant rejects. Then, given τ, the defendant's payoff from going to court is:

$$(\tau + (1-\tau)(1-\zeta_{1cj}) - \Lambda_1)(V_{cjt}(n^d+1) - V_{cjt}(n^d)) \tag{B.32}$$

recalling $\Lambda_1 = 0$ and taking expectation over τ before its realization, we have

$$\begin{aligned} & \mathbb{E}\left[(\tau + (1-\tau)(1-\zeta_{1cj}))(V_{cjt}(n^d+1) - V_{cjt}(n^d))\right] \\ = \quad & (V_{cjt}(n^d+1) - V_{cjt}(n^d))\left(1 - \zeta_{1cj} + \zeta_{1cj}\int_{\tau^l_{1cj}}^{\tau^h_{1cj}} \tau dT_{1cj}(\tau)\right) \\ = \quad & (V_{cjt}(n^d+1) - V_{cjt}(n^d))\left(1 - \zeta_{1cj} + \zeta_{1cj}\frac{\tau^h_{1cj} + \tau^l_{1cj}}{2}\right) \end{aligned} \tag{B.33}$$

Then, given the probability of the plaintiff hiring a legal team, the expected payoff of the defendant conditional on a type 1 patent infringement is

$$
\begin{aligned}
W^{def}_{1,cjt} \quad \equiv \quad & p^{LT}_{1,cjt}(V_{cjt}(n^d+1)-V_{cjt}(n^d))\left(1-\zeta_{1cj}+\zeta_{1cj}\frac{\tau^h_{1cj}+\tau^l_{1cj}}{2}\right) \\
& +(1-p^{LT}_{1,cjt})(V_{cjt}(n^d+1)-V_{cjt}(n^d))
\end{aligned} \tag{B.34}
$$

where the first term is the probability to hire a legal team times the expected returns to the defendant, and the second term is the complementary probability times the value change from adding a product line for certain.

□

B.4. Proof of Proposition 3

Proposition 3. *Suppose $V_{cjt}(n)$ is linear in n and $\Lambda_1 > 0$. When successful innovation by a firm with technology class c in industry j with n^d product lines leads to a type 1 patent infringement on the IP of an incumbent with n product lines, the following are true in the subgame perfect equilibrium of the litigation game:*

1. *The ex-ante probability that the plaintiff hires a legal team is*

$$
\bar{p}^{LT}_{1,cjt} \equiv \mathbb{P}\left(\gamma \leq \left(-\frac{\Lambda_1^2}{2\zeta_{1cj}(\tau^h_{1cj}-\tau^l_{1cj})}-\zeta_{1cj}+\zeta_{1cj}\frac{\tau^h_{1cj}+\tau^l_{1cj}}{2}\right)\frac{(V_{cjt}(n-1)-V_{cjt}(n))}{Y_t}\right) \tag{B.35}
$$

2. *The optimal settlement offer made by the plaintiff is*

$$
s^* \quad \equiv \quad \zeta_{1cj}(1-\tau^l_{1cj})(V_{cjt}(n)-V_{cjt}(n-1)) \tag{B.36}
$$

3. *The defendant accepts the settlement if $\tau \leq \tau^* \equiv \tau^l_{1cj} + \Lambda_1/\zeta_{1cj}$, and rejects otherwise. The ex-ante acceptance probability is $\mathbb{P}(s^* < \bar{s}_{cjt}(n,\tau)) = \frac{\Lambda_1}{\zeta_{1cj}(\tau^h_{1cj}-\tau^l_{1cj})}$.*

4. *The expected payoff of the plaintiff is*

$$
\begin{aligned}
\bar{W}^{plain}_{1,cjt} \quad \equiv \quad & \bar{p}^{LT}_{1,cjt}(V_{cjt}(n-1)-V_{cjt}(n))\left(-\frac{\Lambda_1^2}{2\zeta_{1cj}(\tau^h_{1cj}-\tau^l_{1cj})}+1-\zeta_{1cj}+\zeta_{1cj}\frac{\tau^h_{1cj}+\tau^l_{1cj}}{2}\right) \\
& -Y_t\int_0^{\left(-\frac{\Lambda_1^2}{2\zeta_{1cj}(\tau^h_{1cj}-\tau^l_{1cj})}-\zeta_{1cj}+\zeta_{1cj}\frac{\tau^h_{1cj}+\tau^l_{1cj}}{2}\right)\frac{(V_{cjt}(n-1)-V_{cjt}(n))}{Y_t}}\gamma d\Gamma_{cj}(\gamma) \\
& +(1-\bar{p}^{LT}_{1,cjt})(V_{cjt}(n-1)-V_{cjt}(n))
\end{aligned} \tag{B.37}
$$

5. The expected payoff of the defendant is

$$\begin{aligned}\bar{W}^{def}_{1,cjt} \quad \equiv \quad & \Bigg(\bar{p}^{LT}_{1,cjt}\Bigg[(1-\zeta_{1cj}(1-\tau^{l}_{1cj}))\frac{\Lambda_1/\zeta_{1cj}}{\tau^{h}_{1cj}-\tau^{l}_{1cj}} + (1-\zeta_{1cj}-\Lambda_1)\Bigg(1-\frac{\Lambda_1/\zeta_{1cj}}{\tau^{h}_{1cj}-\tau^{l}_{1cj}}\Bigg) \\ & +\zeta_{1cj}\frac{(\tau^{h}_{1cj})^2-(\tau^{l}_{1cj})^2-2\tau^{l}_{1cj}\frac{\Lambda_1}{\zeta_{1cj}}-\frac{\Lambda_1^2}{\zeta_{1cj}^2}}{2(\tau^{h}_{1cj}-\tau^{l}_{1cj})}\Bigg] + (1-\bar{p}^{LT}_{1,cjt})\Bigg)(V_{cjt}(n^d+1)-V_{cjt}(n^d)) \end{aligned} \tag{B.38}$$

Proof. We consider the decision problem of a plaintiff facing a type 1 patent infringement which means they are the owner of the product line that is facing the risk of creative destruction. We further know that the plaintiff and the defendant share the same technology class c. Unlike a type 2 infringement, this time the plaintiff cares about more than the potential settlement they can extract from the defendant, since settling out of court also means they lose their product line for sure. Assume the distribution $T_{1cj}(\tau)$ is the continuous uniform distribution $U(\tau^{l}_{1cj}, \tau^{h}_{1cj})$ with $0 \le \tau^{l}_{1cj} < \tau^{h}_{1cj} \le 1$. Then the plaintiff's problem is written as

$$\begin{aligned}\max_{s\ge 0} \quad \Bigg\{ \quad & \int_{\tau^{l}_{1cj}}^{1-s/(\zeta_{1cj}(V_{cjt}(n^d+1)-V_{cjt}(n^d)))+\Lambda_1/\zeta_{1cj}} \Big(V_{cjt}(n-1)-V_{cjt}(n)+s\Big)\,dT_{1cj}(\tau) \\ & + \int_{1-s/(\zeta_{1cj}(V_{cjt}(n^d+1)-V_{cjt}(n^d)))+\Lambda_1/\zeta_{1cj}}^{\tau^{h}_{1cj}} (\tau+(1-\tau)(1-\zeta_{1cj}))(V_{cjt}(n-1)-V_{cjt}(n))\,dT_{1cj}(\tau)\Bigg\} \end{aligned} \tag{B.39}$$

where the first integral is the expected payoff from defendants who accept the settlement and the second integral is the expected payoff from those who reject. The term $V_{cjt}(n-1)-V_{cjt}(n)$ is negative, and reflects the cost of losing the product line. In the cases when the defendant accepts, the plaintiff gives up their $(1-\tau)\zeta_{1cj}$ chance of retaining their product line in exchange for a settlement amount s.

Unlike in Proposition 2, this time we assume $\Lambda_1 > 0$ instead of $\Lambda_1 = 0$. While the same inherent adverse selection problem is still present, causing almost all defendants to reject the plaintiff's offer and go to court, defendants with the worst τ draws might now choose to settle out-of-court only to avoid the cost of litigation. This gives the plaintiffs the opportunity to do slightly better than going to court for a limited set of defendants, the measure of which depends on how high Λ_1 is.

To see this, let's reorganize the objective function in equation (B.39):

$$\int_{\tau^l_{1cj}}^{1-s/(\zeta_{1cj}(V_{cjt}(n^d+1)-V_{cjt}(n^d)))+\Lambda_1/\zeta_{1cj}} \left((1-\tau)\zeta_{1cj}(V_{cjt}(n-1)-V_{cjt}(n))+s\right) dT_{1cj}(\tau)$$
$$+\int_{\tau^l_{1cj}}^{\tau^h_{1cj}} (\tau+(1-\tau)(1-\zeta_{1cj}))(V_{cjt}(n-1)-V_{cjt}(n))dT_{1cj}(\tau) \tag{B.40}$$

Note that the second integral is now completely independent of the choice of the settlement amount s, whereas the first integral captures the previously described tradeoff: the plaintiff giving up their $(1-\tau)\zeta_{1cj}$ chance of retaining their product line in exchange for a settlement amount s. The second integral can be moved out of the maximization, so we restrict focus to the first integral.

Define $v^p = V_{cjt}(n)-V_{cjt}(n-1)$ and $v^d = V_{cjt}(n^d+1)-V_{cjt}(n^d)$ for brevity. By the linearity of $V_{cjt}(n)$ in n, we have $v^p = v^d = \hat{v} \equiv V_{cjt}(n)/n$. Define the threshold below which the defendant accepts the settlement offer as $\tau^*(s) = 1-s/(\zeta_{1cj}\hat{v})+\Lambda_1/\zeta_{1cj}$. The first integral can be rewritten as:

$$\begin{aligned}
& \left(\zeta_{1cj}\hat{v}\int_{\tau^l_{1cj}}^{\tau^*(s)} \tau dT_{1cj}(\tau)\right)+\left((s-\zeta_{1cj}\hat{v})\int_{\tau^l_{1cj}}^{\tau^*(s)} dT_{1cj}(\tau)\right) \\
= & \left(\zeta_{1cj}\hat{v}\frac{(\tau^*(s))^2-(\tau^l_{1cj})^2}{2(\tau^h_{1cj}-\tau^l_{1cj})}\right)+\left((s-\zeta_{1cj}\hat{v})\frac{\tau^*(s)-\tau^l_{1cj}}{\tau^h_{1cj}-\tau^l_{1cj}}\right) \\
= & \left(\zeta_{1cj}\hat{v}\frac{\left(1+\frac{\Lambda_1}{\zeta_{1cj}}\right)^2-2\left(1+\frac{\Lambda_1}{\zeta_{1cj}}\right)\frac{s}{\zeta_{1cj}\hat{v}}+\frac{s^2}{\zeta_{1cj}^2\hat{v}^2}-(\tau^l_{1cj})^2}{2(\tau^h_{1cj}-\tau^l_{1cj})}\right) \\
& +\left(\frac{\left(1+\frac{\Lambda_1}{\zeta_{1cj}}\right)s-\frac{s^2}{\zeta_{1cj}\hat{v}}-\left(1+\frac{\Lambda_1}{\zeta_{1cj}}\right)\zeta_{1cj}\hat{v}+s-\tau^l_{1cj}s+\zeta_{1cj}\hat{v}\tau^l_{1cj}}{\tau^h_{1cj}-\tau^l_{1cj}}\right) \\
\equiv & \mathfrak{a}s^2+\mathfrak{b}s+\mathfrak{c}
\end{aligned} \tag{B.41}$$

where

$$\begin{aligned}
\mathfrak{a} &= \frac{-1}{2\zeta_{1cj}\hat{v}(\tau^h_{1cj}-\tau^l_{1cj})} \\
\mathfrak{b} &= \frac{1-\tau^l_{1cj}}{\tau^h_{1cj}-\tau^l_{1cj}}
\end{aligned}$$

That is, the expression is a quadratic polynomial, and concave in s since $\mathfrak{a} < 0$. Therefore, the optimal settlement offer s^* is given by:

$$s^* = \frac{-\mathfrak{b}}{2\mathfrak{a}} = \zeta_{1cj}\hat{v}(1-\tau^l_{1cj}) \tag{B.42}$$

Plugging this expression into the threshold $\tau^*(s)$, we get:

$$\tau^*(s^*) = 1 - \frac{\zeta_{1cj}\hat{v}(1-\tau^l_{1cj})}{\zeta_{1cj}\hat{v}} + \frac{\Lambda_1}{\zeta_{1cj}} = \tau^l_{1cj} + \frac{\Lambda_1}{\zeta_{1cj}} \tag{B.43}$$

This means defendants with $\tau \in [\tau^l_{1cj}, \tau^l_{1cj} + \Lambda_1/\zeta_{1cj}]$ will accept to settle out of court. The measure of such defendants is $\frac{\Lambda_1}{\zeta_{1cj}(\tau^h_{1cj}-\tau^l_{1cj})}$, linearly increasing in the defendant litigation cost parameter Λ_1.

Given the optimal s^* and $\tau^*(s^*)$, we can compute the plaintiff's payoff from going to court as:

$$\begin{aligned}
& \int_{\tau^l_{1cj}}^{\tau^*(s^*)} \left(-(1-\tau)\zeta_{1cj}\hat{v} + \zeta_{1cj}\hat{v}(1-\tau^l_{1cj})\right) dT_{1cj}(\tau) + \int_{\tau^l_{1cj}}^{\tau^h_{1cj}} (\tau + (1-\tau)(1-\zeta_{1cj}))(-\hat{v}) dT_{1cj}(\tau) \\
= \; & \zeta_{1cj}\hat{v} \int_{\tau^l_{1cj}}^{\tau^*(s^*)} \tau dT_{1cj}(\tau) - \tau^l_{1cj}\zeta_{1cj}\hat{v} \int_{\tau^l_{1cj}}^{\tau^*(s^*)} dT_{1cj}(\tau) + \int_{\tau^l_{1cj}}^{\tau^h_{1cj}} (\tau + (1-\tau)(1-\zeta_{1cj}))(-\hat{v}) dT_{1cj}(\tau) \\
= \; & \zeta_{1cj}\hat{v} \frac{2\tau^l_{1cj}\frac{\Lambda_1}{\zeta_{1cj}} + \frac{\Lambda_1^2}{\zeta_{1cj}^2}}{2(\tau^h_{1cj}-\tau^l_{1cj})} - \zeta_{1cj}\hat{v} \frac{2\tau^l_{1cj}\frac{\Lambda_1}{\zeta_{1cj}}}{2(\tau^h_{1cj}-\tau^l_{1cj})} - \hat{v}\left(1 - \zeta_{1cj} + \zeta_{1cj}\frac{\tau^h_{1cj}+\tau^l_{1cj}}{2}\right) \\
= \; & \hat{v}\left(\frac{\Lambda_1^2}{2\zeta_{1cj}(\tau^h_{1cj}-\tau^l_{1cj})} - 1 + \zeta_{1cj} - \zeta_{1cj}\frac{\tau^h_{1cj}+\tau^l_{1cj}}{2}\right)
\end{aligned} \tag{B.44}$$

Given this expression, we can turn to the plaintiff's decision to hire a legal team or not. If the plaintiff does not hire a legal team, then their payoff is simply $V_{cjt}(n-1) - V_{cjt}(n) = -\hat{v}$ since they will lose their product line for certain. Therefore, they will strictly prefer to hire a legal team if and only if

$$\begin{aligned}
\hat{v}\left(\frac{\Lambda_1^2}{2\zeta_{1cj}(\tau_{1cj}^h-\tau_{1cj}^l)}-1+\zeta_{1cj}-\zeta_{1cj}\frac{\tau_{1cj}^h+\tau_{1cj}^l}{2}\right)-\gamma Y_t &> -\hat{v} \\
\left(\frac{\Lambda_1^2}{2\zeta_{1cj}(\tau_{1cj}^h-\tau_{1cj}^l)}+\zeta_{1cj}-\zeta_{1cj}\frac{\tau_{1cj}^h+\tau_{1cj}^l}{2}\right)\hat{v} &> \gamma Y_t \\
\left(-\frac{\Lambda_1^2}{2\zeta_{1cj}(\tau_{1cj}^h-\tau_{1cj}^l)}-\zeta_{1cj}+\zeta_{1cj}\frac{\tau_{1cj}^h+\tau_{1cj}^l}{2}\right)(V_{cjt}(n-1)-V_{cjt}(n)) &> \gamma Y_t
\end{aligned} \tag{B.45}$$

where we use the definition of $v^p = \hat{v}$ in the last line to make the expression comparable to that in Proposition 2. As can be seen, the only difference is the first term which is quadratic in Λ_1. Since the term is quadratic, the extra payoff the plaintiff can extract due to positive defendant litigation costs, while positive, is quite small (e.g., a 10% defendant legal fee converts to only a payoff difference of $(10\%)^2 = 1\%$ of the value of a single product line, which is further divided by $2\zeta_{1cj}(\tau_{1cj}^h-\tau_{1cj}^l)$).

