# A Theory of Dynamic Product Awareness and Targeted Advertising*

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**Abstract**

Technological advances in advertising enable firms to contact new customers faster and to better target those most likely to buy their products. To study the aggregate implications, we develop a framework of demand as a network, where heterogeneous consumers dynamically become "aware" of differentiated products. With their advertising choices, firms can affect the rate at which their networks expand ("contacting") and the probability with which they match with high valuation consumers ("targeting"). When calibrating the model to the advent of digital advertising in the United States, we find an increase in aggregate productivity due to improved consumer-firm match quality. Moreover, while both contacting and targeting intensified over this period, firms did not sufficiently increase their targeting investment relative to the social optimum.

**Keywords**: Product Awareness, Advertising, Customer Capital, Information Frictions, Targeting, Choice Sets.

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# 1 Introduction

Advertising helps firms build a customer base by spreading product awareness. To accomplish this goal, firms have traditionally employed a mixture of broad-based advertising (e.g., door-to-door sales, billboard ads, or print and radio advertising) and targeted advertising (e.g., buying mailing lists or sending customer catalogues).

While these practices have been widely used by businesses for over a century, recent technological advances and, in particular, the rise of digital advertising over the past few decades, have improved the efficiency of both broad-based and targeted forms of advertising.[1] These technological changes in advertising are affecting how customers are reached and how markets are structured, and they are likely to have consequences on industry dynamics, and competition. Yet, little is known about their overall effects at the macroeconomic level and, ultimately, about their impact on aggregate social welfare.

This paper aims to fill this gap by uncovering and quantifying the macroeconomic effects of changes in advertising technologies. To do so, we develop a theoretical framework that addresses two important dimensions of advertising seldom considered in the macroeconomics literature: firms reaching out to new potential customers ("contacting"), and firms aiming at those consumers who are more likely to purchase their products ("targeting").

Building these mechanisms into a general-equilibrium model with heterogeneous consumers, and calibrating its parameters to a period of time that saw a rapid increase in advertising efficiency in the United States, we find that improved advertising technologies have several effects: (i) due to reduced costs of both contacting and targeting, firm-consumer match quality improves (i.e., consumers found their most preferred products faster), which increases aggregate productivity, consumption, and welfare; (ii) declining costs in contacting and targeting have opposite effects on competition: faster contacting fosters competition while better targeting allows firms to charge higher markups; (iii) contacting and targeting are substitutes at the firm level: a decrease in the cost of contacting (respectively, targeting) reduces the intensity of targeting (respectively, contacting); and (iv) like firms, the social planner allocates more resources to both contacting and targeting as a result of improved advertising technologies, but the increase in targeting investment by firms is too small relative to the social optimum.

In our model, a fixed measure of consumers seek to purchase products from an expanding set of imperfectly substitutable product categories. In each category, there are

[1] Digital advertising has vastly outgrown any other advertising method: by 2024, it accounted for roughly 60% of total ad revenue worldwide, and it is expected to surpass 70% by 2028 (data by Dentsu, sourced from Statista: https://www.statista.com/statistics/375008/share-digital-ad-spend-worldwide/).

a finite number of firms producing horizontally differentiated products and competing strategically. Consumers are heterogeneous in their idiosyncratic tastes for the different products within each product category. To this standard expanding-varieties model, we introduce information frictions in the form of *limited product awareness*: at each point in time, consumers are only aware of a subset of the products that are being sold in each product category.[2] Idiosyncratic awareness sets evolve endogenously as new consumer-firm connections are formed as a result of the firms' strategic advertising choices. Thus, our framework models demand as an endogenous network between consumers and producers, nesting the standard Dixit-Stiglitz demand structure as a limiting case when networks are fully saturated.

In this economy, each consumer purchases, in equilibrium, at most one product from each product category — the one which offers the best price-to-valuation comparison among the set of known alternatives. A firm's demand is thus not only a function of prices along the intensive margin, i.e., incumbent customers purchase a lower quantity if the price increases, but also along the *extensive margin*: price changes induce some consumers to switch away from their incumbent sellers into firms in their awareness set that offer better price-to-valuation ratios. Throughout the paper, we refer to this extensive margin as the "sorting component of demand".

To this framework, we introduce advertising as a technology that affects how connections are formed between consumers and firms over the product's life-cycle. There are two dimensions to advertising in our model. First, firms can directly affect the rate at which they form new network connections at random, i.e., the rate at which consumers randomly match with firms and learn about them for the first time. We call this *contacting*. Second, firms can distort the probability of meeting a consumer with a high valuation for their product. We call this dimension *targeting*, as it allows firms to reach consumers with higher idiosyncratic preferences earlier on, expanding demand along the intensive margin at given prices. In sum, advertising fulfills two related but distinct roles in our model: it raises product awareness, and it facilitates the formation of better matches with fewer network connections.[3]

To characterize the equilibrium, we assume that firms make their advertising decisions

[2]We label information frictions for consumers within the network with the term "limited awareness" following the empirical literature on this topic (e.g., Goeree (2008) and De Los Santos, Hortaçsu and Wildenbeest (2012)). Limited awareness captures frictions such as a consumer having no knowledge of a product's existence, the idiosyncratic match to their preferences, and/or the location of a distributor.

[3]While our paper focuses on informative advertising, advertising could also serve a *persuasive* role, a dimension that is absent from our baseline model. In Appendix C, however, we propose an extension of the model that incorporates persuasive advertising.

strategically (i.e., offering a best response to the decisions of others) and once-and-for-all at birth. Moreover, firms make their pricing decisions simultaneously in a repeated Bertrand game. While our focus is on a Markov Perfect Equilibrium with symmetric strategies, we derive firms' best response functions by allowing for unilateral, off-equilibrium deviations in both advertising and pricing strategies.

Given advertising strategies, we show that a firm's optimal pricing policy is to set a markup which declines in the price-elasticity of the sorting component of the firm's demand — an object that we label the "extensive-margin demand elasticity". Intuitively, when the consumer is aware of few alternatives to her current seller, a price change is less likely to induce a separation. As a result, firms wield more market power when networks are sparse. However, when product awareness sets are more saturated and consumers know about more firms, demand curves become more price-elastic along the extensive margin, implying lower price markups. We further show that in an equilibrium with symmetric pricing and advertising strategies, the extensive-margin demand elasticity is an increasing function of the average *size* of awareness sets (namely, the number of competitors that the firm is effectively facing) but not of their composition (namely, which firms compete with which). As an implication, markups are monotonically decreasing in the saturation of firm-consumer networks: greater product awareness strengthens competition between firms and lowers markups in the industry.[4]

The core mechanism of our model lies in how the two different dimensions of advertising (contacting and targeting) interact with the firms' pricing strategies just described. On the one hand, a higher contact rate accelerates network formation and, therefore, yields lower consumer misallocation through better sorting: with more (random) contacts per unit of time, consumers have more alternatives to choose from, so they gain faster access to products that they may prefer over those that they are currently purchasing. As networks expand faster, firm competition intensifies earlier on, and markups decline faster as information disseminates. On the other hand, a high degree of targeting allows firms to more rapidly find high willingness-to-pay consumers — those less likely to switch to new products as their networks expand. This allows a firm to extract a higher match surplus while being able to maintain high prices. Hence, targeting lets firms segment their market and more easily find the most profitable and least price-sensitive consumers, weakening competition.[5]

[4]In our parameterization, networks tend to become denser as the product category ages. However, the model admits other relationships between markups and firm age in equilibrium, so long as parameter values are such that networks do not monotonically expand on average with age. Our theory is, therefore, flexible enough to accommodate different age profiles of markups.

[5]To be precise, this is a partial-equilibrium logic that applies to off-equilibrium deviations in targeting from the symmetric strategy. In the symmetric equilibrium, targeting has no direct effect on product awareness nor, therefore, on markups. However, targeting affects the return to contacting, so it indirectly does impact

To make progress on the aggregate effects of advertising, we show that, in spite of the underlying heterogeneity, our model aggregates to a representative-agent neoclassical growth economy with endogenous aggregate TFP. Aggregate TFP, in turn, increases in what we call "aggregate match quality", an endogenous object that encompasses the aggregate effects on productivity of targeting and demand sorting.

Equipped with this characterization, we compare the symmetric decentralized equilibrium allocation to that of a constrained social planner that is subject to the same product market frictions as the agents in the market economy. In this comparison, we uncover various inefficiencies. First, there are static losses in aggregate TFP from market power, and, in particular, from markups being dispersed across product categories, reflecting that both capital and labor are misallocated away from high-markup (i.e., low awareness) categories. Dynamically, markups generate underinvestment in physical capital, which further depresses output relative to the efficient benchmark. Second, firms do not internalize the effects of their advertising choices on aggregate outcomes. On the one hand, as they do not appropriate the full consumer surplus when forming a new connection, there is underinvestment in both contacting and targeting. On the other hand, firms over-invest due to a business-stealing effect: a higher advertising intensity increases the chances of attracting customers of other firms. The planner, instead, understands that more targeting and faster network formation both allow consumers to get matched with their most preferred firm faster, increasing aggregate match quality, productivity, and consumption.[6]

Having characterized the relationship between advertising and industry dynamics theoretically, we then move to quantifying the effects of advertising and information frictions in the product market on consumer sorting, competition, and welfare, by exploiting changes in advertising technologies in the data. Specifically, we focus on the rise of digital advertising in the United States. To implement this exercise, the stationary solution of our model is calibrated separately to two time periods, 2005 and 2014, which saw a rapid increase in the share of digital advertising in total advertising spending.[7] We interpret the advent of digital advertising in recent years as a rise in the effectiveness of targeting, using the click-through rate of targeted vis-a-vis untargeted digital advertising in the data as a proxy for the return to targeting in the model. Under this interpretation, our calibrations show

markups.

[6]While advertising exhibits both positive and negative spillovers theoretically, in all of our quantitative exercises, we consistently find that the limited appropriability externalities are the quantitatively dominant ones.

[7]In the United States, online advertising revenue — including search, social media, and mobile — grew exponentially, increasing from 8.09 billion U.S. dollars in 2000 to 258.6 billion U.S. dollars in 2024 (source: https://www.statista.com/statistics/183816/us-online-advertising-revenue-since-2000/).

that the rise of digital advertising was associated with a decrease in the cost of targeting, but also (though to a lesser extent) in the cost of contacting new consumers. Through the lens of our calibrations, these cost changes led to a surge in targeting investment — matches were of higher quality on average, correlating more strongly with consumers' idiosyncratic preferences —, but also to higher contact rates —awareness sets expanded faster on average. In the aggregate, this caused an increase in aggregate TFP, due in part to the strong effects of increased aggregate match quality (which combines the effects of targeting and sorting) on overall product demand, raising both consumption and total output.

To understand the degree to which these changes stem from cost savings in contacting vis-a-vis targeting, we conduct a series of counterfactual exercises. Starting from the late calibration, we re-compute the economy's stationary equilibrium assuming that the cost parameters related to both advertising technologies are set back to their (higher) levels in the early period, while keeping all the other parameter values fixed at their calibrated values for the late period. Our findings show that, had neither advertising cost declined in this period, firms would have invested less in both targeting and contacting new consumers, leading to a 10.7% lower aggregate match quality and a 10.6% lower aggregate consumption — our headline number for welfare.[8]

The declines in targeting and contacting costs contribute differently to the welfare gains from improved advertising technologies. To make this point, we compute two additional counterfactuals: one in which only the cost of contacting is reverted back to its initial (and higher) level, and another counterfactual in which the same is done for the cost of targeting. We find that welfare gains in consumption-equivalent terms are about 4%, or two-fifths of the total gains, from cheaper contacting, with the remaining gains being due to cheaper targeting.[9]

These exercises also reveal that contacting and targeting have opposite effects on competition and markups. Reverting only the cost of contacting to its early (higher) value decreases the contact rate (by 12.3%), but raises targeting (by 1.6%), whereas reverting the cost of only targeting increases the contact rate (by 6.9%), and lowers targeting (by 38.9%). Therefore, contacting and targeting are substitutes at the firm level: a higher contact rate

[8] Consumption-equivalent welfare in this economy is equal to the percentage change in aggregate consumption. The effect on aggregate consumption, in turn, comes in part from a decrease in the investment of new product creation, leading to a lower measure of product categories. However, the gains of the rise in digital advertising are still substantial, at 3.7%, when using normalized consumption (i.e., consumption per product category) to measure welfare.

[9] In normalized consumption (i.e., consumption per product category), differences are more dramatic: faster contacting contributes one-tenth of the welfare gains (0.4% out of the 3.7% of gains in normalized consumption).

accelerates network formation, and this lowers the return to targeting because it closes the window of opportunity for firms to set high markups via attracting higher valuation consumers earlier on.[10]

Conducting the same counterfactual experiments on the constrained-efficient allocation reveals that the planner's response to the change in advertising costs is also to substitute between contacting and targeting. When compared to the market allocation response, we find that the targeting rate moves further away over time from the constrained-efficient level (widening the gap to the frontier from 1.3% to 3.5%), whereas the degree of underinvestment in contacting remains virtually unchanged (at a gap of around 3.3%). That is, in response to the improved advertising technologies, firms did not sufficiently increase their investment into targeting relative to what would have been socially optimal.