Then the probability of hiring a legal team is given by

$$\bar{p}_{1,cjt}^{LT} \equiv \mathbb{P}\left(\gamma \le \left(-\frac{\Lambda_1^2}{2\zeta_{1cj}(\tau_{1cj}^h-\tau_{1cj}^l)}-\zeta_{1cj}+\zeta_{1cj}\frac{\tau_{1cj}^h+\tau_{1cj}^l}{2}\right)\frac{(V_{cjt}(n-1)-V_{cjt}(n))}{Y_t}\right) \tag{B.46}$$

and the expected payoff of the plaintiff conditional on a type 1 patent infringement minus legal team cost is given as

$$\begin{aligned}
\bar{W}_{1,cjt}^{plain} \equiv\ & \bar{p}_{1,cjt}^{LT}(V_{cjt}(n-1)-V_{cjt}(n))\left(-\frac{\Lambda_1^2}{2\zeta_{1cj}(\tau_{1cj}^h-\tau_{1cj}^l)}+1-\zeta_{1cj}+\zeta_{1cj}\frac{\tau_{1cj}^h+\tau_{1cj}^l}{2}\right) \\
& -Y_t\int_0^{\left(-\frac{\Lambda_1^2}{2\zeta_{1cj}(\tau_{1cj}^h-\tau_{1cj}^l)}-\zeta_{1cj}+\zeta_{1cj}\frac{\tau_{1cj}^h+\tau_{1cj}^l}{2}\right)\frac{(V_{cjt}(n-1)-V_{cjt}(n))}{Y_t}} \gamma d\Gamma_{cj}(\gamma) \\
& +(1-\bar{p}_{1,cjt}^{LT})(V_{cjt}(n-1)-V_{cjt}(n))
\end{aligned} \tag{B.47}$$

where the first term is the probability to hire a legal team times the expected returns to the plaintiff not including legal team costs, the second term is the expected legal team costs, and the third term is the probability not to hire a legal team times the expected returns, which is simply the value change from losing a product line for certain.

Turning to the defendant's side, conditional on a type 1 patent infringement, they will receive a settlement offer only if the plaintiff chooses to hire a legal team, the probability of which is $\bar{p}^{LT}_{1,cjt}$. Conditional on receiving a settlement offer, their expected payoff is

$$
\begin{aligned}
& \int_{\tau^l_{1cj}}^{\tau^*(s^*)} \left(\hat{v} - s^*\right) dT_{1cj}(\tau) + \int_{\tau^*(s^*)}^{\tau^h_{1cj}} [\tau + (1-\tau)(1-\zeta_{1cj}) - \Lambda_1]\hat{v}\, dT_{1cj}(\tau) \\
= \; & \hat{v}(1-\zeta_{1cj}(1-\tau^l_{1cj})) \int_{\tau^l_{1cj}}^{\tau^l_{1cj}+\Lambda_1/\zeta_{1cj}} dT_{1cj}(\tau) \\
& + \hat{v}(1-\zeta_{1cj}-\Lambda_1) \int_{\tau^l_{1cj}+\Lambda_1/\zeta_{1cj}}^{\tau^h_{1cj}} dT_{1cj}(\tau) + \hat{v}\zeta_{1cj} \int_{\tau^l_{1cj}+\Lambda_1/\zeta_{1cj}}^{\tau^h_{1cj}} \tau\, dT_{1cj}(\tau) \\
= \; & \left[(1-\zeta_{1cj}(1-\tau^l_{1cj}))\frac{\Lambda_1/\zeta_{1cj}}{\tau^h_{1cj}-\tau^l_{1cj}} + (1-\zeta_{1cj}-\Lambda_1)\left(1-\frac{\Lambda_1/\zeta_{1cj}}{\tau^h_{1cj}-\tau^l_{1cj}}\right)\right. \\
& \left. + \zeta_{1cj}\frac{(\tau^h_{1cj})^2-(\tau^l_{1cj})^2 - 2\tau^l_{1cj}\frac{\Lambda_1}{\zeta_{1cj}} - \frac{\Lambda_1^2}{\zeta_{1cj}^2}}{2(\tau^h_{1cj}-\tau^l_{1cj})}\right]\hat{v}
\end{aligned} \tag{B.48}
$$

Therefore, the defendant's expected payoff conditional on a type 1 patent infringement and before learning whether they will receive a settlement offer can be written as

$$
\begin{aligned}
\bar{W}^{def}_{1,cjt} \equiv \; & \left(\bar{p}^{LT}_{1,cjt}\left[(1-\zeta_{1cj}(1-\tau^l_{1cj}))\frac{\Lambda_1/\zeta_{1cj}}{\tau^h_{1cj}-\tau^l_{1cj}} + (1-\zeta_{1cj}-\Lambda_1)\left(1-\frac{\Lambda_1/\zeta_{1cj}}{\tau^h_{1cj}-\tau^l_{1cj}}\right)\right.\right. \\
& \left.\left. + \zeta_{1cj}\frac{(\tau^h_{1cj})^2-(\tau^l_{1cj})^2 - 2\tau^l_{1cj}\frac{\Lambda_1}{\zeta_{1cj}} - \frac{\Lambda_1^2}{\zeta_{1cj}^2}}{2(\tau^h_{1cj}-\tau^l_{1cj})}\right] + (1-\bar{p}^{LT}_{1,cjt})\right)(V_{cjt}(n^d+1) - V_{cjt}(n^d))
\end{aligned} \tag{B.49}
$$

where we use the definition of $v^d = \hat{v}$ in the last line to make the expression comparable to that in Proposition 2. Note that the whole expression is linear in the expected change in firm value if they take over the product line, $V_{cjt}(n^d+1) - V_{cjt}(n^d)$. □

B.5. Characterizing the incumbent value function

Having solved the subgame perfect equilibrium of the litigation game for both types of patent infringements, we are now ready to characterize the value function of an incumbent.

The rent flow for a single product line from type 2 patent infringements by others in industry j, R_{cjt}, is given by

$$R_{cjt} = p^{rent}_{cjt} W^{plain}_{2,cjt} \tag{B.50}$$

where p^{rent}_{cjt} is the Poisson arrival rate of a type 2 patent infringement from firms in industry j, calculated in general equilibrium. This arrival rate depends on the innovation choices of all other firms with the same technology class c in industry j, as well as the share of product lines that belong to firms with technology class c across all industries. p^{rent}_{cjt} is increasing in the prior (since more firms innovating means there are more potential infringements) as well as the probabilities of a type 2 infringement $\kappa_{2cj'}$ originating from any industry j', and it is decreasing in the latter (since the same amount of infringements is spread over a larger mass of potentially infringed product lines).

The value difference conditional on successful innovation, but before the litigation subgame, denoted as $V^+_{cjt}(n) - V_{cjt}(n)$, is given by

$$\begin{aligned} V^+_{cjt}(n) - V_{cjt}(n) &= p^{def}_{cjt}\kappa_{1cj}W^{def}_{1,cjt} + (1-p^{def}_{cjt})\kappa_{2cj}W^{def}_{2,cjt} \\ &\quad +[(p^{def}_{cjt}(1-\kappa_{1cj}) + (1-p^{def}_{cjt})(1-\kappa_{2cj})](V_{cjt}(n+1) - V_{cjt}(n)) \end{aligned} \tag{B.51}$$

where p^{def}_{cjt} is the probability that the firm innovates on the product line of another firm with the same technology class c in its industry, which is again determined in general equilibrium. The first term is the probability of a type 1 patent infringement times the associated defendant payoff, $W^{def}_{1,cjt}$, calculated earlier. Likewise, the second term is the probability of a type 2 patent infringement times the associated defendant payoff, $W^{def}_{2,cjt}$. The last term is the probability that no infringement happens times the value change from adding a new product line for certain.

The value difference conditional on being innovated on (i.e., value loss from creative destruction), but before the litigation subgame, denoted as $V^-_{cjt}(n) - V_{cjt}(n)$, is given by

$$\begin{aligned} V^-_{cjt}(n) - V_{cjt}(n) &= p^{plain}_{cjt}\kappa_{1cj}W^{plain}_{1,cjt} + (1-p^{plain}_{cjt})p^{inj}_{cjt}(V_{cjt}(n) - V_{cjt}(n)) \\ &\quad +(1-p^{plain}_{cjt}\kappa_{1cj} - (1-p^{plain}_{cjt})p^{inj}_{cjt})(V_{cjt}(n-1) - V_{cjt}(n)) \end{aligned} \tag{B.52}$$

where p^{plain}_{cjt} is the probability that the incoming innovation belongs to a firm with the same technology class c, in which case a type 1 infringement is possible with probability κ_{1cj}. The first term is this joint probability times the associated plaintiff payoff, $W^{plain}_{1,cjt}$. The second term is the probability that the incoming innovation belongs to a firm with a different technology class, in which case a type 2 infringement is possible with a probability that depends on all $\kappa_{c'j}$ with $c' \neq c$. In such an event, the innovating firm interacts with a third firm whose patent is infringed, and the combined probability of a type 2 infringement, followed by an injunction being granted in the litigation subgame is denoted as p^{inj}_{cjt}. In this case, the incumbent retains its product line, and therefore there is no value loss (i.e., the second term equals zero, and it is kept only for clarity). The last term is the remaining probability times the value change from losing a product line for certain.

B.6. Proof of Theorem 1

Definition 1. *A balanced growth path (BGP) equilibrium of this economy is an equilibrium in which:*

1. *The aggregate variables Y_t, C_t, A_t and the real wage rate w_t grow at the constant rate $g > 0$.*

2. *The real interest rate r, the industry-specific creative destruction rates $\{d_j\}_{j=1}^{J}$, the fraction of product lines owned by technology class c firms $\{M_c\}_{c=1}^{C}$ and the probabilities $\{\{p^{rent}_{cj}, p^{LT}_{1,cj}, p^{LT}_{2,cj}, p^{def}_{cj}, p^{plain}_{cj}, p^{inj}_{cj}\}_{j=1}^{J}\}_{c=1}^{C}$ are time-invariant.*

Theorem 1. *In a BGP equilibrium, the value function of an incumbent firm with technology class c in industry j who is the leader in n product lines at time t is given by*

$$V_{cj}(n) = v_{cj} n Y_t, \tag{B.53}$$

where $v_{cj} > 0$ is an industry- and technology-class-specific time-invariant scalar given by

$$v_{cj} = \frac{\frac{\lambda_c}{1+\lambda_c}\omega_j + \sum_{j'=1}^{J} \hat{R}_{cj'} - \frac{(1-s_{cj})\chi_{cj} x_{cj}^{\psi}}{1+\sigma M_c}}{\rho + \delta - x_{cj} L^{def}_{cj} + d_j L^{plain}_{cj}}. \tag{B.54}$$

In particular, x_{cj} is the time-invariant per-product-line incumbent innovation arrival rate given by

$$x_{cj} = \left(\frac{L^{def}_{cj} v_{cj} (1+\sigma M_c)}{(1-s_{cj})\chi_{cj}\psi} \right)^{\frac{1}{\psi-1}}, \tag{B.55}$$

and $\hat{R}_{cj}$, L^{def}_{cj}, and L^{plain}_{cj} are time-invariant terms that summarize the implications of the *litigation subgame on firm value, defined in Equations (B.57), (B.60), and (B.63), respectively. Likewise, z is the time-invariant entrant innovation arrival rate given by*

$$z = \left(\frac{\sum_{c=1}^{C}\sum_{j=1}^{J}\eta_{cj}L_{cj}^{def}v_{cj}}{(1-s_e)\nu\psi}\right)^{\frac{1}{\psi-1}}. \tag{B.56}$$

Proof. The guess-and-verify method will be used. Suppose the value function takes the specified form. Then, we can plug it into the various terms that show up in Equation (13) and recover the equations that pin down the values of the scalars v_{cj} for all technology classes c and all industries j.

First, consider the expected rent flow from type 2 patent infringements by firms in industry j on our firm's IP, nR_{cjt}. Using Equations (B.5) and (B.50), we get:

$$\begin{aligned}
R_{cjt} &= p_{cj}^{rent}\Bigg(p_{2,cj}^{LT}\frac{(1-\tau_{2cj}^{l}+\Lambda_2/\zeta_{2cj})^2\zeta_{2cj}(V_{cjt}(n^d+1)-V_{cjt}(n^d))}{4(\tau_{2cj}^{h}-\tau_{2cj}^{l})} \\
&\quad -Y_t\int_0^{\frac{(1-\tau_{2cj}^{l}+\Lambda_2/\zeta_{2cj})^2\zeta_{2cj}(V_{cjt}(n^d+1)-V_{cjt}(n^d))}{4(\tau_{2cj}^{h}-\tau_{2cj}^{l})Y_t}}\gamma d\Gamma_{cj}(\gamma)\Bigg) \\
R_{cjt} &= p_{cj}^{rent}\Bigg(p_{2,cj}^{LT}\frac{(1-\tau_{2cj}^{l}+\Lambda_2/\zeta_{2cj})^2\zeta_{2cj}(v_{cj}(n^d+1)Y_t-v_{cj}n^dY_t)}{4(\tau_{2cj}^{h}-\tau_{2cj}^{l})} \\
&\quad -Y_t\int_0^{\frac{(1-\tau_{2cj}^{l}+\Lambda_2/\zeta_{2cj})^2\zeta_{2cj}(v_{cj}(n^d+1)Y_t-v_{cj}n^dY_t)}{4(\tau_{2cj}^{h}-\tau_{2cj}^{l})Y_t}}\gamma d\Gamma_{cj}(\gamma)\Bigg) \\
R_{cjt} &= p_{cj}^{rent}\left(p_{2,cj}^{LT}\frac{(1-\tau_{2cj}^{l}+\Lambda_2/\zeta_{2cj})^2\zeta_{2cj}v_{cj}}{4(\tau_{2cj}^{h}-\tau_{2cj}^{l})}-\int_0^{\frac{(1-\tau_{2cj}^{l}+\Lambda_2/\zeta_{2cj})^2\zeta_{2cj}v_{cj}}{4(\tau_{2cj}^{h}-\tau_{2cj}^{l})}}\gamma d\Gamma_{cj}(\gamma)\right)Y_t \\
nR_{cjt} &= p_{cj}^{rent}\left(p_{2,cj}^{LT}\frac{(1-\tau_{2cj}^{l}+\Lambda_2/\zeta_{2cj})^2\zeta_{2cj}v_{cj}}{4(\tau_{2cj}^{h}-\tau_{2cj}^{l})}-\int_0^{\frac{(1-\tau_{2cj}^{l}+\Lambda_2/\zeta_{2cj})^2\zeta_{2cj}v_{cj}}{4(\tau_{2cj}^{h}-\tau_{2cj}^{l})}}\gamma d\Gamma_{cj}(\gamma)\right)nY_t \\
nR_{cjt} &\equiv \hat{R}_{cj}nY_t
\end{aligned} \tag{B.57}$$

where the last line implicitly defines the normalized term $\hat{R}_{cj}$ for convenience.

Second, consider the value difference conditional on successful innovation, but before the litigation subgame, $V_{cjt}^{+}(n)-V_{cjt}(n)$. As gleaned from Equation (B.51), we must first obtain the expected payoffs of the defendant conditional on type 1 and type 2 patent infringements, denoted as $W^{def}_{1,cjt}$ and $W^{def}_{2,cjt}$, respectively. Plugging the guess in Equation (B.22) yields:

$$\begin{aligned}
W^{def}_{1,cjt} &= p^{LT}_{1,cj}(V_{cjt}(n^d+1)-V_{cjt}(n^d))\left(1-\zeta_{1cj}+\zeta_{1cj}\frac{\tau^h_{1cj}+\tau^l_{1cj}}{2}\right) \\
&\quad +(1-p^{LT}_{1,cj})(V_{cjt}(n^d+1)-V_{cjt}(n^d)) \\
&= p^{LT}_{1,cj}(v_{cj}(n^d+1)Y_t-v_{cj}n^dY_t)\left(1-\zeta_{1cj}+\zeta_{1cj}\frac{\tau^h_{1cj}+\tau^l_{1cj}}{2}\right) \\
&\quad +(1-p^{LT}_{1,cj})(v_{cj}(n^d+1)Y_t-v_{cj}n^dY_t) \\
&= \left(p^{LT}_{1,cj}\left(1-\zeta_{1cj}+\zeta_{1cj}\frac{\tau^h_{1cj}+\tau^l_{1cj}}{2}\right)+(1-p^{LT}_{1,cj})\right)v_{cj}Y_t \\
&\equiv \hat{W}^{def}_{1,cj}v_{cj}Y_t
\end{aligned} \tag{B.58}$$

where the last line implicitly defines the normalized term $\hat{W}^{def}_{1,cj}$ which depends on the probability $p^{LT}_{1,cj}$. Likewise, plugging the guess in Equation (B.6) yields:

$$\begin{aligned}
W^{def}_{2,cjt} &= \Bigg((1-p^{LT}_{2,cj})+p^{LT}_{2,cj}\Bigg[\left(1-\frac{(1-\tau^l_{2cj}+\Lambda_2/\zeta_{2cj})\zeta_{2cj}}{2}\right)\left(\frac{1}{2}\frac{1-\tau^l_{2cj}+\Lambda_2/\zeta_{2cj}}{\tau^h_{2cj}-\tau^l_{2cj}}\right) \\
&\quad +(1-\zeta_{2cj}-\Lambda_2)\left(1-\frac{1}{2}\frac{1-\tau^l_{2cj}+\Lambda_2/\zeta_{2cj}}{\tau^h_{2cj}-\tau^l_{2cj}}\right) \\
&\quad +\frac{\zeta_{2cj}}{2(\tau^h_{2cj}-\tau^l_{2cj})}\left((\tau^h_{2cj})^2-\frac{(1+\tau^l_{2cj}+\Lambda_2/\zeta_{2cj})^2}{4}\right)\Bigg]\Bigg)v_{cj}Y_t \\
&\equiv \hat{W}^{def}_{2,cj}v_{cj}Y_t
\end{aligned} \tag{B.59}$$

where the last line implicitly defines the normalized term $\hat{W}^{def}_{2,cj}$ which depends on the probability $p^{LT}_{2,cj}$. Using Equations (B.51), (B.58), and (B.59), we get:

$$
\begin{aligned}
V^{+}_{cjt}(n) - V_{cjt}(n) &= p^{def}_{cjt}\kappa_{1cj}W^{def}_{1,cjt} + (1-p^{def}_{cjt})\kappa_{2cj}W^{def}_{2,cjt} \\
&\quad + [(p^{def}_{cj}(1-\kappa_{1cj}) + (1-p^{def}_{cj})(1-\kappa_{2cj})](V_{cjt}(n+1) - V_{cjt}(n)) \\
&= \Big(p^{def}_{cj}\kappa_{1cj}\hat{W}^{def}_{1,cj} + (1-p^{def}_{cj})\kappa_{2cj}\hat{W}^{def}_{2,cj} \\
&\quad + [(p^{def}_{cj}(1-\kappa_{1cj}) + (1-p^{def}_{cj})(1-\kappa_{2cj})]\Big)v_{cj}Y_t \\
&\equiv L^{def}_{cj}v_{cj}Y_t \qquad \text{(B.60)}
\end{aligned}
$$

where the last line implicitly defines L^{def}_{cj}. Notice that, in the absence of any patent infringement – that is, $\kappa_{1cj} = \kappa_{2cj} = 0, \forall c, j$ – we have $L^{def}_{cj} = 1$, and the whole expression simplifies to $v_{cj}Y_t$ alone. Therefore, $1 - L^{def}_{cj}$ captures the fraction of the value of a successful innovation that is lost due to the risk of infringing on other firms' IP.

Given Equation (B.60), we can calculate the optimal innovation rate $x_{cjt}(n)$ using Equation (14) as:

$$
\begin{aligned}
x_{cjt}(n) &= \left(\frac{\Big(V^{+}_{cjt}(n) - V_{cjt}(n)\Big)(1+\sigma M_{ct})}{(1-s_{cj})\chi_{cj}\psi Y_t}\right)^{\frac{1}{\psi-1}} \\
x_{cjt}(n) &= \left(\frac{L^{def}_{cj}v_{cj}(1+\sigma M_c)}{(1-s_{cj})\chi_{cj}\psi}\right)^{\frac{1}{\psi-1}} \equiv x_{cj} \qquad \text{(B.61)}
\end{aligned}
$$

Note that this optimal innovation rate is independent of both the number of product lines owned by the firm, n, and time, t. The latter owes to the fact that the term M_{ct} must be time-invariant in a BGP equilibrium since the firm distribution across industries, technology classes, and number of product lines is stationary.

Third, consider the value difference conditional on being innovated on (i.e., value loss from creative destruction), but before the litigation subgame, denoted as $V^{-}_{cjt}(n) - V_{cjt}(n)$. As gleaned from Equation (B.52), we must first obtain the expected payoff of the plaintiff conditional on type 1 patent infringement, denoted as $W^{plain}_{1,cjt}$. Plugging the guess in Equation (B.21) yields:

$$
\begin{aligned}
W^{plain}_{1,cjt} &= p^{LT}_{1,cj}(V_{cjt}(n-1)-V_{cjt}(n))\left(1-\zeta_{1cj}+\zeta_{1cj}\frac{\tau^h_{1cj}+\tau^l_{1cj}}{2}\right) \\
&\quad -Y_t\int_0^{\left(-\zeta_{1cj}+\zeta_{1cj}\frac{\tau^h_{1cj}+\tau^l_{1cj}}{2}\right)\frac{(V_{cjt}(n-1)-V_{cjt}(n))}{Y_t}}\gamma d\Gamma_{cj}(\gamma) \\
&\quad +(1-p^{LT}_{1,cj})(V_{cjt}(n-1)-V_{cjt}(n)) \\
&= p^{LT}_{1,cj}(-v_{cj}Y_t)\left(1-\zeta_{1cj}+\zeta_{1cj}\frac{\tau^h_{1cj}+\tau^l_{1cj}}{2}\right)-Y_t\int_0^{\left(-\zeta_{1cj}+\zeta_{1cj}\frac{\tau^h_{1cj}+\tau^l_{1cj}}{2}\right)(-v_{cj})}\gamma d\Gamma_{cj}(\gamma) \\
&\quad +(1-p^{LT}_{1,cj})(-v_{cj}Y_t) \\
&= \left(p^{LT}_{1,cj}\left(1-\zeta_{1cj}+\zeta_{1cj}\frac{\tau^h_{1cj}+\tau^l_{1cj}}{2}\right)+\frac{1}{v_{cj}}\int_0^{\left(-\zeta_{1cj}+\zeta_{1cj}\frac{\tau^h_{1cj}+\tau^l_{1cj}}{2}\right)(-v_{cj})}\gamma d\Gamma_{cj}(\gamma)\right. \\
&\quad \left.+(1-p^{LT}_{1,cj})\right)(-v_{cj}Y_t) \\
&\equiv \hat{W}^{plain}_{1,cj}(-v_{cj}Y_t) \qquad \text{(B.62)}
\end{aligned}
$$

where the last line implicitly defines the normalized term $\hat{W}^{plain}_{1,cj}$ which depends on the probability $p^{LT}_{1,cj}$ and v_{cj}. Using Equations (B.52) and (B.62), we get:

$$
\begin{aligned}
V^-_{cjt}(n)-V_{cjt}(n) &= p^{plain}_{cj}\kappa_{1cj}W^{plain}_{1,cjt}+(1-p^{plain}_{cj}\kappa_{1cj}-(1-p^{plain}_{cj})p^{inj}_{cj})(V_{cjt}(n-1)-V_{cjt}(n)) \\
&= p^{plain}_{cj}\kappa_{1cj}\hat{W}^{plain}_{1,cj}(-v_{cj}Y_t)+(1-p^{plain}_{cj}\kappa_{1cj}-(1-p^{plain}_{cj})p^{inj}_{cj})(-v_{cj}Y_t) \\
&= \left(p^{plain}_{cj}\kappa_{1cj}\hat{W}^{plain}_{1,cj}+(1-p^{plain}_{cj}\kappa_{1cj}-(1-p^{plain}_{cj})p^{inj}_{cj})\right)(-v_{cj}Y_t) \\
&\equiv L^{plain}_{cj}(-v_{cj}Y_t) \qquad \text{(B.63)}
\end{aligned}
$$

where the last line implicitly defines L^{plain}_{cj}. Notice that, in the absence of any patent infringement – that is, $\kappa_{1cj}=\kappa_{2cj}=0, \forall c,j$ – we have $L^{plain}_{cj}=1$, and the whole expression simplifies to $-v_{cj}Y_t$ alone.[25] Therefore, $1-L^{plain}_{cj}$ captures the value gain to the owner of a product line from the possibility of using a patent infringement case to fight off an entrant, and by doing so, retain the ownership of their product line.

Before we move on to the HJB equation, there are a few additional expressions that need to be computed. First, notice that the summation of the static profit flows from owned product

[25] This is because $p^{inj}_{cj}=0$ when $\kappa_{2cj}=0, \forall c,j$.

lines is simply:

$$\sum_{m=1}^{n} \frac{\lambda_c}{1+\lambda_c}\omega_j Y_t = \frac{\lambda_c}{1+\lambda_c}\omega_j n Y_t \tag{B.64}$$

Second, the time derivative of the value function is:

$$\dot{V}_{cjt}(n) = \frac{d}{dt}(v_{cj} n Y_t) = v_{cj} n \frac{dY_t}{dt} = g v_{cj} n Y_t \tag{B.65}$$

Third, the total R&D bill is given as:

$$\sum_{m=1}^{n} \frac{(1-s_{cj})\chi_{cj} x_{mcjt}^{\psi} Y_t}{1+\sigma M_{ct}} = \frac{(1-s_{cj})\chi_{cj} x_{cj}^{\psi}}{1+\sigma M_c} n Y_t \tag{B.66}$$

Given all the previous derivations, we are now ready to plug in all expressions to the HJB equation given in Equation (13). This yields:

$$\begin{aligned}
r_t V_{cjt}(n) - \dot{V}_{cjt}(n) = \max_{\{x_{mcjt}\}_{m=1}^{n}} \quad & \Bigg\{ \quad \sum_{m=1}^{n} \frac{\lambda_c}{1+\lambda_c}\omega_j Y_t + n \sum_{j'=1}^{J} R_{cj't} \\
& - \sum_{m=1}^{n} \frac{(1-s_{cj})\chi_{cj} x_{mcjt}^{\psi} Y_t}{1+\sigma M_{ct}} + \left(\sum_{m=1}^{n} x_{mcjt} \right) \left(V_{cjt}^{+}(n) - V_{cjt}(n) \right) \\
& + n d_{jt} \left(V_{cjt}^{-}(n) - V_{cjt}(n) \right) + \delta \left(0 - V_{cjt}(n) \right) \Bigg\} \\
(r-g) v_{cj} n Y_t \quad = \quad & \frac{\lambda_c}{1+\lambda_c}\omega_j n Y_t + \sum_{j'=1}^{J} \hat{R}_{cj'} n Y_t \\
& - \frac{(1-s_{cj})\chi_{cj} x_{cj}^{\psi}}{1+\sigma M_c} n Y_t + x_{cj} L_{cj}^{def} v_{cj} n Y_t \\
& - d_j L_{cj}^{plain} v_{cj} n Y_t - \delta v_{cj} n Y_t
\end{aligned}$$

As can be seen, all the terms are linear in nY_t. Dividing both sides by nY_t and reorganizing, we get:

$$
\begin{aligned}
(r-g)v_{cj} &= \frac{\lambda_c}{1+\lambda_c}\omega_j + \sum_{j'=1}^{J}\hat{R}_{cj'} - \frac{(1-s_{cj})\chi_{cj}x_{cj}^{\psi}}{1+\sigma M_c} + x_{cj}L_{cj}^{def}v_{cj} \\
&\quad -d_jL_{cj}^{plain}v_{cj} - \delta v_{cj} \\
\left(r-g+\delta-x_{cj}L_{cj}^{def}+d_jL_{cj}^{plain}\right)v_{cj} &= \frac{\lambda_c}{1+\lambda_c}\omega_j + \sum_{j'=1}^{J}\hat{R}_{cj'} - \frac{(1-s_{cj})\chi_{cj}x_{cj}^{\psi}}{1+\sigma M_c} \\
v_{cj} &= \frac{\frac{\lambda_c}{1+\lambda_c}\omega_j + \sum_{j'=1}^{J}\hat{R}_{cj'} - \frac{(1-s_{cj})\chi_{cj}x_{cj}^{\psi}}{1+\sigma M_c}}{\rho+\delta-x_{cj}L_{cj}^{def}+d_jL_{cj}^{plain}} \qquad \text{(B.67)}
\end{aligned}
$$

where the last line uses $r-g=\rho$ that must hold in a BGP equilibrium due to the Euler equation of the representative household. Given the probabilities $p_{cj}^{rent}, p_{1,cj}^{LT}, p_{2,cj}^{LT}, p_{cj}^{def}, p_{cj}^{plain}, p_{cj}^{inj}$, the growth rate g, the fraction of product lines owned by technology class c firms M_c, and the creative destruction rate d_j, Equation (B.67) pins down the exact values of the scalars v_{cj} for all c and j, and thus concludes the proof for the incumbents.