**Literature review** Our paper relates to several strands of the literature. Most directly, we contribute to a literature in macroeconomics and international trade that studies the implications of customer capital for firm and industry dynamics. Contributions to this literature include Fishman and Rob (2003), Luttmer (2006), Arkolakis (2010, 2016), Dinlersoz and Yorukoglu (2012), Drozd and Nosal (2012), and Gourio and Rudanko (2014a,b).[11] In these models, firms grow via the accumulation of idiosyncratic demand, which the empirical literature has found to be an important determinant of both the overall dispersion of firm sales and the growth dynamics of firms (see e.g., Foster, Haltiwanger and Syverson (2008) for evidence in the manufacturing sector, and Hottman, Redding and Weinstein (2016) and Einav, Klenow, Levin and Murciano-Goroff (2021) for retail markets). Our paper contributes to this literature by showing that it is not just accumulated customers that matter, but also the features of the interconnected network that aggregate to form the firm's customer capital.

Our interpretation for the slow-moving process of demand accumulation is related to the idea that consumers accumulate information slowly about the producers that they can purchase from. Goeree (2008) uses a similar notion of awareness as limited information sets, but in a largely static fashion. Like us, Guthmann (2025, 2024) explores the dynamic implications of limited awareness, but through word-of-mouth dynamics among buyers and price-posting strategies on the side of sellers similar to those in Butters (1977). Our main contribution in the limited awareness literature is to provide a connection with advertising

[10]This is a quantitative result. Theoretically, we argue that the cross-derivative of the returns to advertising can have a positive sign as well.

[11]Some earlier models of advertising are due to Dorfman and Steiner (1954), Butters (1977), Stegeman (1991) and Becker and Murphy (1993). For a survey of the advertising literature in economics, see Bagwell (2007).

choices along two margins, the speed at which new customers are contacted and the quality of the firm-customer matches via targeting.[12] Moreover, unlike previous papers, we analyze how this process shapes market dynamics and aggregate welfare through its effects on competition and sorting in a general-equilibrium setting.

Various studies lend support to our assumptions regarding demand formation. First, for there to be significant quantitative consequences of our information friction, networks must remain relatively sparse and choice sets cannot be large. Empirical studies able to connect individuals to choice sets consistently show that consumers choose between few options, a surprising finding in light of the rapid advances in advertising technology. For example, De Los Santos *et al.* (2012) find that 35% of consumers only visit a single online bookstore during 18 months of data, while Honka and Chintagunta (2016) document average choice sets of size two to three in the market for auto insurance.[13] We rely on these types of findings to model demand as a network in which connections between firms and consumers are formed slowly in response to the advertising decisions, and awareness sets remain relatively small.

Numerous studies have also documented that the entry and exit of products into household consumption baskets plays an important role empirically, e.g., Broda and Weinstein (2010), Argente, Fitzgerald, Moreira and Priolo (2025), and Michelacci, Paciello and Pozzi (2021). Our model can be seen as offering a micro-foundation for these dynamics.[14] Finally, in our model the prospect of accumulating demand also shapes the incentives to create new products through innovation, as in Cavenaile and Roldan-Blanco (2021) and Ignaszak and Sedláček (2024). Therefore, changes in the advertising technology also affect the number of products that enter the economy, which has consequences for welfare.

Part of the literature has emphasized the role of price dynamics in models with consumer markets, such as in Klemperer (1995), Bergemann and Välimäki (2006), Shi (2016), Roldan-Blanco and Gilbukh (2021), and Rudanko (2025), among others. In our paper, incomplete

[12] In our framework, firms can access better matches not through prices, but through (nonpecuniary) advertising. Cheaper advertising technologies can lead to increased market power because targeting may diminish the incentives to invest in contacting, which fosters competition. Relatedly, Menzio (2023) offers an alternative explanation for the decrease in competition in spite of better sorting, namely that firms can segment markets by designing more specialized varieties that provide higher utility to a smaller measure of buyers.

[13] Looking at direct relationships of buyers and sellers using Colombian export data, Eaton, Eslava, Jinkins, Krizan and Tybout (2025) also find very small networks, of around 1.5 buyers per exporter and 4 sellers per buyer, on average. Bernard, Dhyne, Magerman, Manova and Moxnes (2022), Fitzgerald, Haller and Yedid-Levi (2024) and Lenoir, Martin and Mejean (2023) also document that the customer margin plays an important role in export markets among Belgian, Irish and French firms, respectively.

[14] Like in our model, Paciello, Pozzi and Trachter (2019) build a theory in which firms respond to changes in their extensive-margin demand, though in that model this occurs because customers can choose to search for other suppliers when faced with a price change.

information on the side of buyers has implications for welfare due to misallocation effects in the aggregate. In this sense, we also relate to papers where advertising and customer markets give rise to misallocation through market power. Along these lines, Cavenaile, Celik, Roldan-Blanco and Tian (2025) show that advertising can have beneficial effects on allocative efficiency, thereby alleviating static welfare losses from input misallocation, albeit at the cost of crowding out R&D resources. Relatedly, De Ridder (2024) argues that a more intensive use of intangible investments might give rise to increases in concentration, and Afrouzi, Drenik and Kim (2023) show that firms increase market shares through the number of customers but exert market power through non-pricing activities, consistent with our setting where customer accumulation is driven by advertising. We contribute to this literature by proposing a new mechanism for market power coming from the endogenous formation of consumer-firm networks induced by advertising activities.

Finally, we also relate to a small but nascent literature that studies the effects of the rise of digital advertising on the aggregate economy. Similar to our paper, though through a very different setting, Baslandze, Greenwood, Marto and Moreira (2023) focus on how the advent of digital advertising may have welfare implications;[15] Rachel (2024) argues that the emergence of leisure-enhancing technologies, e.g., through media platforms financed by advertising, may have had an adverse effect on hours worked and aggregate TFP; and Greenwood, Ma and Yorukoglu (2025) argue that the rise in digital advertising may have had positive effects on welfare because it alleviates an under-provision inefficiency problem in media goods, which are valued by consumers because they increase utility through non-market activities. Complementing these studies, we argue that digital advertising has positive effects from increased match quality, i.e., consumers having earlier access to products that they value more.

# 2 Model

## 2.1 Environment

Time is continuous, runs forever, and is indexed by $t$. The economy is populated by a measure-one continuum of infinitely-lived and heterogeneous consumers indexed by $j \in [0, 1]$, with preferences over a continuum of product categories. The measure of product

[15] In their case, welfare effects come through an increase in product categories. While this effect is also present in our quantitative exercises, we argue that the welfare effect is to a significant extent also due to the increase in match quality.

categories is endogenous and denoted by $\boldsymbol{M}_t > 0$.[16] Each category $m \in [0, \boldsymbol{M}_t]$ is populated by the same exogenous number $N \in \mathbb{Z}_+$ of identical single-product firms, indexed by $i = 1, \ldots, N$, who are born at the time at which the product category is created and exit only when the product category is destroyed. The $N$ firms repeatedly interact strategically within their product category in various ways, as detailed below.

A synthetic final good can be used either for consumption or investment.[17] There are three types of investment: in physical capital, in advertising, and in product category creation. Consumers supply labor inelastically in a frictionless labor market at a wage $w_t$, and they own the stock $\boldsymbol{K}_t$ of physical capital in the economy. Capital is rented to firms at the perfectly competitive rental rate $R_t^K$. Consumers also receive dividends from the firms' profits, and trade in financial assets that pay the net interest rate $r_t$.

### 2.1.1 Consumers

Consumer $j \in [0, 1]$ maximizes lifetime utility:

$$\int_0^{+\infty} e^{-\rho t} \frac{C_{jt}^{1-\gamma}}{1-\gamma} dt, \tag{1}$$

where $\rho > 0$ is the time discount rate, $\gamma > 1$ is the coefficient of relative risk aversion, and $C_{jt}$ is individual $j$'s level of consumption. Every individual $j$ can purchase from each and every one of the $N$ different firms $i = 1, \ldots, N$ in every product category $m \in [0, \boldsymbol{M}_t]$, provided that the consumer is *aware* of the product (as described below). Henceforth, the pair $(i, m)$ uniquely identifies a product.

Consumers are heterogeneous along two dimensions. First, there are permanent heterogeneous preferences across products, captured by a time-invariant, consumer-product specific preference shifter $\xi_{imj} > 0$. We make the following assumption regarding the distribution of these idiosyncratic preferences:

**Assumption 1** (Idiosyncratic preferences)**.** *Preference shifters $(\xi_{imj})$ for the population of consumers are independent and identically distributed (i.i.d.) across consumers, product categories and firms according to a Gumbel distribution, with location parameter equal to zero and scale parameter equal to one.*

[16] Throughout, we use the following convention on notation: lowercase letters and symbols refer to variables at the firm level, uppercase letters and symbols are variables at the product category level, and uppercase ones in boldface refer to aggregate variables.

[17] The synthetic final good is formally defined later on in equation (12). It shares the same CES aggregator as the consumption bundle defined in equation (2).

The second dimension over which consumers are heterogeneous is the set of firms that they are aware of within any given product category at each point in time. Consumer $j$ may only purchase goods from the subset $A_{mjt} \subseteq \{1, \ldots, N\}$ of firms that produce in product category $m$ that the consumer is aware of at time $t$. The evolution of the awareness sets $A_{mjt}$ over time is endogenous, stochastic, and idiosyncratic to each consumer-category pair, as described below.

With these assumptions in place, we define the individual-specific consumption bundle $C_{jt}$ from equation (1) as a constant-elasticity-of-substitution (CES) composite of the consumption of the different products in the consumer's awareness set:

$$C_{jt} = \left[ \int_0^{\boldsymbol{M}_t} \left( \sum_{i \in A_{mjt}} \overline{\Gamma} e^{\sigma \xi_{imj}} c_{imjt} \right)^{\frac{\kappa-1}{\kappa}} dm \right]^{\frac{\kappa}{\kappa-1}}, \tag{2}$$

where $\kappa > 1$ is the elasticity of substitution between different categories, $\sigma \in \left(0, \frac{1}{\kappa-1}\right)$ measures the degree of preference differentiation between firms within a product category, and $\overline{\Gamma} \equiv \Gamma(1 - \sigma(\kappa - 1))^{\frac{1}{1-\kappa}}$ is a normalizing constant, where $\Gamma(\cdot)$ is the Gamma function.[18]

Equation (2) shows that, up to idiosyncratic tastes, products within a category are perfect substitutes. More precisely, the total consumption in category $m$ is the weighted sum of the consumption of each individual firm $i$ that the consumer is aware of at that time. The resulting utility is shifted by the $\xi_{imj}$ idiosyncratic taste, which we refer to as the *quality* of the match between firm and consumer.

### 2.1.2 Firms

Firm $i = 1, \ldots, N$ in product category $m \in [0, \boldsymbol{M}_t]$ produces with technology:

$$y_{imt} = z k_{imt}^{\alpha} l_{imt}^{1-\alpha}, \tag{3}$$

where $\alpha \in (0, 1)$, $k_{imt}$ is capital, $l_{imt}$ is labor, and $z > 0$ is a common and time-invariant productivity component. Firm heterogeneity is driven entirely by the network of connections to consumers —in particular, by the joint distribution of idiosyncratic preferences and awareness sets of those consumers who have that particular firm in their choice sets.

Firms compete strategically, engaging in a repeated static Bertrand pricing game within the product category. In this game, firms simultaneously choose their price as a best response

[18]The Gamma function is defined over positive real numbers by $\Gamma : x \mapsto \int_0^{+\infty} t^{x-1} e^{-t} dt$. The constant $\overline{\Gamma}$ simplifies some algebraic expressions in equilibrium but carries no economic intuition.

to their competitors' prices, which they take as given. Firms also compete strategically in advertising, offering best responses to their competitors' advertising choices, which they take as given. Each firm $i$ makes two advertising choices, $\theta_i$ and $\mu_{0,i}$, whose effects are explained below. These choices are made once-and-for-all at firm birth, but they affect firm-level profits at every future period because they shape the evolution of consumer awareness sets within the product category, as described below.

In both the repeated pricing and the time-zero advertising games, we solve for a Nash equilibrium with symmetric strategies. However, to compute this symmetric equilibrium, we allow for unilateral, off-equilibrium deviations in both advertising and pricing. Unilateral deviations in time-zero advertising generate subsequent asymmetry in awareness, which in turn renders the repeated static Bertrand pricing game also asymmetric.

### 2.1.3 Evolution of awareness sets

Awareness sets $A_{mjt}$ evolve endogenously and stochastically for each consumer-category pair, $(m, j) \in [0, \boldsymbol{M}_t] \times [0, 1]$, as a result of firms' advertising choices. In principle, this would require us to specify a law of motion for each and every consumer and product category in the continuum, a complicated object. However, as we show in Proposition 2, a sufficient statistic to calculate firm profits and prices in the symmetric equilibrium is the distribution of the *count of firms* in consumer awareness sets at each product category age. In anticipation of this result, we lay out assumptions regarding the law of motion of this distribution as the product category ages.

Let $a \in \mathbb{R}_+$ denote the age of a product category. Define the proportion of consumers aware of $n \in \{0, 1, \ldots, N\}$ firms at product category age $a$ as $f_n(a)$, a probability mass function with $f_n(a) \geq 0$ and $\sum_{n=0}^{N} f_n(a) = 1$, for any $a \geq 0$. Let us present these probability mass functions as a row vector $\vec{f}(a) \equiv [f_0(a), f_1(a), \ldots, f_N(a)]$. The evolution of this distribution is assumed to follow a continuous-time Markov process, as stated in our next assumption.

**Assumption 2** (Evolution of awareness). *The law of motion of $\vec{f}(a)$ is:*

$$\partial_a \vec{f}(a) = \vec{f}(a) \cdot \mathcal{Q}(\theta), \tag{4}$$

*given an initial condition $\vec{f}(0) \in [0, 1]^{N+1}$, where the infinitesimal generator matrix $\mathcal{Q}(\theta)$ is the following tridiagonal matrix:*[19]

[19] It is worth pointing that nothing in our theory is tightly connected to this particular generator matrix, which we are taking as a model primitive. Richer versions, as well as limiting cases (e.g., $N \to +\infty$ and $\zeta = 0$, which relates to a pure counting process) could be used.

$$\mathcal{Q}(\theta) \equiv \frac{1}{N} \begin{bmatrix} -N\theta & N\theta & 0 & \dots & 0 & 0 & 0 \\ \zeta & -\zeta-(N-1)\theta & (N-1)\theta & \dots & 0 & 0 & 0 \\ 0 & 2\zeta & -2\zeta-(N-2)\theta & \dots & 0 & 0 & 0 \\ 0 & 0 & 3\zeta & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & (N-1)\zeta & -(N-1)\zeta-\theta & \theta \\ 0 & 0 & 0 & \dots & 0 & N\zeta & -N\zeta \end{bmatrix}. \tag{5}$$

Assumption 2 states the following regarding network formation and destruction.[20] First, we assume that with (exogenous) intensity $\zeta/N \geq 0$, the consumer loses the link with any one firm that she was already aware of. Second, each consumer has an intensity $\theta_i > 0$ of becoming aware of firm $i$ in the product category. As $\theta_i = \theta$ in a symmetric equilibrium, then when the consumer is already aware of $n \leq N$ firms, the intensity with which she becomes aware of a new firm (i.e., of a firm that was not already in her awareness set) is equal to $\frac{N-n}{N}\theta$.

### 2.1.4 Advertising: Contacting and Targeting

The rate $\theta$ of link formation introduced in Assumption 2 is chosen by firms at birth (i.e., at product category age $a = 0$).[21] This choice entails a time-zero cost in units of the synthetic final good, which we introduce below. As this rate affects the speed at which firms contact new random customers, we call this form of advertising "contacting".

Targeting, in contrast, allows firms to affect the quality of their matches directly. As seen above, our assumption on preferences for the *population* of consumers is that they are Gumbel-distributed with location parameter equal to zero (Assumption 1). We further assume that, at any product category age $a \geq 0$, the preference shifters of consumers *who are aware* of a specific firm $i$ are also distributed according to a Gumbel distribution from the firm's perspective, except with a location parameter $\widetilde{\mu}_i(a)$. Targeting allows firms to

[20] This discussion presumes that every firm chooses the same $\theta_i = \theta$, as is the case in the symmetric equilibrium. When we solve for this equilibrium in Section 2.2, however, we consider unilateral deviations, which alter the way in which awareness sets evolve from the perspective of the deviating firm. For this firm, the $\mathcal{Q}(\theta)$ matrix that is used to evaluate payoffs is no longer given by equation (5), and a new generator matrix that takes unilateral deviations into account must be introduced. See Appendix A.2 for the full details.

[21] We assume that firms commit to this choice, and cannot revise their decision after age zero. This assumption, which we make for tractability, does not imply, however, that firms do not internalize the dynamic implications of their advertising choices. To the contrary, in equilibrium, a firm's advertising decisions take into account the effects that these decisions have on future profits through the evolution of product awareness and match quality.

pick the *initial* (age-zero) value of this location parameter, $\widetilde{\mu}_{0,i} \equiv \widetilde{\mu}_i(0) \in [0,1]$, directly affecting the quality of the initial match: a higher choice for $\widetilde{\mu}_{0,i}$ makes it more likely that firm $i$ draws a high-valuation (i.e., high $\xi_{imj}$) consumer at age zero.

This targeting choice is costly and, like the contacting choice, it is made once-and-for-all at birth. Thereafter, targeting evolves according to the law of motion stated in our next assumption. To state this assumption, we need to introduce two more pieces of notation. First, we define the set of awareness sets that contain a certain firm $i = 1, \ldots, N$ by $\mathcal{A}_i \equiv \{A \in \mathbb{A} | i \in A\}$, where $\mathbb{A} \equiv \mathbf{2}^{\{1,\ldots,N\}}$ is the power-set of firm indices.[22] Second, we denote by $\widehat{f}(a, A)$ the marginal density of awareness set $A \in \mathbb{A}$ at age $a$, with the property $\sum_{A \in \mathbb{A}} \widehat{f}(a, A) = 1$, $\forall a > 0$.

**Assumption 3** (Evolution of targeting)**.** *At product category age $a$, the location parameter for the Gumbel distribution from which firm $i$ draws new consumers equals:*

$$\widetilde{\mu}_i(a) = \widetilde{\mu}_{0,i}\big(1 - s_i(a)\big), \tag{6}$$

*where $s_i(a)$ denotes the degree of* market saturation *for firm $i$ at age $a$, defined by:*

$$s_i(a) \equiv \sum_{A \in \mathcal{A}_i} \widehat{f}(a, A), \tag{7}$$

Assumption 3 states that the degree of targeting drops in proportion to the rate at which the firm's network saturates, where by saturation we mean the proportion of awareness sets that contain the firm.[23] Intuitively, for a given choice $\widetilde{\mu}_{0,i}$, when the firm is contained in the awareness set of a large share of consumers, the likelihood with which it draws consumers with high preference for its product relative to the population mean is lower. In the limit at which every consumer is aware of every firm ($s_i(a) \to 1$), it is no longer possible to have any systematic selection, and the distribution of preferences matches the unconditional distribution (i.e., $\widetilde{\mu}_i(a) \to 0$).[24]

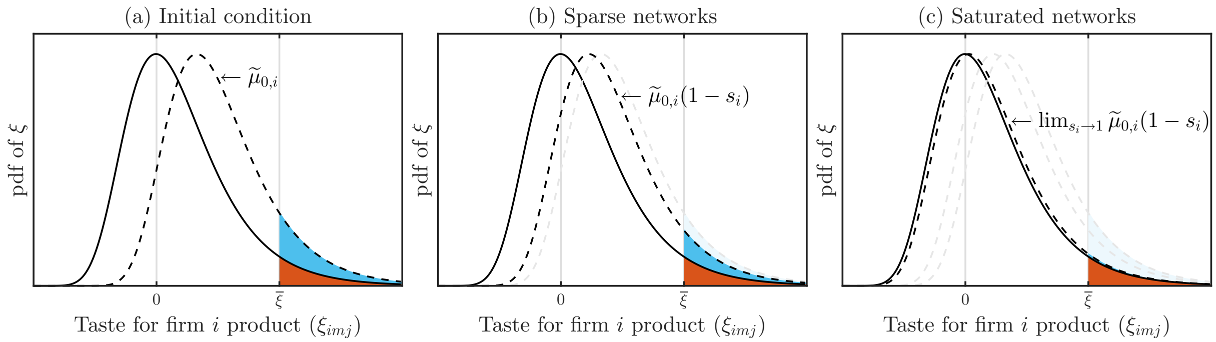
Figure 1 provides an illustration of how targeting works in this economy by plotting the

[22]For example, when $N = 3$, then $\mathbb{A} = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$. So, for instance, $A_1 = \{\{1\}, \{1,2\}, \{1,3\}, \{1,2,3\}\}$. The cardinality of the power-set is $|\mathbb{A}| = \sum_{k=0}^{N} \binom{N}{k} = 2^N$.

[23]Note that this assumption implicitly defines some primitives regarding the meeting probabilities of particular consumers with the firm (a higher $\widetilde{\mu}_{0,i}$ assigns more probability to meeting consumers with higher $\xi_{imj}$). This modeling choice allows the derivation of some closed-form solutions, as is common in the discrete-choice literature.

[24]It should be pointed out that, even though targeting changes the distribution of firm draws at a certain point in time, it respects the underlying Gumbel distribution in the full population for the range $\widetilde{\mu}_{0,i} \in [0,1]$. To be specific, $sF(x, \widetilde{\mu}_{0,i}(1-s)) \leq F(x, 0), \forall x$ and $\forall s \in [0,1]$ where $F(x, \widetilde{\mu}) = \exp(-(x - \widetilde{\mu}) - \exp(-(x - \widetilde{\mu})))$ is the probability density function.

Figure 1: Targeting technology for different network saturation levels.




**Notes:** This figure plots the distribution of tastes in the population (solid curves, centered at zero) and the distribution from which firms take draws (dashed curves, centered at $\widetilde{\mu} > 0$), as firm-consumer networks become saturated. Panel (a) shows the situation at entry. Panel (b) shows an intermediate case in which networks are still relatively sparse, $0 < s_i(a) < 1$: the distribution from which the firm takes its draws is still distorted relative to the distribution in the population of consumers, but not as much as in panel (a). Panel (c) shows the limiting case in which all consumers are aware of all firms.

distribution of idiosyncratic tastes in the population of consumers (solid lines), which are centered at zero, and the distributions from which the firms draw their matches (dashed lines). We plot these two distributions for three different configurations: at firm birth (panel (a)), for an incompletely saturated network (panel (b)), and for a fully saturated network, $s(a) \to 1$ (panel (c)). Initially, the sampling distribution is shifted relative to the population distribution by $\widetilde{\mu}_{0,i}$, allowing the firm to sample consumers with tastes $\xi_{imj} > \overline{\xi}$ with a higher probability (the blue area) than they would otherwise (the orange area). When the firm is contained in a larger share of awareness sets, however, this likelihood shrinks (the blue area becomes smaller, as seen in panel (b)). In the limit at which the firm's network is fully saturated, targeting is ineffective.

As stated, both contact and targeting advertising choices entail output costs for the firm. The advertising choices are paid upfront (at the inception of the product category) in units of the synthetic final good. For the remainder of the paper, we use the change of variables $\mu_{0,i} \equiv \exp(\widetilde{\mu}_{0,i})$.[25] With this notation, the advertising cost function at the firm level is:

$$d(\theta_i, \mu_{0,i}) = \nu\theta_i^2 + \eta(\mu_{0,i} - 1)^2 \tag{8}$$

where $\nu > 0$ and $\eta > 0$ are cost scale parameters which are common to all firms.

[25] This re-centers the Gumbel draws for idiosyncratic tastes, which is convenient for the derivation of the equilibrium conditions.

### 2.1.5 Investment in capital and product category creation

On top of being used for consumption and advertising, the synthetic final good is also used for investment technologies that increase the measure of product categories, $\boldsymbol{M}_t$, and the stock of physical capital, $\boldsymbol{K}_t$.[26]

The technology to create new categories generates a Poisson arrival rate $z_M \boldsymbol{I}_t^M$ of new products categories, and its cost in terms of the synthetic final good is quadratic, equal to $(\boldsymbol{I}_t^M)^2$, and paid upfront. When a new product category is created, the owner of the blueprint sells, at fair market value, perpetual licenses to the $N$ entering firms for using the newly created production technology. After its creation, there is no further entry or exit of firms over the lifetime of a product category.

The investment technology for physical capital transforms synthetic final good into physical capital one for one. Physical capital depreciates at an instantaneous rate $\delta_K > 0$, and product categories become obsolete at an exogenous rate $\delta_M > 0$. Therefore:

$$\partial_t \boldsymbol{K}_t = \boldsymbol{I}_t^K - \delta_K \boldsymbol{K}_t, \tag{9}$$

$$\partial_t \boldsymbol{M}_t = z_M \boldsymbol{I}_t^M - \delta_M \boldsymbol{M}_t, \tag{10}$$

where $\boldsymbol{I}_t^K$ and $\boldsymbol{I}_t^M$ denote the respective investments, in units of the synthetic final good.

The synthetic final good, denoted $\boldsymbol{Y}_t$, is used for consumption, advertising, and investment in physical capital and product category creation. The resource constraint is:

$$\boldsymbol{Y}_t = \boldsymbol{C}_t + \boldsymbol{I}_t^K + \left(\boldsymbol{I}_t^M\right)^2 + \boldsymbol{D}_t, \tag{11}$$

where $\boldsymbol{D}_t$ denotes aggregate advertising expenditures.[27]

[26]This implies investors are subject to the same product awareness frictions as the consumers. Alternatively, we could assume that investment directly uses labor and/or capital. This would not fundamentally alter the main mechanisms in our model.