Given the value function of incumbents, the optimal entrant innovation arrival rate z chosen by the entrepreneurs can also be calculated in closed-form. Using Equation (16), we get

$$
\begin{aligned}
z_t &= \left(\frac{\sum_{c=1}^{C}\sum_{j=1}^{J}\eta_{cj}(V_{cjt}^{+}(0)-V_{cjt}(0))}{(1-s_e)\nu\psi Y_t}\right)^{\frac{1}{\psi-1}} \\
&= \left(\frac{\sum_{c=1}^{C}\sum_{j=1}^{J}\eta_{cj}L_{cj}^{def}v_{cj}Y_t}{(1-s_e)\nu\psi Y_t}\right)^{\frac{1}{\psi-1}} \\
&= \left(\frac{\sum_{c=1}^{C}\sum_{j=1}^{J}\eta_{cj}L_{cj}^{def}v_{cj}}{(1-s_e)\nu\psi}\right)^{\frac{1}{\psi-1}} \equiv z \qquad \text{(B.68)}
\end{aligned}
$$

which is time-invariant and the same for all entrepreneurs.

To compute the full BGP equilibrium, the values of these endogenous probabilities must also be calculated. Two of these, the litigation probabilities $p_{1,cj}^{LT}$ and $p_{2,cj}^{LT}$ can be computed without any reference to the stationary distribution of firms. Using Equation (B.20), we have:

$$p^{LT}_{1,cj} = \mathbb{P}\left(\gamma \leq \left(-\zeta_{1cj} + \zeta_{1cj}\frac{\tau^h_{1cj} + \tau^l_{1cj}}{2}\right)\frac{(V_{cjt}(n-1) - V_{cjt}(n))}{Y_t}\right)$$
$$= \mathbb{P}\left(\gamma \leq \left(-\zeta_{1cj} + \zeta_{1cj}\frac{\tau^h_{1cj} + \tau^l_{1cj}}{2}\right)(-v_{cj})\right) \tag{B.69}$$

Likewise, using Equation (B.3), we have:

$$p^{LT}_{2,cj} = \mathbb{P}\left(\gamma \leq \frac{(1 - \tau^l_{2cj} + \Lambda_2/\zeta_{2cj})^2 \zeta_{2cj}(V_{cjt}(n+1) - V_{cjt}(n))}{4(\tau^h_{2cj} - \tau^l_{2cj})Y_t}\right)$$
$$= \mathbb{P}\left(\gamma \leq \frac{(1 - \tau^l_{2cj} + \Lambda_2/\zeta_{2cj})^2 \zeta_{2cj} v_{cj}}{4(\tau^h_{2cj} - \tau^l_{2cj})}\right) \tag{B.70}$$

The remaining endogenous variables must be computed numerically, consistent with the stationary firm distribution in the economy. □

B.7. Proof of Proposition 4

Proposition 4. *In a BGP equilibrium, the following are true:*

1. *The industry-specific creative destruction rate d_j in industry j is*

$$d_j = \sum_{c=1}^{C}(\mu_{cj}x_{cj} + \eta_{cj}z) \tag{B.71}$$

2. *The probability for plaintiffs of type (c,j) that the incoming innovation belongs to a firm with the same technology class, p^{plain}_{cj}, is*

$$p^{plain}_{cj} = \frac{\mu_{cj}x_{cj} + \eta_{cj}z}{\sum_{c'=1}^{C}(\mu_{c'j}x_{c'j} + \eta_{c'j}z)} \tag{B.72}$$

3. *The probability for defendants of type (c,j) to innovate on the product line of another firm with the same technology class c in its industry, p^{def}_{cj}, is*

$$p^{def}_{cj} = \mu_{cj} \tag{B.73}$$

4. *The Poisson arrival rate of a type 2 patent infringement for plaintiffs with technology class c from firms in industry j, p^{rent}_{cj}, is*

$$p_{cj}^{rent} = \frac{(\mu_{cj}x_{cj} + \eta_{cj}z)(1-\mu_{cj})\kappa_{2cj}}{\sum_{j'=1}^{J}\mu_{cj'}} \tag{B.74}$$

5. *The combined probability that a type 2 infringement occurs and an injunction is granted from the perspective of the owner of the product line, p_{cj}^{inj}, is*

$$p_{cj}^{inj} = \frac{\sum_{c'\neq c}(\mu_{c'j}x_{c'j} + \eta_{c'j}z)\kappa_{2c'j}p_{c'j}^{inj2}}{\sum_{c'\neq c}(\mu_{c'j}x_{c'j} + \eta_{c'j}z)} \tag{B.75}$$

where $p_{c'j}^{inj2}$ is defined in equation (B.85).

6. *The time-invariant output growth rate g is given by*

$$g = \sum_{j=1}^{J}\omega_j \sum_{c=1}^{C}\mu_{cj}f_{cj} \tag{B.76}$$

where

$$\begin{aligned} f_{cj} &= (\mu_{cj}x_{cj} + \eta_{cj}z)\left[1-\kappa_{1cj}p_{1,cj}^{LT}\left(1-\frac{\tau_{1cj}^{h}+\tau_{1cj}^{l}}{2}\right)\zeta_{1cj}\right]\ln(1+\lambda_c) \\ &\quad + \sum_{c'\neq c}(\mu_{c'j}x_{c'j} + \eta_{c'j}z)\left[1-\kappa_{2c'j}p_{c'j}^{inj2}\right]\ln(1+\lambda_c) \end{aligned} \tag{B.77}$$

7. *Define $P(\Theta,\Theta')$ as the transition rate from product lines of type $\Theta = (c,j)$ (origin) to $\Theta' = (c',j')$ (destination). The stationary values of μ_{cj} are pinned down by the following linear system of equations*

$$P^T\mu = \mu \tag{B.78}$$

$$\sum_{c=1}^{C}\mu_{cj} = 1, \forall j \tag{B.79}$$

which consists of $CJ + J$ equations.

Proof. To close the model, we need to derive the equations that pin down the values of endogenous variables in a BGP equilibrium, such as the growth rate g, the stationary product line distribution across industries and technology classes $\{\{\mu_{cj}\}_{c=1}^{C}\}_{j=1}^{J}$, and the associated probabilities of various events discussed earlier.

Recall that $\mu_{cjt} \in [0,1]$ denotes the measure of all product lines in industry j for which the leader has technology class c at time t, with $\sum_{c=1}^{C} \mu_{cjt} = 1$. In a stationary equilibrium, μ_{cjt} are time-invariant, so time subscripts will be suppressed from here on. Under this definition, total incumbent innovation by firms of technology class c in industry j is $\mu_{cj} x_{cj}$, and the total entrant innovation for the same is $\eta_{cj} z$.

The industry-specific creative destruction rate d_j in industry j depends on total innovation in that industry by both incumbents and entrants with any technology class. This is given by

$$d_j = \sum_{c=1}^{C} (\mu_{cj} x_{cj} + \eta_{cj} z) \tag{B.80}$$

The probability for plaintiffs of type (c, j) that the incoming innovation belongs to a firm with the same technology class, denoted p_{cj}^{plain}, can be calculated as

$$p_{cj}^{plain} = \frac{\mu_{cj} x_{cj} + \eta_{cj} z}{\sum_{c'=1}^{C} (\mu_{c'j} x_{c'j} + \eta_{c'j} z)} \tag{B.81}$$

which is the fraction of total innovation in industry j originating from firms of type (c, j) to that of total innovation in industry j irrespective of technology class.

The probability for defendants of type (c, j) to innovate on the product line of another firm with the same technology class c in its industry, p_{cj}^{def} is simply

$$p_{cj}^{def} = \mu_{cj} \tag{B.82}$$

since $\sum_{c=1}^{C} \mu_{cj} = 1$.

To calculate the Poisson arrival rate of a type 2 patent infringement for plaintiffs with technology class c from firms in industry j, denoted p_{cj}^{rent}, we need to do an accounting of the measure of type 2 patent infringements that happen in technology class c in industry j, and the measure of eligible plaintiffs across all industries. The prior is calculated as

$$(\mu_{cj} x_{cj} + \eta_{cj} z)(1 - \mu_{cj}) \kappa_{2cj} \tag{B.83}$$

where the first factor is the total innovation in industry j originating from firms of type (c, j), the second factor is the probability that such innovation lands on a product line with technology class $c' \neq c$, and the third factor is the probability of a type 2 patent infringement occurring under this scenario. The latter is simply the sum of all product lines belonging to firms with technology class c across all industries, i.e., $\sum_{j=1}^{J}\mu_{cj}$. Then we can calculate p_{cj}^{rent} as

$$p_{cj}^{rent} = \frac{(\mu_{cj}x_{cj} + \eta_{cj}z)(1-\mu_{cj})\kappa_{2cj}}{\sum_{j'=1}^{J}\mu_{cj'}} \tag{B.84}$$

Recall that the combined probability that a type 2 infringement occurs and an injunction is granted from the perspective of the owner of the product line was denoted p_{cj}^{inj}. In type 2 infringements, the technology class c' of the innovating firm matters for the injunction probability, since it also influences the rents the third-party plaintiff can extract. Define $p_{c'j}^{inj2}$ as the probability of an injunction conditional on the innovating firm having technology class $c' \neq c$. Then, this probability is calculated as

$$\begin{aligned} p_{c'j}^{inj2} &= p_{2,c'j}^{LT}\left(\int_{\tau_{2c'j}^{l}}^{\tau^*} 0 dT_{2c'j}(\tau) + \int_{\tau^*}^{\tau_{2c'j}^{h}}(1-\tau)dT_{2c'j}(\tau)\right)\zeta_{2c'j} \\ &= p_{2,c'j}^{LT}\left[\left(\tau_{2c'j}^{h} - \tau^* - \frac{(\tau_{2c'j}^{h})^2}{2} + \frac{(\tau^*)^2}{2}\right)\frac{1}{\tau_{2c'j}^{h} - \tau_{2c'j}^{l}}\right]\zeta_{2c'j} \\ &= p_{2,c'j}^{LT}\left[\left(\tau_{2c'j}^{h} - \frac{1+\tau_{2c'j}^{l} + \Lambda_2/\zeta_{2c'j}}{2} - \frac{(\tau_{2c'j}^{h})^2}{2} + \frac{(1+\tau_{2c'j}^{l}+\Lambda_2/\zeta_{2c'j})^2}{8}\right)\frac{1}{\tau_{2c'j}^{h} - \tau_{2c'j}^{l}}\right]\zeta_{2c'j} \end{aligned} \tag{B.85}$$

where the first factor is the probability that the plaintiff hires a legal team, the second factor is the probability that the defendant rejects the settlement offer and loses at court, and the third factor is the probability that an injunction is granted. Given this, we can calculate p_{cj}^{inj} as

$$p_{cj}^{inj} = \frac{\sum_{c'\neq c}(\mu_{c'j}x_{c'j} + \eta_{c'j}z)\kappa_{2c'j}p_{c'j}^{inj2}}{\sum_{c'\neq c}(\mu_{c'j}x_{c'j} + \eta_{c'j}z)} \tag{B.86}$$

To calculate the growth rate of the economy, we must tally not only successful innovations, but also the rate at which successful innovations convert to product line takeovers (i.e., the fraction of successful innovations that are not blocked by an injunction), and the technology classes of both the incumbent and the innovator, since the productivity gains λ_c are heterogeneous, and so are the markups charged over marginal cost.

From the definition of the production technology, we have:

$$
\begin{aligned}
\ln Y_t &= \sum_{j=1}^{J} \omega_j \left(\int_0^1 \ln y_{ijt} di \right) \\
\frac{\ln Y_{t+\Delta t} - \ln Y_t}{\Delta t} &= \sum_{j=1}^{J} \omega_j \left(\int_0^1 \frac{\ln y_{ijt+\Delta t} - \ln y_{ijt}}{\Delta t} di \right) \\
g_t &= \lim_{\Delta t \to 0} \sum_{j=1}^{J} \omega_j \left(\int_0^1 \frac{\ln y_{ijt+\Delta t} - \ln y_{ijt}}{\Delta t} di \right) \qquad \text{(B.87)}
\end{aligned}
$$

Hence, to figure out the output growth rate g_t, we must focus on how log output in each product line $\ln y_{ijt}$ changes over time. From the incumbent firm's static problem, we know

$$
\begin{aligned}
\ln y_{ijt} &= \ln \left(\frac{\omega_j Y_t q_{ijt}}{w_t (1+\lambda_c)} \right) \\
&= \ln \omega_j + \ln \left(\frac{Y_t}{w_t} \right) + \ln q_{ijt} - \ln(1+\lambda_c) \qquad \text{(B.88)}
\end{aligned}
$$

The first term is the function of a parameter, and thus constant. The second term is a function of the relative wage w_t/Y_t, which is time-invariant in a BGP equilibrium. The third term is log productivity, which increases upon successful innovation that is not blocked. The fourth term is the markup distortion, which can change upon successful innovation that is not blocked if the innovator has a different technology class $c' \neq c$.

Now, consider the case of some product line i in industry j owned by a firm with technology class c. The probability that the product line is lost to a firm with the same technology class c over a small time interval Δt is

$$
(\mu_{cj} x_{cj} + \eta_{cj} z) \Delta t \left[1 - \kappa_{1cj} p_{1,cj}^{LT} \left(1 - \frac{\tau_{1cj}^h + \tau_{1cj}^l}{2} \right) \zeta_{1cj} \right] \qquad \text{(B.89)}
$$

where the term outside the brackets is the probability of a successful innovation, whereas the term inside the brackets is the probability that an injunction is not granted. An injunction is only granted if there is an infringement (prob. κ_{1cj}), the plaintiff pays the legal team cost (prob. $p_{1,cj}^{LT}$), the defendant loses (prob. $1 - (\tau_{1cj}^h + \tau_{1cj}^l)/2$), and the court grants an injunction (prob. ζ_{1cj}). In this scenario, since both firms have the same technology class, the markup distortion is unchanged. However, log productivity increases by $\ln(1+\lambda_c)$.

For any technology class $c' \neq c$, the probability that the product line is lost to a firm with the technology class c' over a small time interval Δt is

$$(\mu_{c'j}x_{c'j}+\eta_{c'j}z)\Delta t\left[1-\kappa_{2c'j}p^{inj2}_{c'j}\right] \tag{B.90}$$

where the term outside the brackets is the probability of a successful innovation, whereas the term inside the brackets is the probability that an injunction is not granted, which uses the $p^{inj2}_{c'j}$ defined in Equation (B.85). In this scenario, the markup distortion changes from $\ln(1+\lambda_c)$ to $\ln(1+\lambda_{c'})$. Log productivity also increases by $\ln(1+\lambda_{c'})$. The net effect on log output for the product line is therefore $\ln(1+\lambda_{c'})+\ln(1+\lambda_c)-\ln(1+\lambda_{c'})=\ln(1+\lambda_c)$, same as the previous scenario.