[27]Just as the consumption goods are a synthetic composite aggregating demand of individual products for each consumer (equation (2)), we conjecture that the final good, with which investment decisions are made, has a comparable aggregation. This conjecture allows us to pose the aggregate resource constraint of the economy in equation (11). In the equilibrium section, we solve for demand using a synthetic final good (equation (12)), which is different for every consumer, and show that, in the symmetric Markov Perfect equilibrium, all consumers have the same real income. Using this result, we then show that these individual outputs aggregate up to a single final good, confirming our conjecture that the equilibrium admits a single aggregate resource constraint, as written in equation (11).

## 2.2 Equilibrium

In this section, we solve for the Markov Perfect Equilibrium of the economy. We begin by characterizing consumer choices (Section 2.2.1) and firm choices (Section 2.2.2). For the latter, we restrict attention to a symmetric equilibrium in which firms choose best responses taking their competitors actions as given. When computing these choices, and before imposing symmetry, we allow for unilateral off-equilibrium deviations in both pricing and advertising strategies. Having solved for all firm choices, we then show (in Section 2.2.3) that the economy aggregates to a representative-agent Neoclassical growth model with endogenous TFP and an endogenous number of product categories. Exploiting this finding allows us to compute firm value, derive the time-zero optimal advertising choices of the firms, and clear all markets (Section 2.2.4).

### 2.2.1 Consumer Problem

By equation (11), the synthetic final good is used for consumption, advertising, and investment in physical capital and product category creation. In the stationary equilibrium, a constant share of final output is devoted to these various investments. In anticipation of this result, we solve the individual's intra-temporal allocation problem across categories, and of products within a category, as a static problem on quantities purchased, $y_{imjt}$, rather than consumed, $c_{imjt}$.[28]

Mirroring equation (2), define the synthetic final good for consumer $j$ at time $t$ as:

$$Y_{jt} \equiv \left[ \int_0^{\boldsymbol{M}_t} \left( \sum_{i \in A_{mjt}} \overline{\Gamma} e^{\sigma \xi_{imj}} y_{imjt} \right)^{\frac{\kappa-1}{\kappa}} dm \right]^{\frac{\kappa}{\kappa-1}}. \tag{12}$$

Moreover, define $\Omega_{jt}$ as the *real* income available to consumer $j$ at time $t$, and $P_{jt}$ as the price index of her bundle of purchases.[29] Taking her awareness sets $\left(A_{mjt} : m \in [0, \boldsymbol{M}_t]\right)$ and her *nominal* income $P_{jt}\Omega_{jt}$ as given, the objective of consumer $j \in [0, 1]$ is to choose purchases $y_{imjt}$ for each $i \in A_{mjt}$ and all $m \in [0, \boldsymbol{M}_t]$ to maximize purchased quantity $Y_{jt}$

[28]The inter-temporal allocation of resources to consumption is relegated to Section 2.2.3, after we show that the economy aggregates to a representative-household model.

[29]Both $\Omega_{jt}$ and $P_{jt}$ are solved in general equilibrium later (Proposition 5 and Appendix A.5), but for now it suffices to express them generically as state variables in the consumer's problem.

subject to the budget constraint:

$$\int_0^{\boldsymbol{M}_t} \sum_{i \in A_{mjt}} \widehat{p}_{imt} y_{imjt} dm \leq P_{jt} \Omega_{jt}, \tag{13}$$

where $\widehat{p}_{imt}$ is the (nominal) price of product $(i, m)$ at time $t$. The following proposition describes the solution to this static resource allocation problem:

**Proposition 1** (Product demand)**.** *Given awareness sets* $(A_{mjt} : m \in [0, \boldsymbol{M}_t])$*, real income* $\Omega_{jt}$*, and nominal prices* $(\{\widehat{p}_{imt}\}_{i \in A_{mjt}} : m \in [0, \boldsymbol{M}_t])$*:*

*1.* (Extensive demand) *In product category* $m \in [0, \boldsymbol{M}_t]$*, consumer* $j$ *purchases from firm* $i$ *and from no other firm in her awareness set if, and only if,*

$$\ln \left( \frac{\widehat{p}_{i'mt}}{\widehat{p}_{imt}} \right) > \sigma \left( \xi_{i'mj} - \xi_{imj} \right), \quad \forall i' \in A_{mjt} \backslash \{i\}. \tag{14}$$

*2.* (Intensive demand) *Suppose* $i \in A_{mjt}$ *satisfies condition (14). Then, consumer* $j$*'s demand for firm* $i$ *is:*

$$y^d_{imjt} = \overline{\Gamma}^{\kappa-1} e^{\sigma(\kappa-1)\xi_{imj}} p_{imjt}^{-\kappa} \Omega_{jt}, \tag{15}$$

*where* $p_{imjt} \equiv \widehat{p}_{imt} / P_{jt}$ *denotes the real price, and*

$$P_{jt} = \overline{\Gamma}^{-1} \left( \int_{\mathcal{M}_{jt}} \left( e^{-\sigma \xi_{i(m)mj}} \widehat{p}_{i(m)mt} \right)^{1-\kappa} dm \right)^{\frac{1}{1-\kappa}} \tag{16}$$

*is the price index, where* $\mathcal{M}_{jt} \equiv \{m : A_{mjt} \neq \emptyset\} \subseteq [0, \boldsymbol{M}_t]$ *is the subset of product categories for which consumer* $j$ *is aware of at least one firm at time* $t$ *and, for every* $m \in \mathcal{M}_{jt}$*,* $i(m)$ *denotes the firm in* $A_{mjt}$ *that satisfies equation (14).*

*Proof.* See Appendix A.1.

Proposition 1 states that, for each product category $m$, the consumer demands from only one firm, almost surely, out of the $|A_{mjt}|$ firms in her awareness set at time $t$. Particularly, within each $m$, the consumer demands the product $i(m)$ that satisfies condition (14), i.e., the firm that offers the best price-to-valuation comparison among all firms known to the consumer.[30] Equation (15) then provides the intensive demand for each such product,

[30] Intuitively, this is because firms are perfect substitutes in the eyes of the consumer, up to idiosyncratic tastes $\xi_{i(m)mj}$ (recall equation (2)).

showing that the consumer's quantity demanded is increasing in preferences $\xi_{i(m)mj}$ and decreasing in the real price $p_{i(m)mjt}$.

### 2.2.2 Firm Problem

**Preliminaries** Each firm $i = 1, \ldots, N$ makes various choices. Upon entry, a firm must make an advertising choice for $\theta_i$ and $\mu_{0,i}$. Subsequently, at every age $a > 0$, the firm must choose prices $p_i(a)$ and input quantities $k_i(a)$ and $l_i(a)$. Advertising and pricing choices are strategic, i.e., the firm takes the actions of the other $N-1$ firms as given and offers a best response to them. As mentioned before, we focus on a symmetric equilibrium in which all firms choose the same $\theta$, $\mu_0$ and $p(a)$ at each age $a > 0$. However, when solving for the firm's optimal choices in the Markov Perfect Equilibrium, we allow for unilateral, off-equilibrium deviations from the symmetric advertising and pricing strategies.

Considering off-equilibrium, unilateral deviations from the symmetric age-zero advertising strategy implies that the subsequent evolution of awareness sets as the industry matures becomes asymmetric from the point of view of the deviating firm. Given this asymmetry in awareness, the static Bertrand-Nash Equilibrium (BNE) of the repeated pricing game at each stage is also asymmetric, which the firms considering unilateral deviations must therefore take into account when evaluating future profits and choosing their own optimal time-zero advertising strategies.

To characterize the solution to this complicated problem, we proceed as follows. We start by solving for the demand schedule that is faced by a particular firm at a given point in time (Proposition 2). Then, we present the solution to the pricing problem in the equilibrium that is obtained from imposing symmetry in the optimality conditions that allow for unilateral, off-equilibrium deviations in both advertising and pricing (Proposition 3). Given these, we then derive the firm's optimal choice of labor and capital inputs (Proposition 4). These, in turn, allow us to compute equilibrium profits and firm value at time zero, which the firm evaluates upon entry to make her initial advertising choices (Section 2.2.4).

**Firm demand** To derive demand, we maintain the conjecture (which we verify in Proposition 5) that both the price index and real income are identical across consumers: $P_{jt} = \boldsymbol{P}_t$ and $\Omega_{jt} = \boldsymbol{\Omega}_t$, $\forall j \in [0, 1]$. Under this conjecture, every consumer pays the same real price, i.e., $p_{imjt} = p_{imt}$. Henceforth, we drop the subscript $m$ for brevity and identify the state of the product category by its age, $a$.

Firm $i = 1, \ldots, N$ takes as given aggregate income $\boldsymbol{\Omega}_t$ and the joint distribution of awareness sets and idiosyncratic preferences across those consumers whose awareness sets include the firm. Further, let $\vec{p}(a) \equiv [p_1(a), \ldots, p_N(a)]^\top \in \mathbb{R}^N$ denote the vector of all prices in the product category, of which prices $\{p_{i'}(a)\}_{i' \neq i}$ are taken as given, and price $p \equiv p_i(a)$ is chosen by the firm. Similarly, the firm takes as given the competitors' degree of targeting, $\{\mu_{i'}(a)\}_{i' \neq i}$.

By Proposition 1, the firm faces demand from the subset of consumers whose awareness sets contain its product and who, additionally, choose the firm's product over the other products in the awareness set, if any. In principle, this requires that the firm keeps track of the joint distribution of awareness sets and idiosyncratic preferences across those consumers whose awareness sets contain the firm at each age $a$, a potentially complicated object. However, as we show in Appendix A.2, exploiting Bayes' rule and the fact that preference shifters are independently and identically Gumbel-distributed, the demand schedule of an individual firm can be simplified to:

$$y_{it}(a, \vec{p}) = \big(\mu_i(a)\big)^{\sigma(\kappa-1)} p^{-\kappa} \mathbf{\Omega}_t q_i(a, \vec{p}), \tag{17}$$

with

$$q_i(a, \vec{p}) \equiv \sum_{A \in \mathcal{A}_i} \widehat{f}(a, A) \left(1 + \sum_{i' \in A \setminus \{i\}} \frac{\mu_{i'}(a)}{\mu_i(a)} \left(\frac{p_{i'}(a)}{p}\right)^{-\frac{1}{\sigma}}\right)^{\sigma(\kappa-1)-1}, \tag{18}$$

We provide an economic interpretation of this demand after we state our next proposition. For now, we note that evaluating equation (17) requires that the firm computes $\widehat{f}(a, A)$, the marginal density (as of age $a$) of awareness sets, over all sets that contain the firm, $A \in \mathcal{A}_i$. However, as we explain in detail in Appendix A.2, this density can be computed using the logic of a combinatorial urn-ball problem without replacement. Instead of summing densities across sets, as stated in equation (18), we can instead sum probabilities of drawing sets of different *sizes* that contain the firm. Precisely, the probability of drawing a specific set of size $n \equiv |A|$ is $f_n(a)/\binom{N}{n}$, i.e., the product of the probability of drawing any set of $n$ (out of $N$) firms without replacement (ignoring order), equal to $1/\binom{N}{n}$, times the share of awareness sets of size $n$, defined as $f_n(a)$ in Assumption 3. Thus, since the number of size-$n$ subsets that contain a specific firm equals $\binom{N-1}{n-1}$, the aggregate probability that a consumer's awareness set has size $n$ and contains the firm equals $\widehat{f}(a, A) = f_n(a)\binom{N-1}{n-1}/\binom{N}{n}$, or:

$$\widehat{f}(a, A) = \frac{n}{N} f_n(a). \tag{19}$$

Our next proposition states the expression for demand that results from imposing equation (19) into equation (18). For this, we must introduce one more piece of notation.

We define the expectation at age $a$ of any real-valued function $g$ defined over support $\widehat{n} = 1, \ldots, N$ (excluding zero, i.e., $\widehat{n} \equiv n | n \geq 1$) as follows:

$$\mathbb{E}_a \left[ g(\widehat{n}) \right] \equiv \sum_{n=1}^{N} h_n(a) g(n), \tag{20}$$

where $h_n(a) \equiv \frac{f_n(a)}{1 - f_0(a)}$ is a re-normalized probability mass function that conditions on $n \geq 1$, and thus satisfies $\sum_{n=1}^{N} h_n(a) = 1$. We are now ready to state our main result.

**Proposition 2** (Firm demand with symmetric strategies)**.** *Given real income $\mathbf{\Omega}_t$, in an equilibrium with symmetry in $\theta$, i.e., $\theta_i = \theta$ for all $i = 1, \ldots, N$, and symmetry in competitor prices and targeting, i.e., $p_{i'}(a) = p_{-i}$ and $\mu_{i'}(a) = \mu_{-i}$ for all $i' \neq i$, firm demand is given by*

$$y_i(a, \vec{p}) = \underbrace{\left(1 - f_0(a)\right)}_{\text{Awareness}} \underbrace{\left(\mu_i(a)\right)^{\sigma(\kappa - 1)}}_{\text{Targeting}} \underbrace{p_i^{-\kappa} \frac{\mathbf{\Omega}_t}{N}}_{\substack{\text{Downward-}\\ \text{sloping demand}}} \underbrace{q_i(a, \vec{p})}_{\text{Sorting}}, \tag{21}$$

*where*

$$q_i(a, \vec{p}) \equiv \mathbb{E}_a \left[ \widehat{n} \left( 1 + (\widehat{n} - 1) \frac{\mu_{-i}}{\mu_i} \left( \frac{p_{-i}}{p_i} \right)^{-\frac{1}{\sigma}} \right)^{\sigma(\kappa - 1) - 1} \right], \tag{22}$$

*and $\widehat{n} \equiv n | n \geq 1$.*

*Proof.* See Appendix A.2.