Given these observations, for some product line i in industry j owned by a firm with technology class c, we can write:

$$\begin{aligned}\frac{\ln y_{ijt+\Delta t}-\ln y_{ijt}}{\Delta t} &= (\mu_{cj}x_{cj}+\eta_{cj}z)\left[1-\kappa_{1cj}p^{LT}_{1,cj}\left(1-\frac{\tau^h_{1cj}+\tau^l_{1cj}}{2}\right)\zeta_{1cj}\right]\ln(1+\lambda_c)\\ &\quad+\sum_{c'\neq c}(\mu_{c'j}x_{c'j}+\eta_{c'j}z)\left[1-\kappa_{2c'j}p^{inj2}_{c'j}\right]\ln(1+\lambda_c)\equiv f_{cj}\end{aligned} \tag{B.91}$$

which is called f_{cj} for convenience. Then, we can plug in these expressions in Equation (B.87) to obtain

$$g = \sum_{j=1}^{J}\omega_j\sum_{c=1}^{C}\mu_{cj}f_{cj} \tag{B.92}$$

which pins down the output growth rate in a BGP equilibrium.

Finally, we need to pin down the equations that determine $\mu_{cj},\forall c,j$. To this purpose, define a joint product line type as $\Theta=(c,j)$, and define $P(\Theta,\Theta')$ as the transition rate from product lines of type $\Theta=(c,j)$ (origin) to $\Theta'=(c',j')$ (destination). First, note that no event can change the industry of a product line. Therefore, we have

$$P((c,j),(c',j'))=0,\forall c,\forall j,\forall c',\forall j'\neq j \tag{B.93}$$

Second, if the innovating firm has the same technology class as the incumbent, the type of the product line does not change even if ownership does, so it requires no explicit accounting. So that leaves the third case to consider, with $j=j'$ and $c'\neq c$. In this case, we have:

$$P((c,j),(c',j))=(\mu_{c'j}x_{c'j}+\eta_{c'j}z)\left[1-\kappa_{2c'j}p^{inj2}_{c'j}\right],\forall c,\forall j,\forall c'\neq c \tag{B.94}$$

in agreement with Equation (B.90). Finally, we have the case $j = j'$ and $c = c'$ which is implicitly defined as

$$P((c,j),(c,j)) = 1 - \sum_{c' \neq c} P((c,j),(c',j)) \tag{B.95}$$

Using the transition matrix defined by $P(\Theta, \Theta')$, we can pin down the stationary values of μ_{cj} by solving the linear system of equations

$$P^T \mu = \mu \tag{B.96}$$

$$\sum_{c=1}^{C} \mu_{cj} = 1, \forall j \tag{B.97}$$

which consists of $CJ + J$ equations. □

B.8. Firm size distributions

To compute the firm size distributions, we need to calculate the product line takeover probabilities conditional on successful innovation for every firm type. Define this takeover probability for a firm with technology class c in industry j at time t as $p^{take}_{cjt} \in [0,1]$. This probability is calculated as:

$$\begin{aligned} p^{take}_{cjt} &= p^{def}_{cjt} \kappa_{1cj} \left[p^{LT}_{1,cjt} \left(1 - \zeta_{1cj} + \zeta_{1cj} \frac{\tau^h_{1cj} + \tau^l_{1cj}}{2} \right) + \left(1 - p^{LT}_{1,cjt} \right) \right] \\ &\quad + (1 - p^{def}_{cjt}) \kappa_{2cj} \left\{ p^{LT}_{2,cjt} \left[\left(\frac{1}{2} \frac{1 - \tau^l_{2cj} + \Lambda_2/\zeta_{2cj}}{\tau^h_{2cj} - \tau^l_{2cj}} \right) + (1 - \zeta_{2cj}) \left(1 - \frac{1}{2} \frac{1 - \tau^l_{2cj} + \Lambda_2/\zeta_{2cj}}{\tau^h_{2cj} - \tau^l_{2cj}} \right) \right.\right. \\ &\quad \left.\left. + \frac{\zeta_{2cj}}{2(\tau^h_{2cj} - \tau^l_{2cj})} \left((\tau^h_{2cj})^2 - \frac{(1 + \tau^l_{2cj} + \Lambda_2/\zeta_{2cj})^2}{4} \right) \right] + \left(1 - p^{LT}_{2,cjt} \right) \right\} \\ &\quad + \left(p^{def}_{cjt} (1 - \kappa_{1cj}) + (1 - p^{def}_{cjt})(1 - \kappa_{2cj}) \right) \end{aligned} \tag{B.98}$$

where the first term is the probability of a type 1 infringement times the conditional takeover probability, the second term is the same for type 2 infringements, and the third term is the complementary event that no infringement occurs, in which case the takeover is assured.

We also need to calculate the flow rate of losing a product line for incumbent firms. Define the per product line product line loss flow rate for a firm with technology class c in industry j at time t as $p^{loss}_{cjt} > 0$. This flow rate is calculated as:

$$
\begin{aligned}
p_{cjt}^{loss} &= (\mu_{cjt} x_{cjt} + \eta_{cj} z_t) \left[1 - \kappa_{1cj} p_{1,cjt}^{LT} \left(1 - \frac{\tau_{1cj}^h + \tau_{1cj}^l}{2} \right) \zeta_{1cj} \right] \\
&\quad + \sum_{c' \neq c} (\mu_{c'jt} x_{c'jt} + \eta_{c'j} z_t) \left[1 - \kappa_{2c'j} p_{c'jt}^{inj2} \right]
\end{aligned}
\tag{B.99}
$$

Define the mass of firms with technology class c in industry j at time t that own n product lines as $\varphi_{cjt}(n) \geq 0$. Using previously-calculated expressions, we can write the ordinary differential equations that govern the evolution of these expressions. Due to new firm entry and endogenous firm exit, $n = 1$ is a special case, which is given by:

$$
\dot{\varphi}_{cjt}(1) = z_t \eta_{cj} + 2 p_{cjt}^{loss} \varphi_{cjt}(2) - (x_{cjt} p_{cjt}^{take} + p_{cjt}^{loss}) \varphi_{cjt}(1) \tag{B.100}
$$

where the first term corresponds to new entrants with a single product line, the second term corresponds to firms with two product lines losing one of them, and the third term corresponds to outflows of firms with a single product line due to both successful takeovers, as well as losses.

For all the other cases with $n \geq 2$, we have the general expression:

$$
\begin{aligned}
\dot{\varphi}_{cjt}(n) &= (n-1) x_{cjt} p_{cjt}^{take} \varphi_{cjt}(n-1) + (n+1) p_{cjt}^{loss} \varphi_{cjt}(n+1) \\
&\quad - n (x_{cjt} p_{cjt}^{take} + p_{cjt}^{loss}) \varphi_{cjt}(n)
\end{aligned}
\tag{B.101}
$$

where the first term corresponds to firms with $n-1$ product lines succeeding in taking over a new product line, the second term corresponds to firms with $n+1$ product lines losing one of them, and the third term corresponds to outflows of firms with n product lines due to both successful takeovers, as well as losses.

In a stationary equilibrium, we have $\dot{\varphi}_{cjt}(n) = 0, \forall c, j, t, n$. Therefore, the firm size distributions are time-invariant; that is, $\varphi_{cjt}(n) \equiv \varphi_{cj}(n), \forall c, j, t, n$. Using the previous equations, we can pin down these time-invariant firm size distributions. For any technology class c and industry j, we have the following equations:

$$
0 = z \eta_{cj} + 2 p_{cj}^{loss} \varphi_{cj}(2) - (x_{cj} p_{cj}^{take} + p_{cj}^{loss}) \varphi_{cj}(1) \tag{B.102}
$$

$$
\begin{aligned}
0 &= (n-1) x_{cj} p_{cj}^{take} \varphi_{cj}(n-1) + (n+1) p_{cj}^{loss} \varphi_{cj}(n+1) \\
&\quad - n (x_{cj} p_{cj}^{take} + p_{cj}^{loss}) \varphi_{cj}(n), \forall n \geq 2
\end{aligned}
\tag{B.103}
$$

In addition, we also know

$$
\begin{aligned}
z\eta_{cj} &= p_{cj}^{loss}\varphi_{cj}(1) && \text{(B.104)} \\
\sum_{n=1}^{\infty} n\varphi_{cj}(n) &= \mu_{cj} && \text{(B.105)}
\end{aligned}
$$

where the first equation is due to firm entry being equal to firm exit in a stationary equilibrium, and the second equation is an accounting identity that ensures that the total number of product lines owned by firms with technology class c in industry j equals μ_{cj}. Together, equations (B.102), (B.103), and (B.104) pin down $\varphi_{cj}(n), \forall n \geq 1$.

B.9. Computing output and welfare

We would like to compute social welfare in counterfactual economies and compare them against the estimated equilibrium. To calculate welfare, we need to compute the consumption stream of the representative household. In a BGP equilibrium, two components must be known: the growth rate of consumption g, and the initial consumption level C_0. This requires us to compute initial output Y_0 and aggregate spending on R&D. In turn, computing initial output requires computing the (time-invariant) relative wage rate w_t/Y_t. We will compute these in reverse order.

To calculate the relative wage rate, we will use the labor market clearing condition. First, recall that the output y_{ijt} of firm i in industry j at time t is given by:

$$
y_{ijt} = \frac{\omega_j Y_t}{p_{ijt}} = \frac{\omega_j Y_t q_{ijt}}{w_t(1+\lambda_c)} \qquad \text{(B.106)}
$$

Then, the labor demand of this firm becomes

$$
l_{ijt} = \frac{y_{ijt}}{q_{ijt}} = \frac{\omega_j Y_t}{w_t(1+\lambda_c)} \qquad \text{(B.107)}
$$

which is independent of the firm's productivity q_{ijt}. Since the representative household supplies labor $L = 1$ inelastically, labor market clearing requires:

$$
\begin{aligned}
1 &= \sum_{j=1}^{J}\int_0^1 l_{ijt}\,di \\
1 &= \sum_{j=1}^{J}\int_0^1 \frac{\omega_j Y_t}{w_t(1+\lambda_c)}di \\
\frac{w_t}{Y_t} &= \sum_{j=1}^{J}\omega_j\int_0^1 \frac{1}{(1+\lambda_c)}di \\
\frac{w_t}{Y_t} &= \sum_{j=1}^{J}\omega_j\sum_{c=1}^{C}\frac{\mu_{cj}}{(1+\lambda_c)} \qquad \text{(B.108)}
\end{aligned}
$$

which delivers the time-invariant relative wage rate.

The level of output Y_t at time t is given by:

$$
\begin{aligned}
\ln Y_t &= \sum_{j=1}^{J}\omega_j\left(\int_0^1 \ln y_{ijt}\,di\right) \\
\ln Y_t &= \sum_{j=1}^{J}\omega_j\left(\int_0^1 \ln\left(\frac{\omega_j Y_t q_{ijt}}{w_t(1+\lambda_c)}\right)di\right) \\
\ln Y_t &= -\ln\frac{w_t}{Y_t}+\sum_{j=1}^{J}\omega_j\ln(\omega_j)+\sum_{j=1}^{J}\omega_j\left(\int_0^1 \ln\left(\frac{q_{ijt}}{(1+\lambda_c)}\right)di\right) \\
\ln Y_t &= -\ln\frac{w_t}{Y_t}+\sum_{j=1}^{J}\omega_j\ln(\omega_j)-\sum_{j=1}^{J}\omega_j\sum_{c=1}^{C}\mu_{cj}\ln(1+\lambda_c)+\sum_{j=1}^{J}\omega_j\left(\int_0^1 \ln q_{ijt}\,di\right) \qquad \text{(B.109)}
\end{aligned}
$$

where the last term is the log productivity level of the economy at time t, i.e., the weighted sum of the log productivity level in each industry j, where the weights are the Cobb-Douglas shares ω_j. In our counterfactual experiments, we shall hold the initial log productivity level at time $t = 0$ fixed across economies. Without loss of generality, it is normalized to zero.[26]

Let L_{cj} denote the normalized per product line expected litigation cost for product lines owned by firms in industry j with technology class c:

[26] This is equivalent to setting all $q_{ij0} = 1$.

$$
\begin{aligned}
L_{cj} \quad = \quad & d_j p_{cj}^{plain} \kappa_{1cj} \left(\int_0^{\left(-\zeta_{1cj}+\zeta_{1cj}\frac{\tau_{1cj}^h+\tau_{1cj}^l}{2}\right)(-v_{cj})} \gamma d\Gamma_{cj}(\gamma) \right) \\
& + \sum_{j'=1}^{J} \left(p_{cj'}^{rent} \int_0^{\frac{(1-\tau_{2cj}^l+\Lambda_2/\zeta_{2cj})^2 \zeta_{2cj} v_{cj'}}{4(\tau_{2cj}^h-\tau_{2cj}^l)}} \gamma d\Gamma_{cj}(\gamma) \right) \\
& + x_{cj}(1-p_{cj}^{def})\kappa_{2cj} p_{2,cjt}^{LT} \left(1 - \frac{1}{2} \frac{1-\tau_{2cj}^l+\Lambda_2/\zeta_{2cj}}{\tau_{2cj}^h-\tau_{2cj}^l} \right) \Lambda_2 v_{cj}
\end{aligned} \tag{B.110}
$$

Then the aggregate litigation spending in the whole economy is calculated as

$$
\sum_{j=1}^{J} \sum_{c=1}^{C} \mu_{cj} L_{cj} Y_t \tag{B.111}
$$

From the goods market clearing, we can compute the time-invariant consumption to output ratio C_t/Y_t as follows:

$$
\begin{aligned}
Y_t \quad &= \quad C_t + \sum_{j=1}^{J} \sum_{c=1}^{C} \mu_{cj} \frac{\chi_{cj} x_{cj}^{\psi} Y_t}{1+\sigma M_c} + \nu z^{\psi} Y_t + \sum_{j=1}^{J} \sum_{c=1}^{C} \mu_{cj} L_{cj} Y_t \\
\frac{C_t}{Y_t} \quad &= \quad 1 - \sum_{j=1}^{J} \sum_{c=1}^{C} \mu_{cj} \frac{\chi_{cj} x_{cj}^{\psi}}{1+\sigma M_c} - \nu z^{\psi} - \sum_{j=1}^{J} \sum_{c=1}^{C} \mu_{cj} L_{cj}
\end{aligned} \tag{B.112}
$$

where the second and third terms are the total incumbent and entrant R&D spending to output ratios, respectively, and the last term is the aggregate litigation spending to output ratio. Then, the initial consumption level is simply $C_0 = Y_0(C_0/Y_0)$.

We are now ready to compute social welfare in a BGP equilibrium. From the utility function of the representative household in equation (1), we have:

$$
W = \int_0^{\infty} e^{-\rho t} \ln C_t dt = \int_0^{\infty} e^{-\rho t} \ln(e^{gt} C_0) dt = \frac{\ln C_0}{\rho} + \frac{g}{\rho^2} \tag{B.113}
$$

which shows how the welfare depends on the initial level of consumption C_0 and the growth rate of the economy g.