Proposition 2 shows that, in an equilibrium with symmetry in competitor strategies, demand is composed of four multiplicative terms.

First, demand is increasing in $\left(1 - f_0(a)\right)$, the proportion of consumers that are aware of at least one firm and, thus, a measure of overall awareness. Intuitively, a single firm's demand is higher when more consumers are aware of the product category as a whole.

Second, a firm's demand is increasing in its degree of targeting, $\mu_i(a)$: as the distribution from which firms samples new consumers is right-shifted relative to the population's distribution (Figure 1), higher targeting implies higher willingness to pay at given prices. Precisely, recall that at age $a$, $\mu_i(a) = \mu_{0,i}^{1 - s_i(a)}$, where $\mu_{0,i}$ is the age-zero targeting choice of the firm and $s_i(a) = \sum_{A \in \mathcal{A}_i} \widehat{f}(a, A)$ is the firm's network saturation as of age $a$. Under symmetry, all firms face the same market saturation which, using equation (19), can be written as:

$$s(a) = \frac{1}{N} \sum_{n=1}^{N} n f_n(a) = (1 - f_0(a)) \mathbb{E}_a \left[ \frac{\widehat{n}}{N} \right]. \tag{23}$$

Thus, equation (21) shows that, by choosing a higher degree of targeting investment initially, and for as long as not all consumers are aware of all firms (i.e., $f_N(a) < 1$, so $s(a) < 1$), a firm is able to target consumers with (on average) higher preference for the product, and thereby increase intensive demand at given prices. The impact of targeting on firm demand is controlled by the elasticity $\sigma(\kappa - 1)$: demand is more responsive to targeting if products are more substitutable and/or if idiosyncratic preferences are more dispersed (for in this case there is a larger scope to match with high-valuation consumers).

Third, demand is shifted by $p_i^{-\kappa}\mathbf{\Omega}_t/N$, a standard component in Dixit-Stiglitz demand systems: the incumbent customers of the firm increase the quantity demanded of the firm's product when their available income per product $\mathbf{\Omega}_t/N$ is higher and/or when the price is lower. Along this margin, the price-elasticity of demand coincides with the elasticity of substitution between product categories, $\kappa$, as in standard Dixit-Stiglitz demand systems.

Finally, demand is increasing in $q_i(a, \vec{p})$, a term which we label "sorting".[31] This term, central to our analysis, summarizes demand along the *extensive margin*. It captures changes in the set of consumers that, other things equal, choose to sort into the firm out of all of the alternatives that are available to them. Intuitively, because consumers purchase from only one firm in their awareness set —the one providing the best price-to-valuation comparison, as shown in Proposition 1—, then if a firm were to undercut its price, it would induce some consumers to switch away from their suppliers and toward the firm in question, whose customer base would expand as a result. This can be seen directly in equation (22): a firm's demand increases via $q(a)$ when the firm undercuts its price relative to its competitors, $p_i < p_{-i}$. A higher relative degree of targeting ($\mu_i > \mu_{-i}$) triggers a similar effect.

Importantly, these effects on extensive demand are a function of how many firms consumers are aware of, $\widehat{n}$. When a consumer knows of no alternatives to its current seller ($\widehat{n} = 1$), she is trivially insensitive to price changes from firms other than her own. In contrast, when consumers have a lot of firms to choose from, firms can create larger swings in extensive demand by inducing consumers to sort into them via undercutting prices or marginally increasing their targeting.

In an equilibrium with full symmetry, which is our focus, relative prices and relative targeting becomes irrelevant for sorting. Setting $p_i/p_{-i} = \mu_i/\mu_{-i} = 1$ in equation (22) yields:

[31]There is a slight abuse of notation here. As we have re-scaled the $f_n$ distribution to consider only non-empty awareness sets and be able to use the $\mathbb{E}_a$ operator, the $q_i$ defined in equation (22) is also a re-scaled version of the one defined in equation (18), by a factor of proportionality $(1 - f_0(a))/N$. This is inconsequential for our analysis: as we will see, the key object of interest is the elasticity of sorting, but this elasticity is invariant to $(1 - f_0(a))/N$.

$$q(a) = \mathbb{E}_a\left[\widehat{n}^{\sigma(\kappa-1)}\right]. \tag{24}$$

In this case, sorting effects on demand are still present, but only through the size of awareness sets, $\widehat{n}$. Intuitively, when awareness sets are on average larger, firms face higher demand because consumers, who have a wider range of alternatives to choose from, can sort towards products that yield a better match to their preferences. For the firm, this means on average more customers at given prices and targeting.

In equilibrium, firms exploit these properties of demand to wield market power over their customers, a question to which we turn next.

**Demand elasticity and markups** To characterize firm's optimal pricing policies, it is first useful to note, that because the firm's technology is Cobb-Douglas, a firm's marginal cost is constant in the level of output and only a function of real input prices $\left(w_t, R_t^K\right)$, which the firms take as given. A firm's input choice problem reads:

$$\mathbf{TC}_t(y) \equiv \min_{k,l}\left\{(r_t + \delta_K)k + w_t l\,,\ \text{ such that } y = zk^{\alpha}l^{1-\alpha}\right\}, \tag{25}$$

where $r_t = R_t^K - \delta_K$ is the interest rate in the economy. As we show in Appendix A.4, the marginal cost is common across all firms and product categories, and given by

$$\mathbf{mc}_t \equiv \partial_y \mathbf{TC}_t(y) = \frac{1}{z}\left(\frac{r_t + \delta_K}{\alpha}\right)^{\alpha}\left(\frac{w_t}{1-\alpha}\right)^{1-\alpha}. \tag{26}$$

Firms in a product category compete to maximize profits by playing a repeated Bertrand game, choosing a price policy taking as given the pricing decisions of the other firms. As the evolution of awareness sets is not directly affected by pricing decisions, only indirectly through general-equilibrium effects, there are no dynamic incentives in this pricing game, and we solve for the case of a repeated static Nash equilibrium in pure strategies. Recall, however, that we allow for unilateral, off-equilibrium deviations in initial advertising choices $\theta$ and $\mu_0$, which alter the perceived evolution of awareness sets from the point of view of the deviating firm and, in turn, may trigger subsequent unilateral deviations in the pricing strategies of this, as well as other, firms. These deviations impact the symmetric pricing solution because they affect how a firm that unilaterally deviates from the $(\theta, \mu_0)$ symmetric strategy evaluates subsequent period profits. In Appendix A.3 we fully characterize the solution of the Bertrand-Nash equilibrium with deviations, while in the main text we restrict attention to the implications that follow from imposing symmetry on the optimality conditions that arise from the pricing problem with deviations.[32]

Taking aggregate income $\mathbf{\Omega}_t$, the marginal cost $\mathbf{mc}_t$, and competitor prices $\{p_{i'}\}_{i' \neq i}$ as given, in the pure-strategy Bertrand Nash-equilibrium at age $a$, firm $i$ chooses

$$p_i(a) = \arg\max_p \left\{ \Big(p - \mathbf{mc}_t\Big) y_i(a, \vec{p}) \right\}, \tag{27}$$

where $y_i(a, \vec{p})$ is given by equation (17). The solution to this problem is characterized in the following proposition.

**Proposition 3** (Equilibrium markups). *Taking $\mathbf{\Omega}_t$, $\mathbf{mc}_t$, and $\{p_{i'}\}_{i' \neq i}$ as given, firm $i$ sets a price $p_{it}(a) = \Lambda_i(a, \vec{p})\mathbf{mc}_t$. The markup is equal to:*

$$\Lambda_i(a, \vec{p}) = \frac{\mathcal{E}_i(a, \vec{p})}{\mathcal{E}_i(a, \vec{p}) - 1} \tag{28}$$

*where $\mathcal{E}_i(a, \vec{p})$ is the price-elasticity demand for firm $i$ at age $a$, equal to:*

$$\mathcal{E}_i(a, \vec{p}) = \underbrace{\kappa}_{\substack{\text{Intensive-margin} \\ \text{price elasticity}}} + \underbrace{\left( -\frac{\partial q_i(a, \vec{p})}{\partial p} \frac{p}{q_i(a, \vec{p})} \right)}_{\substack{\text{Extensive-margin} \\ \text{price elasticity}}} \tag{29}$$

*In an equilibrium with symmetric pricing and targeting, the extensive-margin elasticity equals*

$$\frac{\partial q_i(a, \vec{p})}{\partial p} \frac{p}{q_i(a, \vec{p})} = \frac{\sigma(\kappa - 1) - 1}{\sigma} \left[ 1 - \frac{\mathbb{E}_a \left[ \widehat{n}^{\sigma(\kappa-1)-1} \right]}{\mathbb{E}_a \left[ \widehat{n}^{\sigma(\kappa-1)} \right]} \right] < 0, \tag{30}$$

*so the markup is constant across firms within the product category, $\Lambda_i(a, \vec{p}) = \Lambda(a)$.*

*Proof.* See Appendix A.3.

Proposition 3 states that each firm sets a price markup over the marginal cost, which is equal to a decreasing function of the firm's price elasticity of demand, denoted by $\mathcal{E}_i(a, \vec{p})$. While this result is standard, the forces that make demand elasticities endogenous are new.[33]

[32] In our equilibrium with symmetric pricing strategies, there is price dispersion between product categories: product markets with denser consumer networks have higher levels of competition and lower markups, similar to Burdett and Judd (1983). However, in our theory consumers do not search, so they cannot engage in price experimentation: in equilibrium, consumers only purchase from their most preferred firm within their awareness set (Proposition 1). This rules out such strategic pricing considerations within product categories that can result in price dispersion between firms.

[33] The negative relationship between the price-elasticity of demand and markups is present in most models of variable markups, which obtain this relationship from, e.g., oligopolistic competition à la Atkeson and

Particularly, equation (30) shows that the price-elasticity of demand has two additively separable components: (i) an *intensive-margin* elasticity, coming from changes in intensive demand from captive customers of the firm, and equal to the elasticity of substitution across categories $\kappa$; and (ii) and *extensive-margin* elasticity, equal to the price-elasticity of $q_i(a, \vec{p})$, the sorting component of demand that we defined in Proposition 2. While the intensive-margin component is familiar from models using CES demand systems, the extensive-margin component is new to our paper.

Following our intuition in the previous section, the extensive-margin elasticity $\frac{\partial q_i(a,\vec{p})}{\partial p} \frac{p}{q_i(a,\vec{p})}$ governs how sensitive consumers are to price changes when choosing which firm in the awareness set to sort into.[34] In a symmetric equilibrium, all firms set the same price and the consumer chooses the product that yields the highest price-to-valuation ratio (Proposition 1). If a firm were to marginally increase its price from this symmetric strategy, it would lose those customers that can find cheaper taste-adjusted prices elsewhere in their awareness set (equation (14)). Thus, a firm can set a higher price markup when its customers are not aware of too many alternatives, for this is when the sensitivity of consumer switching due to marginal price changes is lowest.