For two economies A and B, we can define a consumption equivalent welfare change measure (ϖ) which corresponds to the percentage increase in lifetime consumption that an agent in economy A would need to be indifferent between being in economy A or B:

$$W_B = \frac{\ln(C_0^A(1+\varpi))}{\rho} + \frac{g^A}{\rho^2} \tag{B.114}$$

Solving for ϖ, we get:

$$\varpi = \exp\left(\left(W_B - \frac{g^A}{\rho^2}\right)\rho - \ln(C_0^A)\right) - 1 \tag{B.115}$$

C. Model Validation and Robustness

C.1. External validity

A well-specified structural model should generate empirical patterns beyond those used to discipline its parameters. We assess this by examining moments and patterns that were not directly targeted in estimation.

First, regarding the response of innovation to the *eBay* ruling, our model predicts an increase in innovation of approximately 3%, as shown in Table 6. This quantity is consistent with Mezzanotti (2021)'s finding of a roughly 3% increase in innovation following the ruling, despite this moment not being targeted in estimation. The model's ability to match this quasi-experimental estimate is particularly reassuring, as the *eBay* ruling operates through changes in injunctive relief standards rather than through the variation used to identify our model parameters.

Second, while we target the mean level of litigation costs as a fraction of firm revenue and its correlation with firm profitability, the skewness of this ratio is not targeted. The model nevertheless produces a skewness of 0.920, close to the data value of 1.111, indicating that the model replicates the heavy right tail of the litigation cost distribution. This feature of the data is important: a small number of technology class–industry pairs face disproportionately large litigation costs relative to revenue, and our model generates this concentration through the interaction of heterogeneous technology class characteristics and endogenous litigation behavior.

Third, we verify that our model reproduces several patterns documented by Lanjouw and Schankerman (1997) that we did not directly target in estimation. These patterns are well-established in the empirical patent litigation literature and provide a natural benchmark for assessing the external validity of structural models of patent disputes. Lanjouw and Schankerman (1997) show that litigated patents receive roughly twice as many forward citations as non-litigated patents, suggesting that higher-value patents are more likely to be involved in litigation. Our model captures this pattern: litigation probabilities increase with firm value, generating a positive relationship between stakes and litigation. The correlation between the probability of being a plaintiff and firm value is 0.239 in the model and 0.275 in the data. Lanjouw and Schankerman (1997) also find that patents in crowded technology areas face significantly higher litigation risk. Our model generates this pattern through the spillover term M_c: technology class–industry pairs with a larger mass of firms experience higher litigation rates due to greater innovation overlap. The correlation between the litigation rate and the fraction of firms in each technology class–industry pair is 0.285 in the model and 0.293 in the data.

Our model generates the correct directional predictions and reasonable magnitudes for these untargeted patterns, reassuring us that the underlying economic mechanisms are well-specified. These validation exercises span innovation responses to policy changes, distributional features of litigation costs, and cross-sectional relationships between litigation, firm value, and market structure, testing distinct aspects of the model. The *eBay* counterfactual tests the policy response margin, the cost skewness tests the distributional fit, and the Lanjouw and Schankerman (1997) patterns test the cross-sectional implications of equilibrium litigation behavior.

C.2. Sensitivity to defendant litigation costs

Our baseline model assumes that defendants face negligible litigation costs ($\Lambda_2 = 0$), which was a simplification for tractability. In practice, defending a patent infringement lawsuit can be costly, which affects defendants' settlement decisions. We extend the model to include defendants' costs: when a case goes to trial, defendants pay a fraction Λ_2 of the expected value of winning as legal fees, incurred regardless of the trial outcome.

For Type 2 infringements, this modification changes the settlement decision throughout. Defendants are more willing to settle when litigation costs are high, and the optimal settlement offer adjusts accordingly. For Type 1 infringements, we maintain $\Lambda_1 = 0$ for tractability. Positive values of Λ_1 would severely complicate the closed-form expressions throughout the model for negligible quantitative gain. As shown in Proposition 3 in Internet Appendix B.4, reasonable positive values of Λ_1 do not have a large quantitative impact both in terms of the fraction of out-of-court settlements, and the impact on plaintiff payoff, and consequently, plaintiff litigation probability. So we focus on an experiment for Λ_2. In the baseline estimation, we set $\Lambda_2 = 0$ due to a lack of data to reliably estimate what fraction of the expected payoff defendants spend on legal fees. Here, we examine how results change when we vary defendant legal costs.

Table IA.6 in the Internet Appendix presents results for various values of $\Lambda_2 \in \{0, 0.05, 0.10, 0.15\}$, representing different assumptions about defendant legal costs as a fraction of case value.

We believe that these are realistic values for Λ_2 based on the 2023 American Intellectual Property Law Association (AIPLA) Annual Report. Specifically, in a survey conducted by the AIPLA for this report, patent litigation professionals report that defendants in larger cases (those in which the patent value is estimated to be larger than $25 million) spend 14.5% of the estimated patent value on legal fees in the median case. This figure includes all legal costs through all phases of the lawsuit (including trial). This estimate is based on the median cost ($3.625 million) for lawsuits with at least $25 million at risk, the largest size considered in the report. The proportional cost (legal fees divided by value at risk) appears to decline with case size, and the large Compustat firms in our sample likely have cases with a value at risk even higher than $25 million. Accordingly, we view 14.5% as an upper bound for typical cases in our sample.[27]

Applying different assumptions regarding Λ_2, our main welfare conclusions remain robust: defendant-friendly reforms generate positive welfare gains across all specifications.

Table IA.6 in the Internet Appendix reveals an interesting asymmetry in how defendant legal fees affect the welfare gains from different policies. Although both reducing injunction rates and increasing filing costs are "defendant-friendly" policies, their welfare effects move in opposite directions as Λ_2 increases: the gain from reducing injunctions rises from 2.98% to 3.02%, while the gain from increasing filing costs falls from 1.85% to 1.79%.

The intuition lies in how defendant costs affect the composition of lawsuits. When Λ_2 is high, defendants settle more generously to avoid litigation costs. This raises expected settlements, making lawsuits more attractive and drawing in additional plaintiffs.

The filing cost policy deters lawsuits by making them more expensive. But when Λ_2 is high and expected settlements are large, plaintiffs are willing to pay higher filing costs: the lawsuits are simply too lucrative to abandon. The policy's deterrence effect accordingly weakens as Λ_2 rises, yielding smaller welfare gains.

In contrast, the injunction policy reduces expected damages conditional on litigation. When Λ_2 is high, and more cases proceed to litigation, reducing damages per case benefits a larger pool of lawsuits, generating larger welfare gains precisely when the litigation burden is most severe.

The R&D subsidy follows a similar pattern: its welfare gain rises with Λ_2 because direct

[27] See page 67 of the report at `https://fundamentalpatlit.com/wp-content/uploads/2024/02/AIPLA.pdf` for more details.

innovation subsidies become more valuable when the baseline litigation burden on innovators is larger.

Table IA.7 in the Internet Appendix reports the impact of rising patent trolls in an alternative economy with higher defendant litigation costs ($\Lambda_2 = 0.15$). Under this parameterization, the welfare loss from patent trolls is larger at 2.90% compared to 2.82% in the baseline, reflecting the greater resource waste when defendants face higher litigation costs.

C.3. The role of financial constraints

Financial constraints are an important channel through which litigation costs can affect innovation, particularly given the delays and uncertainty in income streams caused by patent disputes. Our model captures several dimensions of this channel. Litigation costs enter directly into firm decision-making, and the financial burden of patent disputes is a central mechanism through which litigation affects innovation investment in our framework. Moreover, our estimation disciplines the model to match heterogeneous litigation behavior across firms with different financial positions: more profitable firms face both higher conditional infringement probabilities and higher plaintiff litigation rates, so that our structural parameters reflect the empirical relationship between a firm's financial position and its litigation behavior. Our counterfactual experiments therefore account for differences in how firms respond to litigation reforms based on their financial resources.

Explicitly modeling financial constraints within our current framework would significantly increase computational complexity. Our model already incorporates substantial cross-industry and technological heterogeneity, as well as general equilibrium and endogenous growth features, and adding a financial-constraints state variable would require tracking the joint distribution of firm productivity and financial resources. We believe our current approach, which captures the direct financial costs of litigation and aligns with the empirical relationship between firms' financial characteristics and litigation behavior, provides a tractable framework that accounts for the first-order effects of financial constraints on litigation and innovation decisions.

To further explore this channel, we split firms into financially unconstrained and constrained subsamples based on the median probability of default (Bharath and Shumway, 2008),[28] and separately estimate the model for each group. We then conduct our key coun-

[28] Because Farre-Mensa and Ljungqvist (2016) find that the Merton distance-to-default implied default

terfactual experiments within each subsample and compare the welfare implications across groups.

Parameter estimates across subsamples. Table IA.8 in the Internet Appendix reports the estimated parameter values for unconstrained and constrained firms. Several patterns emerge that are consistent with financial constraints amplifying the burden of patent litigation.

First, and most importantly, the litigation environment differs markedly across subsamples. Constrained firms face higher litigation costs: the litigation cost scale parameter $\beta_{\xi,0}$ is 15.52 for constrained firms versus 18.05 for unconstrained firms, implying higher expected per-case costs for constrained firms (since lower ξ_{cj} corresponds to higher expected cost draws). This pattern is consistent with constrained firms having fewer resources for legal defense and being less able to absorb litigation expenses. Constrained firms also face higher infringement risk: the baseline type-1 infringement probability $\beta_{\kappa_1,0}$ is 0.580 versus 0.545, and the baseline type-2 infringement probability $\beta_{\kappa_2,0}$ is 0.930 versus 0.870. The higher type-2 infringement rate for constrained firms is consistent with these firms having weaker patent portfolios and fewer resources to mount a credible legal defense, making them more susceptible to rent-seeking claims.

Second, the innovation parameters are similar across subsamples, with estimates close to their full-sample values. The mean innovation step size μ_λ is 0.181 for unconstrained versus 0.182 for constrained, and the R&D cost parameters μ_χ (6.032 versus 6.013) and σ_χ (2.275 versus 2.335) show only modest variation.

Welfare effects across subsamples. Table IA.9 in the Internet Appendix reports the consumption-equivalent welfare change (CEWC) for each policy experiment, estimated separately for each subsample. The results reveal that financially constrained firms benefit more from all three policy reforms.

The welfare gain from reducing injunction rates is 3.18% for constrained firms versus 2.75% for unconstrained firms. Similarly, the welfare gain from increasing filing costs is 2.06% for constrained firms versus 1.68% for unconstrained firms. These results are intuitive: constrained firms face both higher infringement rates ($\beta_{\kappa_1,0}$, $\beta_{\kappa_2,0}$) and substantially higher

probability best predicts financial constraints in their setting, we opted for that as our financial constraints proxy. Specifically, we follow the approach of Bharath and Shumway (2008) to calculate the distance to default, then calculate a probability of default.

per-case litigation costs (lower $\beta_{\xi,0}$), so defendant-friendly reforms that reduce litigation exposure generate disproportionately large welfare gains for these firms. The larger benefit from filing cost increases is particularly noteworthy, as it suggests that policies that deter frivolous lawsuits are especially valuable for firms that lack the financial resources to absorb litigation expenses.

The R&D subsidy also generates larger welfare gains for constrained firms (1.93% versus 1.88% for unconstrained), though the differential is modest relative to the litigation reforms. The direction of this result is intuitive: constrained firms face higher per-case litigation costs, which suppresses their innovation investment, so R&D subsidies that offset this underinvestment generate larger gains.

Taken together, these results support the view that financial constraints amplify the welfare costs of patent litigation. Importantly, because our baseline model does not explicitly incorporate financial frictions, the welfare effects we estimate likely represent a *lower bound* on the true differential impact of litigation reforms across financially constrained and unconstrained firms. With explicit financial constraints, litigation costs would drain cash reserves and tighten borrowing constraints, further reducing investment capacity. This additional channel would amplify the benefits of defendant-friendly reforms for constrained firms beyond what our current framework captures. However, since severe financial constraints are relatively rare for public firms (Farre-Mensa and Ljungqvist, 2016) and the impacts of financial constraints appear to be modest, we expect our current estimates to be quantitatively similar to those from a more complex model featuring financial constraints.

C.4. Reputation effects in litigation

In models such as Hovenkamp (2013), asymmetric information about a firm's willingness to pursue litigation creates an additional incentive to litigate: doing so establishes a reputation for toughness. In such a model, patent holders may pursue a lawsuit with a negative expected monetary value to create a perception that they are litigious, deterring infringers. Similarly, defendants may resist settlement and insist on a costly trial, even when doing so has a negative expected monetary value, to create a perception among potential future plaintiffs that they are expensive to sue. These reputation-building incentives introduce option value beyond the immediate payoff of any individual lawsuit.

Explicitly modeling reputation building in our framework would be intractable. It would require modeling reputation as a state variable in a repeated game, with an information structure governing how potential infringers update beliefs based on observed litigation outcomes, a substantially different modeling exercise that calls for more granular data to estimate.

Luckily, however, our model likely already captures the welfare implications of a reputation-building model. Several features of our model allow for litigation prevalence and litigation incentives to vary heterogeneously across firm characteristics that correspond to reputation-building incentives. Thus, if the true data-generating process for our data involves reputation concerns, our estimation should correctly capture that litigation reforms disproportionately impact innovation for reputation-sensitive firms. In this sense, our model predictions should be isomorphic to the predictions of a reputation model.

Specifically, our model is a dynamic framework with endogenous growth, forward-looking innovation decisions, and firm entry and exit, so firms' litigation and innovation choices already reflect the ongoing consequences of the litigation environment. Reputation in litigation operates through two channels, both of which are captured by the heterogeneity in our structural parameters.

First, our model features a *deterrence channel*: firms with established reputations for aggressive patent enforcement discourage potential infringers, reducing the probability of infringement in equilibrium. We capture this channel by allowing the conditional infringement probability κ_{cj} to vary with firm characteristics, and we estimate that R&D-intensive firms face lower conditional infringement probabilities ($\beta_{\kappa_1,2} < 0$, $\beta_{\kappa_2,2} < 0$), consistent with these firms' investment in patent protection and enforcement building a reputation that deters future infringement.

Second, our model features an *incentive channel*: firms with more at stake, those with greater profitability and larger innovation portfolios, have stronger incentives to invest in building reputations by litigating more frequently. We find that more profitable firms face higher conditional infringement probabilities ($\beta_{\kappa_1,1} > 0$, $\beta_{\kappa_2,1} > 0$), reflecting that they are more attractive targets, and they exhibit higher plaintiff litigation rates in both the data and the model. R&D-intensive firms also litigate more frequently overall: while their conditional infringement probability is lower, their higher innovation rates generate more products exposed to potential infringement, and the volume effect dominates. The net result is that R&D-intensive firms are more frequent litigators, the same pattern that a reputation model would predict. Our structural parameters are estimated to match all of these empirical patterns, so that our counterfactual experiments correctly reflect the heterogeneous litigation behavior of firms with differing reputation-building incentives.