This sensitivity is time-varying as networks evolve. To see this, Figure 2 plots the life-cycle of the density of awareness sets (panel (a)), the sorting component of demand (panel (b)), the extensive-margin elasticity of demand (panel (c)), and the markup (panel (d)), as a function of age. In this parameterization, the same that we use to calibrate the model in Section 3.1, no consumer knows of any firm at the product category's inception (i.e., $f_0(0) = 1$). Therefore, shortly after the entry of a new product category, awareness sets are still sparse: consumers know at most one firm (panel (a)), so the sorting component of demand is inactive (i.e., $q(a) = 1$, panel (b)) because the customers of any given firm know of no other firm that they could buy from. Effectively, firms act as monopolists: their demand functions are perfectly inelastic along the extensive margin, i.e., $\frac{\partial q}{\partial p} \frac{p}{q} \approx 0$ (see panel (c)), and they set the unconstrained monopolist markup, $\Lambda = \frac{\kappa}{\kappa - 1}$ (panel (d)).[35] In this calibration, networks expand with time, which intensifies competition between firms as the product category matures.[36] Over time, consumers learn about new firms (panel

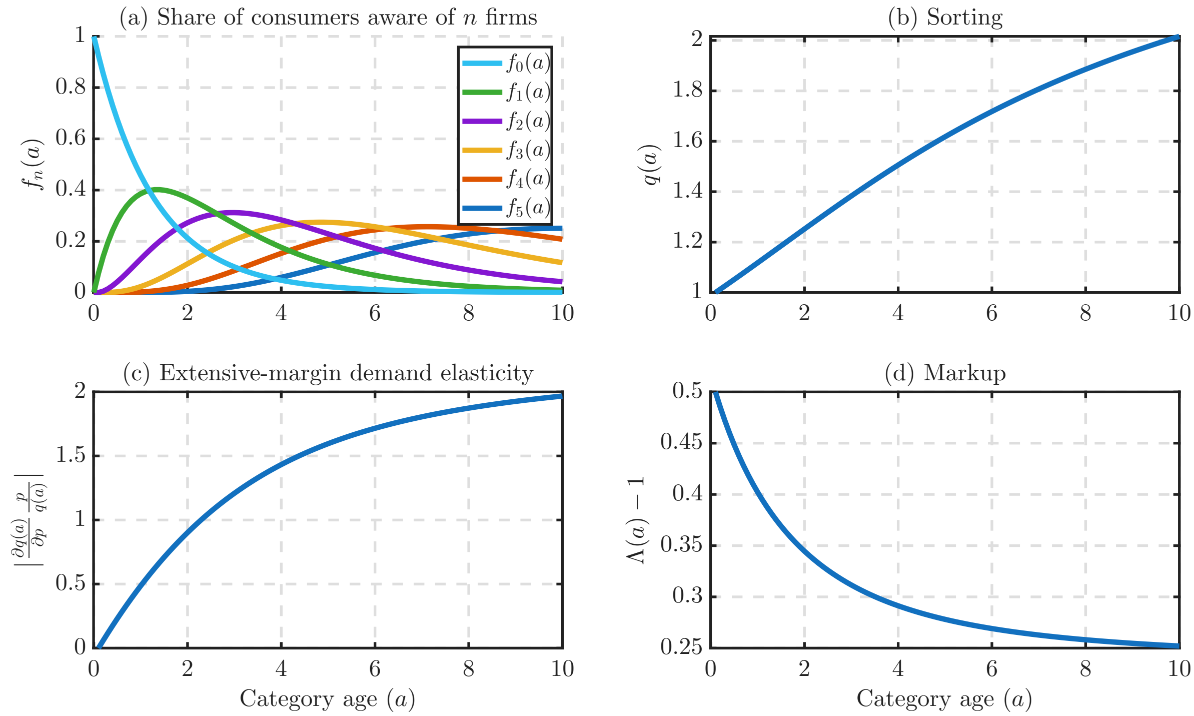
Burstein (2008), or non-CES (e.g., Kimball) preference aggregators, as in Boar and Midrigan (2025).

[34] This result is reminiscent of Paciello *et al.* (2019), where demand is also imperfectly elastic along the extensive margin. In that model, this occurs because consumers can search for alternative suppliers in response to price changes.

[35] In this example, $\kappa = 3$, so the unconstrained monopoly markup is 50%.

[36] Importantly, the relationship between network formation and age is not a theoretical prediction of the model. Our assumption is that markups are higher in categories where networks are more saturated, but these categories need not be the older ones. To see this, Appendix Figure D.1 shows an example in which the initial distribution of awareness is uniform, $f_n(0) = \frac{1}{N+1}$ for all $n = 0, 1, \ldots, N$. For this case, markups may

Figure 2: Life-cycle dynamics of a product category.




**Notes:** This figure plots the life-cycle of awareness counts (panel (a)), the sorting component of demand $q(a)$ (panel (b)), the extensive-margin elasticity of demand (panel (c)) and the markup (panel (d)), in the symmetric equilibrium. This figure uses the calibrated parameter values from the "early calibration", discussed in Section 3.1.

(a)) and the set of alternatives available to them expands (panel (b)), making their sorting behavior more sensitive (panel (c)). This, in turn, reduces firms' market power (panel (d)).

How do advertising choices alter these dynamics? On the one hand, faster contacting (higher $\theta$) accelerates the rate at which firms meet new consumers at random.[37] As the average consumer has more alternatives to choose from, demand is more elastic and firms set lower markups at each point in time when $\theta$ is higher (ceteris paribus).[38] On the other hand, stronger targeting does not directly affect sorting $q$ (nor, therefore, markups) in a symmetric equilibrium, because it shifts demand only via the intensive margin. However, targeting alters the returns to contacting, changing the equilibrium dynamics. We relegate this discussion to the section presenting our quantitative results (specifically, in Section

decrease, be flat or increase in age, depending on the rate of link destruction ($\zeta$). Empirically, there is no systematic evidence on the behavior of markups in firm age, though some studies find no dynamics in specific settings (see, e.g., Fitzgerald *et al.* (2024) and Argente *et al.* (2025)).

[37]Recall that $\theta$ determines the evolution of $f_n(a)$ directly: a higher $\theta$ makes $f_n(a)$ decline (rise) faster with age for lower (higher) $n$'s, making networks saturate faster.

[38]An illustration of this direct effect of contacting on market dynamics is provided in the context of our counterfactual experiments (see, e.g., Figure 6).

3.2.2).

**Input choice** Having described the pricing problem, we can now solve for the optimal labor and capital input choices of the firm.

**Proposition 4** (Firm's input demands)**.** *Given real input prices* $(w_t, r_t)$*, real income* $\mathbf{\Omega}_t$ *and marginal cost* $\mathbf{mc}_t$*, the firm's demand for labor and capital inputs is given by:*

$$l(a) = \frac{1-\alpha}{w_t}\mathbf{mc}_t y(a), \tag{31a}$$

$$k(a) = \frac{\alpha}{r_t + \delta_K}\mathbf{mc}_t y(a), \tag{31b}$$

*In a symmetric equilibrium, firm output* $y(a)$ *is equal to:*

$$y(a) = (1 - f_0(a))\mu(a)^{\sigma(\kappa-1)}\mathbf{mc}_t^{-\kappa}\Lambda(a)^{-\kappa}q(a)\frac{\mathbf{\Omega}_t}{N}. \tag{32}$$

*Proof.* See Appendix A.4.

An implication of this proposition is that the labor and capital shares of firm sales are only a function of the markup, $w_t\frac{l(a)}{p(a)y(a)} = (1-\alpha)\Lambda(a)^{-1}$ and $(r_t + \delta_K)\frac{k(a)}{p(a)y(a)} = \alpha\Lambda(a)^{-1}$. Using equation (32) and $p(a) = \Lambda(a)\mathbf{mc}_t$, we can write the period profits of the firm as a function of the marginal cost, the markup, and the different components of firm demand identified above:

$$\pi(a) = (1 - f_0(a))(\Lambda(a) - 1)\Lambda(a)^{-\kappa}\big(\mu(a)\big)^{\sigma(\kappa-1)}\mathbf{mc}_t^{1-\kappa}\frac{\mathbf{\Omega}_t}{N}q(a) \tag{33}$$

### 2.2.3 Aggregation

Having found the optimal choices of consumers and firms, and before deriving the optimal advertising choices, we next characterize the dynamics of product categories and of the aggregate economy. To this end, we first show that, in spite of the rich underlying household heterogeneity, the model aggregates to a representative-agent neoclassical growth economy in which limited awareness at the microeconomic level is embedded in wedges to aggregate TFP at the macroeconomic level.

To arrive at this result, we aggregate up from the product category level. Let us denote by $\Phi_t(a)$ the cumulative density function (cdf) of the age distribution of product categories as of time $t$, with $\Phi_t(0) = 0$ and $\lim_{a\to+\infty}\Phi_t(a) = 1$, for all $t \in \mathbb{R}_+$. Let $\phi_t(a)$ be the probability density function (pdf) associated to this distribution. Since the instantaneous rate of product category creation (i.e., the flow of new product categories per unit of time) equals $z_M \boldsymbol{I}_t^M$, the law of motion for the age distribution of product categories is given by the Kolmogorov Forward Equation:

$$\partial_t \widehat{\Phi}_t(a) = -\underbrace{\partial_a \widehat{\Phi}_t(a)}_{\substack{\text{Categories}\\ \text{aging}}} - \underbrace{\delta_M \widehat{\Phi}_t(a)}_{\text{Obsolescence}} + \underbrace{z_M \boldsymbol{I}_t^M}_{\substack{\text{New category}\\ \text{creation}}}, \tag{34}$$

where we have defined $\widehat{\Phi}_t(a) \equiv \boldsymbol{M}_t \Phi_t(a)$.[39]

We define the total labor demand in a product category of age $a$ by $L(a) \equiv Nl(a)$, with $l(a)$ given by equation (31a). As there is a unit supply of labor and a measure $\boldsymbol{M}_t$ of product categories, the total labor demand in the economy is $\boldsymbol{L}_t \equiv \boldsymbol{M}_t \int_0^{+\infty} L(a)\phi_t(a)da$. The labor market clearing condition reads $\boldsymbol{L}_t = 1$. We may also define the aggregate stock of capital by $\boldsymbol{K}_t \equiv \boldsymbol{M}_t \int_0^{+\infty} K(a)\phi_t(a)da$, where $K(a) \equiv Nk(a)$ is the product category's demand for capital and $k(a)$ is firm-level capital demand, given by equation (31b). Likewise, we define aggregate profits by $\boldsymbol{\Pi}_t \equiv \boldsymbol{M}_t \int_0^{+\infty} \Pi(a)\phi_t(a)da$, where $\Pi(a) \equiv N\pi(a)$ are product category-level profits and $\pi(a)$ are firm-level profits, given by equation (33).

Finally, we define the following objects, whose interpretation we provide below:

$$\boldsymbol{Q}_t \equiv \left( \int_0^{+\infty} (1 - f_0(a))\Lambda(a)^{1-\kappa}\mu(a)^{\sigma(\kappa-1)} q(a)\phi_t(a)da \right)^{\frac{1}{\kappa-1}}, \tag{35a}$$

$$\boldsymbol{B}_t \equiv \frac{\int_0^{+\infty} (1 - f_0(a))\Lambda(a)^{-\kappa}\mu(a)^{\sigma(\kappa-1)} q(a)\phi_t(a)da}{\int_0^{+\infty} (1 - f_0(a))\Lambda(a)^{1-\kappa}\mu(a)^{\sigma(\kappa-1)} q(a)\phi_t(a)da}. \tag{35b}$$

With these definitions, we arrive at our main aggregation result:

**Proposition 5** (Aggregation)**.** *The economy aggregates to a representative-agent neoclassical growth model, in the following sense:*

1. *The price index and real income are identical across consumers:* $P_{jt} = \boldsymbol{P}_t$ *and* $\Omega_{jt} = \boldsymbol{\Omega}_t$, $\forall j \in [0,1]$. *Moreover, real income equals total output from the composite good defined in equation (12), i.e.,* $\boldsymbol{\Omega}_t = Y_{jt} = \boldsymbol{Y}_t$, *and it can be expressed as follows:*

$$\boldsymbol{Y}_t = \boldsymbol{Z}_t \boldsymbol{K}_t^{\alpha} \boldsymbol{L}_t^{1-\alpha}, \tag{36}$$

[39] Equivalently, $\partial_t \Phi_t(a) = -\partial_a \Phi_t(a) + (1 - \Phi_t(a)) \frac{z_M \boldsymbol{I}_t^M}{\boldsymbol{M}_t}$.

*where $\boldsymbol{Z}_t$ is aggregate TFP, defined by:*

$$\boldsymbol{Z}_t \equiv z\boldsymbol{M}_t^{\frac{1}{\kappa-1}}\boldsymbol{Q}_t\boldsymbol{B}_t^{-1}. \tag{37}$$

*The marginal cost of the economy equals* $\mathbf{mc}_t = \boldsymbol{M}_t^{\frac{1}{\kappa-1}}\boldsymbol{Q}_t$.

2. *Real income is exhausted by labor, capital and profit income payments, i.e.,*

$$\boldsymbol{Y}_t = w_t\boldsymbol{L}_t + (r_t+\delta_K)\boldsymbol{K}_t + \boldsymbol{\Pi}_t, \tag{38}$$

*with the following income shares:* $\frac{w_t\boldsymbol{L}_t}{\boldsymbol{Y}_t} = (1-\alpha)\boldsymbol{B}_t$, $\frac{(r_t+\delta_K)\boldsymbol{K}_t}{\boldsymbol{Y}_t} = \alpha\boldsymbol{B}_t$, *and* $\frac{\boldsymbol{\Pi}_t}{\boldsymbol{Y}_t} = 1-\boldsymbol{B}_t$.

*Proof.* See Appendix A.5.

This proposition states that the symmetric equilibrium of our economy collapses to that of a representative-agent neoclassical growth model, in which the micro-level frictions (in the form of slow-moving product awareness sets) are implicit in aggregate TFP.

Aggregate TFP, defined in equation (37), has different components. First, it scales with the common physical productivity of firms, $z$. Second, it increases with $\boldsymbol{M}_t$, the measure of product categories in the economy: since product categories are strong substitutes in preferences ($\kappa > 1$), the introduction of new categories boosts aggregate productivity, as is standard in expanding-variety models with love-for-variety. Third, aggregate TFP is shifted by $\boldsymbol{Q}_t\boldsymbol{B}_t^{-1}$, a term that is new to our model and which summarizes the aggregate effects of limited awareness in equilibrium.