The remaining distinction between our approach and an explicit reputation model is that we capture these effects through cross-sectional variation in infringement and litigation parameters rather than through a firm-level reputation state variable that evolves with litigation history. From a welfare perspective, what matters is the equilibrium mapping from firm characteristics to litigation rates and innovation decisions, which our model matches to the data. Nonetheless, a full quantitative treatment of dynamic reputation incentives is an important direction for future research.

D. Additional Tables

Table IA.1: Standardized Sensitivity Matrix

Notes: Each cell reports the elasticity ε_{jk}: the percent change in moment j (row) when parameter k (column) changes by one percent, following the sensitivity analysis approach of Andrews et al. (2017). Parameters are grouped by economic function: (i) *Productivity Growth* parameters govern the distribution of innovation step sizes across technology classes which in turn affects the mean and standard deviation of growth rate; (ii) *Innovation and Industry Dynamics* parameters determine R&D costs, entry, and knowledge spillovers; (iii) *Litigation Incidence* parameters govern infringement probabilities and filing costs, which jointly determine the probability and composition of lawsuits; and (iv) *Case Outcomes* parameters determine defendant win rates and settlement behavior. Highlighted cells along the diagonal indicate primary identifying relationships. The block-diagonal structure confirms that each parameter is identified by economically intuitive moments within its category. All parameters are estimated jointly via GMM.

	Growth		*Innovation and Industry Dynamics*					*Litigation Incidence*								*Case Outcomes*					
Moment	μ_λ	σ_λ	μ_χ	σ_χ	ψ	ν	σ	$\beta_{\kappa_1,0}$	$\beta_{\kappa_1,1}$	$\beta_{\kappa_1,2}$	$\beta_{\kappa_2,0}$	$\beta_{\kappa_2,1}$	$\beta_{\kappa_2,2}$	$\beta_{\xi,0}$	$\beta_{\xi,1}$	$\mu_{\bar{\tau}_1}$	$\sigma_{\bar{\tau}_1}$	$\mu_{\tau_2^l}$	$\sigma_{\tau_2^l}$	$\mu_{\tau_2^h}$	$\sigma_{\tau_2^h}$
Growth																					
Output growth rate	**1.043**	0.287	−0.565	0.045	3.144	−0.107	0.007	−0.120	−0.020	−0.008	−0.077	−0.005	−0.008	−0.036	−0.002	0.157	0.000	0.006	0.031	−0.022	−0.003
Stdv. of sales growth	0.189	**0.342**	−1.014	−0.043	3.449	0.389	0.023	0.026	0.003	0.000	−0.060	−0.009	−0.019	0.050	0.000	−0.047	−0.006	0.009	0.067	−0.025	−0.004
Innovation and Industry Dynamics																					
Mean of R&D intensity	0.792	0.222	**−0.650**	0.055	1.513	0.239	0.008	−0.092	−0.014	−0.005	−0.099	−0.006	−0.008	−0.028	−0.001	0.117	0.004	0.009	0.042	0.010	0.001
Stdv. of R&D intensity	0.518	0.464	−0.658	**0.063**	1.295	0.234	0.009	−0.117	−0.019	−0.007	−0.091	−0.007	−0.010	−0.017	−0.001	0.155	0.009	0.010	0.040	0.015	0.002
Skewness of R&D intensity	−0.302	0.201	0.212	−0.162	**0.327**	−0.052	0.013	−0.121	−0.025	−0.013	0.301	0.011	0.020	0.025	0.001	0.180	0.032	0.048	0.138	−0.055	−0.008
Entry rate	0.333	0.095	−0.298	0.041	2.499	**−0.330**	0.003	−0.080	−0.013	−0.005	−0.066	−0.004	−0.006	−0.028	−0.001	0.105	0.000	0.003	0.015	−0.003	−0.000
β(R&D spending, tech-class share)	−1.432	1.174	−0.540	0.817	−3.932	−0.218	**−0.035**	0.080	0.026	0.017	−0.680	−0.050	−0.076	−0.038	−0.004	0.006	0.064	−0.098	−1.032	0.378	0.048
Litigation Incidence																					
Mean prob. of being a plaintiff	0.521	0.085	−0.681	0.070	2.761	0.160	0.008	**0.349**	0.053	0.023	0.436	0.023	0.036	0.341	0.016	−0.082	−0.006	−0.050	−0.445	−0.018	−0.003
β_1(lit. prob. ~ profit, R&D int.)	0.405	−1.471	−4.994	−0.449	8.691	4.523	0.165	−1.405	**−0.118**	−0.187	4.712	0.359	0.491	−1.685	−0.048	2.278	−0.221	−0.327	−1.218	−0.820	−0.096
β_2(lit. prob. ~ profit, R&D int.)	−0.179	−0.124	−0.427	−0.026	1.598	0.375	0.011	0.667	0.092	**0.068**	−0.196	−0.007	−0.020	0.518	0.026	−0.656	−0.090	0.032	0.366	0.135	0.019
Fraction of same-industry lawsuits	−0.005	0.006	−0.019	0.004	0.005	0.011	0.000	0.042	0.006	0.002	**−0.044**	−0.003	−0.004	−0.007	−0.000	−0.014	−0.000	−0.001	−0.007	−0.158	−0.023
β_1(frac. same-ind. ~ profit, R&D int.)	−0.252	−0.432	−0.086	−0.081	1.725	−0.289	−0.021	0.606	0.244	0.040	−0.622	**−0.071**	−0.154	−0.774	−0.038	0.334	0.004	−0.025	−0.106	1.228	0.313
β_2(frac. same-ind. ~ profit, R&D int.)	−0.743	−0.176	−0.384	0.260	−3.419	0.879	0.006	−0.437	−0.065	0.033	0.502	0.030	**0.061**	0.724	0.030	−0.014	0.014	−0.081	−0.414	−6.426	−0.788
Mean litigation costs/revenue	0.775	0.176	−0.268	0.024	1.999	−0.102	0.003	0.466	0.071	0.027	0.408	0.022	0.034	**−0.165**	−0.009	−0.429	−0.014	−0.048	−0.408	−0.079	−0.012
β_1(lit. costs/rev. ~ profit)	−7.683	5.352	−2.379	2.459	−4.772	−1.204	−0.083	1.585	0.406	0.169	−1.942	−0.026	−0.048	−3.371	**−0.166**	0.041	−0.071	−0.447	−3.055	−0.286	−0.007
Case Outcomes																					
P(plaintiff win | same-industry)	0.025	−0.017	−0.016	−0.019	0.115	0.027	0.002	0.006	0.000	−0.000	0.017	0.001	0.001	−0.004	−0.000	**−0.966**	−0.001	−0.001	0.025	−0.014	−0.003
Stdv. of P(plaintiff win | same-industry)	−0.001	0.012	−0.054	0.004	0.007	0.048	0.001	−0.053	−0.000	0.001	0.040	−0.001	−0.004	0.010	−0.000	−0.521	**0.520**	0.046	0.276	0.028	0.030
P(plaintiff win | diff.-industry)	−0.209	0.179	−0.195	0.201	−0.764	−0.013	−0.006	0.026	0.006	0.003	−0.174	−0.011	−0.019	−0.017	−0.001	−0.017	0.008	**−0.084**	−0.609	−0.122	−0.018
Stdv. of P(plaintiff win | diff.-industry)	0.000	−0.000	−0.000	−0.000	0.000	0.000	0.000	−0.000	−0.000	−0.000	0.000	0.000	0.000	0.000	0.000	0.000	−0.000	0.095	**0.871**	−0.047	−0.001
P(settlement | being a plaintiff)	−0.041	0.016	0.057	0.029	−0.210	−0.067	−0.002	−0.336	−0.050	−0.019	0.297	0.018	0.029	0.037	0.003	0.109	0.004	−0.084	−0.754	**−0.198**	−0.030
Stdv. of P(settlement | being a plaintiff)	0.127	−0.109	0.049	−0.128	0.640	0.050	0.006	0.318	0.050	0.020	−0.247	−0.015	−0.024	−0.075	−0.004	−0.068	−0.007	0.116	1.057	0.228	**0.029**
Overidentifying Restrictions																					
Stdv. prob. of being a plaintiff	−0.446	0.822	−2.030	0.816	0.018	0.728	−0.005	0.235	0.044	0.018	0.227	0.013	−0.010	0.112	0.007	−0.083	−0.008	−0.060	−0.362	0.024	0.004
Corr(litigation prob., sales growth)	−0.362	0.307	0.064	−0.166	0.828	0.061	0.006	0.319	0.048	0.022	−0.600	−0.016	−0.008	−0.061	0.006	−0.117	−0.030	−0.065	−0.645	−0.076	−0.006
Corr(litigation prob., R&D intensity)	−0.218	0.152	0.175	−0.203	0.959	0.019	0.008	0.311	0.049	0.022	−0.423	−0.005	0.012	−0.053	0.007	−0.070	−0.007	−0.063	−0.639	−0.010	0.003

Table IA.2: Decomposing the Impact of Reducing the Injunction Rate by Infringement Types

Notes: This table decomposes the impact of reducing the injunction rate while keeping other parameters unchanged at their baseline values. We evaluate its effects on innovation, firm values, firm dynamics, litigation, growth, and social welfare. Column (1) reports the values of the variables of interest for the baseline model. In column (2), we reduce the injunction rate for both type-1 and type-2 infringements to post-eBay levels. In column (3), we reduce the injunction rate parameter for type-1 patent infringement to post-eBay levels while keeping the rate for type-2 patent infringement unchanged at pre-eBay levels. Finally, in column (4), we reduce the injunction rate parameter for type-2 patent infringement to post-eBay levels while keeping the rate for type-1 patent infringement unchanged at pre-eBay levels.

	(1)	(2)		(3)		(4)	
	Baseline	**Both Types**		**Type 1 Only**		**Type 2 Only**	
		Counterfactual	**% Change**	**Counterfactual**	**% Change**	**Counterfactual**	**% Change**
incumbent innovation	0.0609	0.0627	2.984%	0.0619	1.729%	0.0616	1.143%
avg. R&D intensity	4.278%	4.504%	5.282%	4.433%	3.636%	4.352%	1.734%
entrant innovation	0.1217	0.1250	2.739%	0.1235	1.481%	0.1233	1.341%
avg. entry rate	5.258%	5.504%	4.680%	5.429%	3.241%	5.326%	1.293%
contribution of entrants to growth	33.377%	33.004%	−1.117%	33.166%	−0.631%	33.272%	−0.313%
creative destruction rate	9.132%	9.397%	2.902%	9.282%	1.647%	9.242%	1.209%
avg. incumbent value	0.2186	0.2152	−1.547%	0.2195	0.410%	0.2142	−2.015%
avg. entrepreneur value	0.1347	0.1430	6.179%	0.1392	3.314%	0.1388	3.000%
avg. product line value	0.1199	0.1155	−3.623%	0.1181	−1.479%	0.1174	−2.039%
avg. number of product lines	1.8240	1.8633	2.154%	1.8590	1.918%	1.8245	0.024%
stdv. number of product lines	2.3488	2.3973	2.065%	2.3936	1.907%	2.2767	−3.069%
avg. plaintiff prob.	8.664%	8.084%	−6.692%	8.322%	−3.955%	8.445%	−2.531%
per product line plaintiff prob.	5.217%	4.812%	−7.779%	4.902%	−6.044%	5.119%	−1.879%
avg. prob. of hiring legal team	77.809%	70.218%	−9.757%	72.605%	−6.689%	75.292%	−3.235%
output growth rate	2.092%	2.217%	5.974%	2.191%	4.740%	2.116%	1.154%
consumption	0.2332	0.2327	−0.184%	0.2328	−0.140%	0.2331	−0.046%
output	0.2527	0.2527	−0.000%	0.2527	−0.001%	0.2527	−0.000%
CEWC	——	2.984%	——	2.366%	——	0.559%	——

Table IA.3: Decomposing the Impact of Increasing Plaintiff Filing Costs by Infringement Types

Notes: This table decomposes the impact of increasing plaintiff filing costs while keeping other parameters unchanged at their baseline values. We evaluate its effects on innovation, firm values, firm dynamics, litigation, growth, and social welfare. Column (1) reports the values of the variables of interest for the baseline model. In column (2), we halve the ξ_{cj} parameter from its baseline value for both type-1 and type-2 infringements. In column (3), we halve the ξ_{cj} parameter from its baseline value for type-1 infringement, effectively doubling the average expense associated with hiring a legal team for the plaintiff in type-1 infringement cases. In column (4), we halve the ξ_{cj} parameter from its baseline value for type-2 infringement, effectively doubling the average expense associated with hiring a legal team for the plaintiff in type-2 infringement cases.

	(1) **Baseline**	(2) **Both Types**		(3) **Type 1 Only**		(4) **Type 2 Only**	
		Counterfactual	**% Change**	**Counterfactual**	**% Change**	**Counterfactual**	**% Change**
incumbent innovation	0.0622	0.0632	1.581%	0.0627	0.740%	0.0627	0.836%
avg. R&D intensity	4.455%	4.609%	3.442%	4.533%	1.746%	4.532%	1.723%
entrant innovation	0.1244	0.1274	2.410%	0.1254	0.822%	0.1264	1.609%
avg. entry rate	5.442%	5.604%	2.970%	5.539%	1.773%	5.507%	1.186%
contribution of entrants to growth	33.134%	33.278%	0.435%	33.139%	0.016%	33.266%	0.398%
creative destruction rate	9.332%	9.505%	1.857%	9.403%	0.767%	9.434%	1.094%
avg. incumbent value	0.2158	0.2109	−2.251%	0.2153	−0.232%	0.2113	−2.099%
avg. entrepreneur value	0.1414	0.1490	5.422%	0.1440	1.832%	0.1465	3.604%
avg. product line value	0.1166	0.1131	−2.949%	0.1151	−1.265%	0.1145	−1.757%
avg. number of product lines	1.8509	1.8643	0.719%	1.8703	1.046%	1.8445	−0.349%
stdv. number of product lines	2.3066	2.3202	0.587%	2.3168	0.442%	2.2802	−1.143%
avg. plaintiff prob.	8.279%	6.208%	−25.016%	7.379%	−10.863%	7.133%	−13.842%
per product line plaintiff prob.	4.929%	3.615%	−26.655%	4.324%	−12.283%	4.221%	−14.356%
avg. prob. of hiring legal team	72.276%	52.433%	−27.455%	63.518%	−12.117%	61.066%	−15.509%
output growth rate	2.188%	2.270%	3.720%	2.243%	2.526%	2.214%	1.198%
consumption	0.2328	0.2324	−0.203%	0.2326	−0.108%	0.2326	−0.100%
output	0.2527	0.2527	−0.002%	0.2527	−0.001%	0.2527	−0.001%
CEWC	—	1.849%	—	1.282%	—	0.556%	—

Table IA.4: Simulating the Effects of Reducing Patent Infringement Risk

Notes: This table evaluates the impact of reducing patent infringement risk while keeping other parameters fixed at their baseline values. We evaluate its effects on innovation, firm values, firm dynamics, litigation, growth, and social welfare. This analysis is relevant to policies such as the introduction of Patent Trial and Appeal Board (PTAB) and the *Alice* decision in improving patent clarity and reducing litigation risks. Column (1) presents the baseline values for the variables of interest. Column (2) examines the effects of reducing both κ_{1cj} and κ_{2cj} to 75% of their baseline values simultaneously. Column (3) reports the results of reducing the type-1 infringement risk parameter, κ_{1cj}, to 75% of its baseline value. Column (4) reflects the results of reducing the type-2 infringement risk parameter, κ_{2cj}, to 75% of its baseline value.