In this term, $\boldsymbol{Q}_t$, defined in equation (35a), is an endogenous aggregate measure of *match quality*: it incorporates (i) the share of consumers that are aware of firms, $1-f_0(a)$, and the degree of connectedness of awareness sets within different product categories (the sorting component of demand, $q(a)$), both of which increase demand along the extensive margin; (ii) the level of targeting in advertising, $\mu(a)$, which increases demand along the intensive margin for given prices; and (iii) markups $\Lambda(a)$ across product categories, which endogenously respond to the elasticity of demand along both intensive and extensive margins, as described earlier. Aggregate match quality is then adjusted by the term $\boldsymbol{B}_t^{-1} > 1$, which dampens productivity for the existence of market power. Indeed, higher $\boldsymbol{B}_t^{-1}$ implies that a higher share of output is diverted to corporate profits, as $\boldsymbol{\Pi}_t/\boldsymbol{Y}_t = 1-\boldsymbol{B}_t$.

### 2.2.4 Advertising and Dynamic Resource Allocation

Equipped with these results, we are ready to solve for the age-zero advertising choices and the inter-temporal consumption-saving decisions. In the previous section, we showed that all consumers have identical price indices and identical real incomes despite the heterogeneity in their purchases. Therefore, we can study dynamic decisions as though they were made by a representative household that solves the inter-temporal allocation of aggregate resources into consumption and savings using the composite good, $\boldsymbol{Y}_t$.

This household invests in and rents away physical capital $\boldsymbol{K}_t$ to the firms at the rental rate $r_t + \delta_K$, supplies labor inelastically in exchange for the equilibrium wage $w_t$, invests into the creation of new product categories $\boldsymbol{I}_t^M$, and accumulates wealth $\boldsymbol{A}_t$ at the interest rate $r_t$. The household trades in firm shares, so total financial wealth is given by:

$$\boldsymbol{A}_t = \boldsymbol{M}_t \int_0^{+\infty} \left( \sum_{i=1}^{N} \widehat{V}_{it}(a) \right) \phi_t(a) da, \tag{39}$$

where $\widehat{V}_{it}(a)$ denotes the value of firm $i$ at calendar time $t$ and product category age $a$. This value is computed as the present discounted sum of all future stream of profits of the firm:

$$\widehat{V}_{it}(a) \equiv \int_t^{+\infty} e^{-\int_t^{t'} (r_\tau + \delta_M) d\tau} \pi_{it'}(a + t' - t) dt', \tag{40}$$

where $\pi_{it'}(a')$ are the period profits of firm $i$ at time $t'$ and firm age $a' = a + t' - t$.

**Nash equilibrium in advertising choices** When a new product category is created, the $N$ entering firms simultaneously choose once-and-for-all contact rates and targeting. Taking $\{\theta_{i'}\}_{i' \neq i}$ and $\{\mu_{0,i'}\}_{i' \neq i}$ as given, firm $i$ chooses $(\theta_i, \mu_{0,i})$ to maximize

$$V_{it}^0 \equiv \max_{\theta_i, \mu_{0,i}} \left\{ \widehat{V}_{it}(0) - d(\theta_i, \mu_{0,i}) \right\}, \tag{41}$$

where $d(\theta, \mu_0) \equiv \nu\theta^2 + \eta(\mu_0 - 1)^2$ is the firm-level advertising cost function defined in equation (8). Note $\widehat{V}_{it}(0)$ is an implicit function of the advertising choices of *all* firms, as these affect the evolution of awareness sets and directly impact firm demand by affecting match quality, i.e., the combined effects of sorting and targeting (Proposition 2).[40]

[40] In Appendix A.2 we argue that the perceived probability that awareness sets include a given firm depends on whether this firm follows the symmetric Nash-equilibrium strategy or whether it unilaterally deviates from it. While non-deviating firms update this probability using the $\mathcal{Q}(\theta)$ generator matrix introduced in Assumption 2, the unilaterally deviating firm (choosing $\theta_i \neq \theta$) uses instead a larger matrix, which we call $\widetilde{\mathcal{Q}}(\theta_i, \theta)$ in the appendix. We refer the reader to Appendix A.2 for details.

Taking the first-order conditions of problem (41) gives the equilibrium choices:

$$\theta_{it} = \frac{\partial_{\theta_i} \widehat{V}_{it}(0)}{2\nu} \qquad \text{and} \qquad \mu_{0,it} = 1 + \frac{\partial_{\mu_{0,i}} \widehat{V}_{it}(0)}{2\eta}. \tag{42}$$

In a stationary symmetric equilibrium, the contact rate and targeting are constant across firms, product categories and time, so we impose $\theta_{it} = \theta$ and $\mu_{0,it} = \mu_0$, for all $i = 1, \dots, N$, on equation (42). Thus, the value of a firm is the same within and across product categories, $V^0_{it} = V^0_t$. We let $\boldsymbol{V}^0_t \equiv N V^0_t$ denote the value of the whole product category at age zero.[41]

**Dynamic resource allocation** After the advertising choice is made, a license to use the blueprints of the production technology for the newly created product category is sold off to the $N$ firms at fair value. Subsequently, the household solves its dynamic problem.

Given initial conditions $\boldsymbol{A}_0$, $\boldsymbol{K}_0$, $\boldsymbol{M}_0$, and $\Phi_0(a)$, the problem is:

$$\max_{(\boldsymbol{C}_t, \boldsymbol{I}^K_t, \boldsymbol{I}^M_t \geq 0)_{t \in \mathbb{R}_+}} \int_0^{+\infty} e^{-\rho t} \frac{\boldsymbol{C}_t^{1-\gamma}}{1-\gamma} dt, \tag{43a}$$

$$\text{subject to} \quad \partial_t \boldsymbol{A}_t = r_t \boldsymbol{A}_t + w_t + (r_t + \delta_K)\boldsymbol{K}_t - \boldsymbol{C}_t - \boldsymbol{I}^K_t - \left(\boldsymbol{I}^M_t\right)^2 + z_M \boldsymbol{I}^M_t \boldsymbol{V}^0_t, \tag{43b}$$

$$\partial_t \boldsymbol{K}_t = \boldsymbol{I}^K_t - \delta_K \boldsymbol{K}_t. \tag{43c}$$

The flow budget constraint (43b) states that the change in total assets comes from the returns on outstanding assets, income from supplying labor and renting capital to the firms, and the return from creating new product categories, net of consumption and investment expenditures. This problem yields a standard Euler equation:

**Proposition 6** (Euler equation)**.** *The law of motion of aggregate consumption is:*

$$\frac{\partial_t \boldsymbol{C}_t}{\boldsymbol{C}_t} = \frac{r_t - \rho}{\gamma}. \tag{44}$$

*The optimal investment in product creation holds* $\boldsymbol{I}^M_t = \frac{1}{2} z_M \boldsymbol{V}^0_t$.

*Proof.* See Appendix A.6.

[41] In Appendix Figure D.2 we show that, in our calibration, firms' best response functions in advertising are downward-sloping, i.e., advertising choices are *strategic substitutes*: an increase in the advertising investment (in contacting or targeting) of one firm decreases the marginal gain from advertising for other firms. Therefore, the symmetric Nash equilibrium $(\theta, \mu_0)$ is interior.

To close the model, we impose market clearing conditions. The market clearing prices in the capital and labor markets solve $r_t = \alpha z \boldsymbol{M}_t^{\frac{1}{\kappa-1}} \boldsymbol{Q}_t \boldsymbol{K}_t^{\alpha-1} - \delta_K$ and $w_t = (1-\alpha) z \boldsymbol{M}_t^{\frac{1}{\kappa-1}} \boldsymbol{Q}_t \boldsymbol{K}_t^{\alpha}$, respectively. The resource constraint of the economy is given in equation (11) where, in a symmetric equilibrium, aggregate expenditure in advertising $\boldsymbol{D}_t$ is given by:

$$\boldsymbol{D}_t \equiv z_M \boldsymbol{I}_t^M N d(\theta, \mu_0). \tag{45}$$

Advertising expenditures are equal to the upfront per-firm costs $d(\theta, \mu_0)$, times the measure of entering firms at time $t$ (when a new product category is created), equal to $z_M \boldsymbol{I}_t^M N$.

In our quantitative analysis, we focus on a stationary equilibrium, for which $r_t = \rho$ and $w_t = w$, the age distribution of product categories is time-invariant, $\Phi_t(a) = \Phi(a)$, and economic aggregates are constant over time. In this case, we can obtain the following closed-form solution for the invariant distribution of product categories:

**Proposition 7** (Stationary age distribution)**.** *The invariant age distribution is given by:*

$$\Phi(a) = 1 - e^{-\delta_M a}. \tag{46}$$

*Proof.* See Appendix A.7.

## 2.3 Constrained-Efficient Allocation

How do product market frictions distort the allocation of resources? To answer this question, in Appendix B we solve the problem of a social planner that is constrained by the same product market frictions as those faced by consumers in the market economy.

Our results reveal various sources of inefficiencies. First, the symmetric decentralized equilibrium features static misallocation of inputs across product categories, leading to aggregate TFP losses relative to the constrained planner's solution. This misallocation, in turn, is caused by markup dispersion generated by limited awareness (recall Proposition 3 and our subsequent discussion). Intuitively, market power is higher in product markets with low levels of awareness, misallocating inputs away from them, and generating sub-optimal levels of aggregate output and consumption.

Second, there is dynamic underinvestment in both physical capital and the creation of new product categories. On the one hand, profit rents generated by markups lead to underinvestment in physical capital. On the other hand, the value of the firms generated with a new category does not fully capture the full social value from its creation, due to love-for-variety preferences.

Third, and new to our framework, firms in the decentralized economy choose, in general, sub-optimal levels of both the contact rate and targeting. This is because the private gains from advertising (given by the change in firm value, in equation (42)) are not generally equal to the social gains. On the one hand, as firms do not extract the full surplus from their customers, they fail to internalize that their advertising choices raise match quality in the aggregate, a positive "limited appropriability" externality that pushes for under-investment in advertising. The planner, by contrast, chooses $\theta$ and $\mu_0$ taking into account that they both increase productivity through aggregate match quality, equating the marginal cost of advertising to the present-discounted sum of elasticities of match quality to advertising for different cohorts of firms (see equations (B.2.12a)-(B.2.12b) in the Appendix). On the other hand, firms do not internalize the losses that they cause on other firms when they make advertising choices to lure customers away from them in order to increase their own market share and profits, a negative "business-stealing externality" that pushes for over-investment in advertising.

While theoretically there might therefore be both under- and over-investment in advertising, in our quantitative exercises from Section 3, we find that the positive spillovers quantitatively dominate business-stealing forces, so firms under-invest in both contacting and targeting relative to the constrained-efficient allocation.

# 3 Implications of Improved Advertising Technologies

Over the last few decades, the advent and rise of digital advertising has dramatically changed the advertising landscape. One notable difference between digital and more traditional advertising media resides in the degree to which digital advertising can be targeted to consumers who are more likely to purchase the product. In this section, we calibrate the model to analyze how these improvements in advertising technologies have affected consumers' product awareness, match quality, firm dynamics, markups, and welfare over time.

## 3.1 Calibration Strategy

We separately calibrate the model twice, for the years 2005 (henceforth, "early calibration") and 2014 (henceforth, "late calibration"). Over this period, the share of Internet advertising to total advertising increased significantly, from 6.03% to 26.65%.[42] Yet, aggre-

[42] Obtained from Statista, using data from Zenith's Advertising Expenditure Forecasts (December 2021 report): https://www.statista.com/statistics/429036/advertising-expenditure-in-north-america/.

gate advertising spending remained relatively constant as a share of U.S. GDP at around 2.2% (see, e.g., Greenwood *et al.* (2025)). Our purpose in this exercise is to understand the degree to which this compositional shift in advertising technologies may have had consequences for product dynamics and economic aggregates, including welfare.

For each period, we have 13 parameters to calibrate. We externally calibrate several parameters commonly encountered in macroeconomic models. The remaining parameters, which are most closely related to advertising, are internally calibrated.

#### 3.1.1 Externally calibrated parameters

We set the following parameters externally, which are kept constant across the two calibrations. The model's period is one year, and we set the time discount rate to $\rho = 0.04$. The risk aversion parameter is set to $\gamma = 2$, consistent with an elasticity of inter-temporal substitution of 0.5, documented for the U.S. by Havranek, Horvath, Irsova and Rusnak (2015). We normalize the common firm-level productivity to $z = 1$.[43] The capital share as a fraction of non-profit income is set to $\alpha = 1/3$, and the physical capital depreciation rate is set to $\delta_K = 0.069$, as in Celik, Tian and Wang (2022), who in turn compute this number using data from the U.S. NIPA tables. The cross-product-category elasticity of substitution is set to $\kappa = 3$, which is consistent with the range of estimates calculated in Oberfield and Raval (2021) for disaggregated industries.[44]

The parameters governing awareness are newer to the literature. First, we assume as the initial condition for awareness density that $f_0(0) = 1$. That is, the state of the product category at birth is that no consumer is aware of any firm. We set the exogenous rate of losing connections to $\zeta = 0.15$, a mid-point in the range of estimates for consumer separation rates found by Gourio and Rudanko (2014b) and the studies referenced therein.[45] As $\zeta > 0$, in our calibration networks never become fully saturated, so targeting remains active at all ages of the product category (i.e., $s(a) < 1$, and thus $\mu(a) > 1$, for all $a > 0$).