	(1)	(2)		(3)		(4)	
	Baseline	**Both Types**		**Type 1 Only**		**Type 2 Only**	
		Counterfactual	**% Change**	**Counterfactual**	**% Change**	**Counterfactual**	**% Change**
incumbent innovation	0.0622	0.0637	2.377%	0.0629	1.074%	0.0631	1.387%
avg. R&D intensity	4.455%	4.685%	5.149%	4.566%	2.490%	4.576%	2.704%
entrant innovation	0.1244	0.1283	3.117%	0.1259	1.193%	0.1268	1.968%
avg. entry rate	5.442%	5.645%	3.724%	5.564%	2.233%	5.530%	1.612%
contribution of entrants to growth	33.134%	33.225%	0.274%	33.174%	0.120%	33.164%	0.091%
creative destruction rate	9.332%	9.577%	2.624%	9.436%	1.114%	9.479%	1.581%
avg. incumbent value	0.2158	0.2125	−1.500%	0.2167	0.439%	0.2120	−1.735%
avg. entrepreneur value	0.1414	0.1513	7.049%	0.1451	2.665%	0.1476	4.420%
avg. product line value	0.1166	0.1139	−2.287%	0.1158	−0.710%	0.1149	−1.482%
avg. number of product lines	1.8509	1.8658	0.805%	1.8724	1.158%	1.8462	−0.256%
stdv. number of product lines	2.3066	2.2579	−2.112%	2.4006	4.074%	2.2286	−3.382%
avg. plaintiff prob.	8.279%	6.554%	−20.830%	7.465%	−9.828%	7.381%	−10.850%
per product line plaintiff prob.	4.929%	3.804%	−22.817%	4.356%	−11.621%	4.362%	−11.512%
avg. prob. of hiring legal team	72.276%	72.528%	0.349%	71.112%	−1.610%	73.523%	1.725%
output growth rate	2.188%	2.303%	5.234%	2.262%	3.377%	2.233%	2.065%
consumption	0.2328	0.2325	−0.156%	0.2326	−0.100%	0.2327	−0.077%
output	0.2527	0.2527	0.001%	0.2527	−0.000%	0.2527	0.001%
CEWC	——	2.744%	——	1.762%	——	1.058%	——

Table IA.5: Simulating the Impact of Rising Patent Trolls

Notes: This table simulates the potential effects of a rise in patent trolls on innovation, firm values, firm dynamics, litigation, growth, and social welfare. Patent trolls are modeled as a combination of an increase in type-2 infringement probability—cases where the plaintiff is not the owner of the product line and files lawsuits for rent-extraction purposes—along with a rise in the entry rate of low R&D efficiency firms. To simulate these effects, we double the type-2 infringement risk and the entry rate of firms in the lowest tercile of research efficiency (measured by λ_c/χ_{cj}). Column (1) reports the baseline values for the variables of interest. Column (2) presents the effects of a rise in patent trolls, combining both the increased type-2 infringement risk and the higher entry rate of low R&D efficiency firms. Column (3) isolates the effect of a rise in type-2 infringement probability by doubling the type-2 infringement risk. Column (4) shows the impact of a rise in low research efficiency firms by doubling the entry rate of firms in the lowest tercile of research efficiency.

	(1) Baseline	(2) Rise in Patent Trolls		(3) Higher Type-2 Risk		(4) More Low R&D Firms	
		Counterfactual	% Change	Counterfactual	% Change	Counterfactual	% Change
incumbent innovation	0.0622	0.0610	−1.933%	0.0618	−0.699%	0.0614	−1.392%
avg. R&D intensity	4.455%	4.265%	−4.268%	4.403%	−1.175%	4.318%	−3.076%
entrant innovation	0.1244	0.1194	−4.006%	0.1232	−0.921%	0.1206	−3.047%
avg. entry rate	5.442%	5.252%	−3.494%	5.397%	−0.834%	5.291%	−2.789%
contribution of entrants to growth	33.134%	32.955%	−0.539%	33.164%	0.089%	33.008%	−0.381%
creative destruction rate	9.332%	9.087%	−2.624%	9.260%	−0.773%	9.150%	−1.944%
avg. incumbent value	0.2158	0.2120	−1.736%	0.2178	0.936%	0.2101	−2.655%
avg. entrepreneur value	0.1414	0.1290	−8.744%	0.1385	−2.034%	0.1319	−6.710%
avg. product line value	0.1166	0.1142	−2.060%	0.1176	0.896%	0.1134	−2.759%
avg. number of product lines	1.8509	1.8571	0.331%	1.8517	0.039%	1.8529	0.106%
stdv. number of product lines	2.3066	2.2422	−2.793%	2.3475	1.771%	2.1782	−5.568%
avg. plaintiff prob.	8.279%	8.549%	3.263%	8.750%	5.689%	8.081%	−2.386%
per product line plaintiff prob.	4.929%	5.062%	2.707%	5.219%	5.876%	4.774%	−3.140%
avg. prob. of hiring legal team	72.276%	70.882%	−1.928%	71.403%	−1.208%	71.658%	−0.855%
output growth rate	2.188%	2.063%	−5.721%	2.167%	−0.990%	2.080%	−4.927%
consumption	0.2328	0.2334	0.266%	0.2329	0.032%	0.2334	0.240%
output	0.2527	0.2525	−0.080%	0.2527	−0.001%	0.2525	−0.080%
CEWC	——	−2.823%	——	−0.509%	——	−2.425%	——

Table IA.6: Welfare Effects of Policy Experiments under Different Defendant Legal Fee Parameters

Notes: This table reports the consumption-equivalent welfare change (CEWC) for three policy experiments under different values of the defendant legal fee parameter Λ_2. Column (1) reports the welfare effect of reducing the injunction rate from pre-eBay to post-eBay levels. Column (2) reports the welfare effect of increasing plaintiff filing costs by halving the ξ_{cj} parameter. Column (3) reports the welfare effect of doubling R&D subsidies from 8% to 16%. The parameter Λ_2 governs the defendant's litigation cost as a fraction of their value at stake. Higher values of Λ_2 imply greater deadweight loss from litigation.

Λ_2	**Reducing Injunction Rate**	**Increasing Filing Cost**	**R&D Subsidy**
0.00	2.984%	1.849%	1.908%
0.05	2.996%	1.821%	1.910%
0.10	3.003%	1.810%	1.913%
0.15	3.015%	1.785%	1.919%

Table IA.7: Simulating the Impact of Rising Patent Trolls (High Defendant Litigation Cost $\Lambda_2 = 0.15$)

Notes: This table simulates the potential effects of a rise in patent trolls on innovation, firm values, firm dynamics, litigation, growth, and social welfare in an alternative economy with higher defendant litigation cost $\Lambda_2 = 0.15$. Patent trolls are modeled as a combination of an increase in type-2 infringement probability—cases where the plaintiff is not the owner of the product line and files lawsuits for rent-extraction purposes—along with a rise in the entry rate of low R&D efficiency firms. To simulate these effects, we double the type-2 infringement risk and the entry rate of firms in the lowest tercile of research efficiency (measured by λ_c/χ_{cj}). Column (1) reports the baseline values for the variables of interest. Column (2) presents the effects of a rise in patent trolls, combining both the increased type-2 infringement risk and the higher entry rate of low R&D efficiency firms. Column (3) isolates the effect of a rise in type-2 infringement probability by doubling the type-2 infringement risk. Column (4) shows the impact of a rise in low research efficiency firms by doubling the entry rate of firms in the lowest tercile of research efficiency.

	(1) Baseline	(2) Rise in Patent Trolls		(3) Higher Type-2 Risk		(4) More Low R&D Firms	
		Counterfactual	**% Change**	**Counterfactual**	**% Change**	**Counterfactual**	**% Change**
incumbent innovation	0.0610	0.0597	−2.120%	0.0604	−0.926%	0.0602	−1.344%
avg. R&D intensity	4.303%	4.093%	−4.882%	4.236%	−1.551%	4.161%	−3.312%
entrant innovation	0.1216	0.1162	−4.391%	0.1201	−1.237%	0.1178	−3.123%
avg. entry rate	5.329%	5.122%	−3.886%	5.269%	−1.124%	5.175%	−2.876%
contribution of entrants to growth	33.148%	32.842%	−0.924%	33.152%	0.013%	32.934%	−0.646%
creative destruction rate	9.138%	8.875%	−2.875%	9.044%	−1.030%	8.961%	−1.935%
avg. incumbent value	0.2194	0.2150	−1.999%	0.2216	1.009%	0.2130	−2.920%
avg. entrepreneur value	0.1344	0.1215	−9.556%	0.1307	−2.726%	0.1252	−6.872%
avg. product line value	0.1185	0.1157	−2.300%	0.1196	0.956%	0.1149	−3.010%
avg. number of product lines	1.8519	1.8576	0.308%	1.8528	0.052%	1.8536	0.092%
stdv. number of product lines	2.2464	2.1517	−4.219%	2.2705	1.071%	2.1259	−5.364%
avg. plaintiff prob.	9.009%	9.291%	3.137%	9.537%	5.869%	8.770%	−2.649%
per product line plaintiff prob.	5.365%	5.514%	2.772%	5.693%	6.107%	5.190%	−3.255%
avg. prob. of hiring legal team	80.407%	79.564%	−1.049%	80.283%	−0.155%	79.641%	−0.953%
output growth rate	2.141%	2.012%	−6.029%	2.114%	−1.245%	2.035%	−4.965%
consumption	0.2332	0.2339	0.282%	0.2333	0.050%	0.2337	0.235%
output	0.2527	0.2525	−0.083%	0.2527	−0.002%	0.2525	−0.082%
CEWC	——	−2.902%	——	−0.615%	——	−2.394%	——

Table IA.8: Parameter Estimates: Financially Unconstrained vs. Constrained Firms

Notes: This table reports parameter estimates and standard errors for subsamples split by the median naive probability of default (Bharath and Shumway, 2008). Firms with below-median default probability are classified as financially unconstrained; firms with above-median default probability are classified as financially constrained.

		Unconstrained		**Constrained**	
		Values	**Std. Err.**	**Values**	**Std. Err.**
μ_λ	Mean of innov. step size	0.181	(0.0004)	0.182	(0.0004)
σ_λ	Stdv. of innov. step size	0.145	(0.0013)	0.150	(0.0011)
μ_χ	Mean of incumbent R&D cost scale	6.032	(0.4172)	6.013	(0.3552)
σ_χ	Stdv. of incumbent R&D cost scale	2.275	(0.2693)	2.335	(0.2338)
ψ	R&D cost convexity	2.225	(0.0279)	2.210	(0.0226)
ν	Entrant R&D cost scale	2.048	(0.1167)	2.020	(0.0935)
σ	Knowledge spillover strength	0.053	(0.0161)	0.052	(0.0196)
$\beta_{\xi,0}$	Litigation cost scale	18.047	(0.5905)	15.524	(0.6041)
$\beta_{\xi,1}$	Litigation cost, profitability coef.	−19.837	(7.3068)	−8.962	(7.8428)
$\beta_{\kappa_1,0}$	Type-1 infringement prob., constant	0.545	(0.0127)	0.580	(0.0165)
$\beta_{\kappa_1,1}$	Type-1 infringement prob., profitability coef.	1.192	(0.2725)	1.794	(0.3370)
$\beta_{\kappa_1,2}$	Type-1 infringement prob., R&D coef.	−0.649	(0.1755)	−0.666	(0.1818)
$\beta_{\kappa_2,0}$	Type-2 infringement prob., constant	0.870	(0.0183)	0.930	(0.0236)
$\beta_{\kappa_2,1}$	Type-2 infringement prob., profitability coef.	1.108	(0.3703)	0.706	(0.4359)
$\beta_{\kappa_2,2}$	Type-2 infringement prob., R&D coef.	−1.413	(0.2103)	−2.387	(0.2416)
$\mu_{\bar{\tau}_1}$	Mean of type-1 def. win prob.	0.500	(0.0003)	0.497	(0.0004)
$\sigma_{\bar{\tau}_1}$	Stdv. of type-1 def. win prob.	0.115	(0.0005)	0.112	(0.0006)
$\mu_{\tau_2^l}$	Mean of type-2 def. win prob. (lb)	0.089	(0.0073)	0.084	(0.0077)
$\sigma_{\tau_2^l}$	Stdv. of type-2 def. win prob. (lb)	0.480	(0.0046)	0.478	(0.0047)
$\mu_{\tau_2^h}$	Mean of type-2 def. win prob. (ub)	0.479	(0.0029)	0.479	(0.0039)
$\sigma_{\tau_2^h}$	Stdv. of type-2 def. win prob. (ub)	0.160	(0.0078)	0.170	(0.0106)

Table IA.9: Welfare Effects of Policy Experiments: Financially Unconstrained vs. Constrained Firms

Notes: This table reports the consumption-equivalent welfare change (CEWC) for three policy experiments, estimated separately for financially unconstrained and constrained firm subsamples split by the median naive probability of default (Bharath and Shumway, 2008). Firms with below-median default probability are classified as unconstrained; firms with above-median default probability are classified as constrained. "Reducing Injunction Rate" reports the welfare effect of moving from pre-eBay to post-eBay injunction rates. "Increasing Filing Cost" reports the welfare effect of halving the ξ_{cj} parameter. "R&D Subsidy" reports the welfare effect of doubling R&D subsidies from 8% to 16%.

	Unconstrained	**Constrained**
Reducing Injunction Rate	2.749%	3.180%
Increasing Filing Cost	1.680%	2.058%
R&D Subsidy	1.878%	1.926%

Table IA.10: Differential Response of Litigation Types to Policy Changes

Notes: This table reports the percentage change in average firm-level plaintiff probability by infringement type under each policy experiment relative to the baseline. Type 1 infringement occurs when an innovator takes over a product line from a same-industry incumbent who then claims infringement. Type 2 infringement occurs when an innovator inadvertently infringes on patents held by third-party firms in the same technology class. Policy experiments are implemented as follows: (1) Reducing injunction rate simulates the *eBay v. MercExchange* ruling by lowering ζ_{cj} from pre-eBay to post-eBay levels; (2) Increasing filing cost doubles the litigation cost parameter ξ_{cj}; (3) R&D subsidy doubles the subsidy rate s_{cj} from 8% to 16%; (4) PTAB simulates the Patent Trial and Appeal Board and the *Alice Corp. v. CLS Bank* decision by reducing the conditional infringement probability parameters $\kappa_{1,cj}$ and $\kappa_{2,cj}$ by 25%; (5) Patent troll simulates the rise of NPEs by doubling the conditional Type 2 infringement probability parameters $\kappa_{2,cj}$ and the entry of low-research-efficiency firms.

Policy Experiment	Type 1 Δ%	Type 2 Δ%
Reducing Injunction Rate	−11.32%	−2.21%
Increasing Filing Cost	−25.91%	−25.19%
R&D Subsidy	+6.87%	+4.30%
PTAB	−23.79%	−19.25%
Patent Troll	−1.35%	+7.78%