Since our analysis focuses on sectoral-level output and markups, our choice of $N$ is guided by what the empirical literature finds for the "effective number of firms", defined as the inverse of the HHI in an industry. In the symmetric equilibrium of our model, all

[43]This comes without loss of generality because $z$ scales the marginal cost of production faced by firms, which is common to all firms. Thus, changes in aggregates across calibrations and across counterfactual experiments are invariant to $z$.

[44]This choice for $\kappa$ puts an upper bound on net markups at 50%, corresponding to the monopolistically competitive markup (recall our discussion following Proposition 3).

[45]To be precise, in our model consumers separate from their firms for two reasons: because they lose the connection ($\zeta$) or because they find a better alternative. Therefore, the 15% should be considered as a lower bound. Yet, Gourio and Rudanko (2014b) report that annual separation rates are as high as 20-25% in some industries, such as retail, cell phone providers, and online banking.

firms have the same market share within the product category, so the effective number of firms equals $N$. We choose $N = 10$, a conservative choice that is at the upper end of the empirical estimates.[46]

Finally, in our model, a product category obsolescence shock (at rate $\delta_M$) discontinues the production of the category altogether, destroying all the firms at once. Given our choices for $N$ and $\kappa$, we think of product categories as being defined at a low level of disaggregation in the data. Therefore, we choose a low value of $\delta_M = 0.03$, meaning that on average a product category survives for about 33 years, and provide robustness checks on this value in the appendix.[47] This choice provides sufficient time for networks to expand and markups to converge before the product category ceases to exist.

### 3.1.2 Internally calibrated parameters

There are four remaining parameters: the cost of new product category creation ($z_M$), the degree of product differentiation in preferences ($\sigma$), and the advertising cost scale parameters for contacting ($\nu$) and targeting ($\eta$). We calibrate these parameters internally. For each of the two calibrations, we choose values for the four parameters that minimize the distance between model-generated moments and their empirical counterparts.

For both calibrations, we normalize the measure of product categories to $\boldsymbol{M} = 1$, which allows us to pin down the value of the cost of creating new product categories, $z_M$. We then target the ratio of advertising expenditures to GDP, using data from Greenwood *et al.* (2025); the cost-weighted average markup, using estimates from Edmond, Midrigan and Xu (2023); and a measure of the effectiveness of digital advertising. Finding a reliable measure of the effectiveness of digital advertising is challenging due to the lack of non-confidential data that is both representative in the cross-section of firms and consistent over time. To overcome this challenge, we rely on estimates from a field experiment, and combine them with time-series data on the share of digital advertising over the period of analysis. Our implicit assumption in doing this is that digital advertising is targeted whereas contacting

[46] Some examples include Mongey (2021), who reports a median of 40.7 firms per market but a median effective number $1/\text{HHI} \approx 3.73$ in IRI retail data; and Edmond, Midrigan and Xu (2015), who find a median inverse HHI of 3.92 in 7-digit Taiwanese manufacturing data and a median of 10 producers per sector (p. 3201). Thus, $N = 10$ places our model on the competitive side of these effective counts. Consistent with this, Bresnahan and Reiss (1991) document empirically that "once the market has between three and five firms, the next entrant has little effect on competitive conduct."

[47] Tables D.3 and D.4 display the results of a robustness check in which we set $\delta_M = 0.06$, decreasing the average duration of categories to about 17 years. Our results mostly remain qualitatively unchanged. One noteworthy exception is that, despite the cost of both contacting and targeting going down over time, the contact rate decreases between the two calibrations (Columns (1) and (2) of Panel A in Table D.4). This is due to the degree of substitutability between contacting and targeting being strong enough for the contact rate to decrease, as its relative cost increases over time.

advertising is not.

The effectiveness of targeted advertising is based on empirical evidence on the return to targeting reported in Farahat and Bailey (2012). This study finds, using a natural field experiment from ads on the Yahoo! homepage, that targeting increased the click-through rate for brands by 79.9% on average. As the share of digital advertising expenses in total advertising expenditures in the data increased from 6.03% to 26.65% between 2005 and 2014, we weight the return to targeting by the share of digital advertising to be able to compute the average return to targeting. By this measure, the return to targeting goes from 0.048 in 2005 to 0.213 in 2014, nearly a five-fold increase. In the model, we measure the return to targeting by computing the expected increase in a given firm's sales under the assumption that every other firm in the product category does not use targeting at all, i.e., that all other firms in the same market choose $\mu_0 = 1$, which is intended to replicate the experiment conducted in Farahat and Bailey (2012) using our model.[48]

For the late calibration, the internally calibrated parameters are estimated against the empirical values of same set of moments as of 2014. In addition, we also make sure that the growth rate of real GDP per capita between the two calibrations is in line with what is observed in the data.[49]

## 3.2 Calibration Results

Table 1 reports the results for the early and late calibrations in terms of model fit, and provides the corresponding parameter values. The model matches all moments closely. Our calibrations predict little change in consumer preference heterogeneity, from $\sigma = 0.222$ in 2005 to $\sigma = 0.219$ in 2014, due to the fact that the average markup changed very little over this period. Instead, the large changes in the composition of advertising seen in the data are being captured mostly through changes in the advertising technology parameters, $(\nu, \eta)$.

Importantly, our calibrations predict that both the cost of contacting ($\nu$) and the cost of targeting ($\eta$) were lower in the late period relative to the early period. The decrease in the cost of targeting is predicted to have been large, from $\eta = 0.6461$ to $\eta = 0.0800$, an 88% decline. The contact rate cost decreases as well, albeit to a lesser extent, from $\nu = 0.3295$ to $\nu = 0.2392$, a 27% decline. As we argue next, both of these changes are consequential for

[48] In Appendix Tables D.5 and D.6, we explore robustness checks in which our calibration targets a return to targeting that is 50% lower in 2005 and 50% higher in 2014 compared to our baseline calibration. These results show that matching larger changes in the return to targeting over time does not change our main results qualitatively. Quantitatively, most results in our counterfactual experiments (described in Section 3.3) are magnified.

[49] Data on GDP per capita is obtained from the St. Louis Federal Reserve Bank's FRED database, and is available at https://fred.stlouisfed.org/series/A939RX0Q048SBEA#0.

Table 1: Set of internally identified parameters and model fit

| Parameter | | Value | Moment | Data | Model |
|---|---|---|---|---|---|
| *A. Early calibration (2005)* | | | | | |
| Product differentiation | $\sigma$ | 0.2221 | Cost-weighted average markup | 0.247 | 0.247 |
| Product category creation efficiency | $z_M$ | 0.111 | Mass of categories (normalization) | 1 | 1 |
| Contact rate cost | $\nu$ | 0.3295 | Advertising share of GDP | 0.0220 | 0.0220 |
| Targeting cost | $\eta$ | 0.6461 | Return to targeting | 0.0482 | 0.0482 |
| *B. Late calibration (2014)* | | | | | |
| Product differentiation | $\sigma$ | 0.219 | Cost-weighted average markup | 0.246 | 0.244 |
| Product category creation efficiency | $z_M$ | 0.1067 | Mass of categories (normalization) | 1 | 1 |
| Contact rate cost | $\nu$ | 0.2392 | Advertising share of GDP | 0.0224 | 0.0225 |
| Targeting cost | $\eta$ | 0.0800 | Return to targeting | 0.2129 | 0.2127 |
| | | | Real GDP per capita growth | 0.0523 | 0.0523 |

**Notes:** This table reports parameter values and model fit for the early calibration, corresponding to data moments from 2005, and the late calibration, corresponding to 2014.

the effects of improved advertising technologies on productivity, competition, and market dynamics.

### 3.2.1 Effects on Economic Aggregates

Table 2 reports the baseline results for the early and late calibrations on a number of selected variables, including various advertising outcomes and variables related to markups and the sources of aggregate expenditure. As a result of the changes in $\nu$ and $\eta$ between the two calibrations, initial targeting $\mu_0$ goes up strongly between the early and late calibrations, from $\mu_0 = 1.1715$ to $\mu_0 = 1.9036$, a 62% increase, owing to the fact that the return on targeting increased sharply over this period. The contact rate also increases, from $\theta = 0.7573$ to $\theta = 0.8141$, a 7.5% increase between the two periods. Therefore, our calibrations predict that firms have increased the probability of contacting new customers at random over time, but have also become better at targeting those consumers with a greater preference for their products.

As a result of these changes, both overall output and overall consumption go up, by 5.2% and 5.0%, respectively.[50] Correspondingly, the level of aggregate distortions is lower in the late economy: recalling that $\boldsymbol{QB}^{-1}$ is the endogenous component of aggregate TFP,

[50]For now, we abstain from inferring predictions on welfare from these numbers because, in both of these calibrations, the measure of products (a contributor to welfare due to love-for-variety preferences) is kept fixed at $\boldsymbol{M} = 1$. In our counterfactual exercises of Section 3.3, however, we let $\boldsymbol{M}$ adjust freely, which allows us to have a complete picture of the welfare effects from the improvement in advertising technologies.

Table 2: Baseline calibration results

| | | (1) **Early calibration** | (2) **Late calibration** |
|---|---|---|---|
| **A. Advertising and markups** | | | |
| Contact rate | $\theta$ | 0.7573 | 0.8141 |
| Targeting rate | $\mu_0$ | 1.1715 | 1.9036 |
| Average return to targeting | | 0.0482 | 0.2127 |
| Average cost-weighted markup | | 0.2470 | 0.2439 |
| **B. Expenditure shares of GDP** | | | |
| Consumption share | $\boldsymbol{C}/\boldsymbol{Y}$ | 0.7830 | 0.7814 |
| Advertising share | $\boldsymbol{D}/\boldsymbol{Y}$ | 0.0220 | 0.0225 |
| Category creation investment share | $(\boldsymbol{I}^M)^2/\boldsymbol{Y}$ | 0.0258 | 0.0265 |
| Capital investment share | $\boldsymbol{I}^K/\boldsymbol{Y}$ | 0.1692 | 0.1696 |
| **C. Income shares of GDP** | | | |
| Labor share | $w\boldsymbol{L}/\boldsymbol{Y}$ | 0.5346 | 0.5360 |
| Capital share | $(r+\delta_K)\boldsymbol{K}/\boldsymbol{Y}$ | 0.2673 | 0.2680 |
| Profit share | $\boldsymbol{\Pi}/\boldsymbol{Y}$ | 0.1981 | 0.1961 |
| **D. Economic aggregates** | | | |
| Wage | $w$ | 1.5159 | 1.5993 |
| Consumption level | $\boldsymbol{C}$ | 2.2202 | 2.3316 |
| Match quality | $\boldsymbol{Q}$ | 1.1913 | 1.2346 |
| Distortion-adjusted match quality | $\boldsymbol{QB}^{-1}$ | 1.4856 | 1.5357 |
| **E. Constrained-efficient allocation** | | | |
| Efficient contact rate | $\theta^*$ | 0.7836 | 0.8411 |
| Efficient targeting rate | $\mu_0^*$ | 1.1871 | 1.9718 |
| Efficient match quality | $\boldsymbol{Q}^*$ | 1.4923 | 1.5433 |

**Notes:** Results from our calibrations on selected equilibrium variables. Column (1) reports the baseline results for the early calibration (2005). Column (2) reports results for the late calibration (2014).

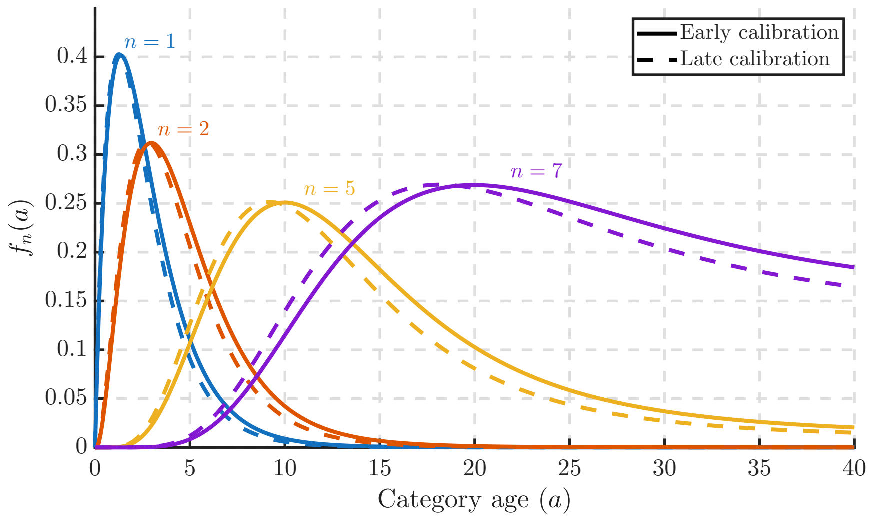
we obtain that $\boldsymbol{QB}^{-1}$ increases by 3.4%, from $\boldsymbol{QB}^{-1} = 1.486$ in 2005 to $\boldsymbol{QB}^{-1} = 1.536$ in 2014, a combined result of the change in aggregate match quality ($\boldsymbol{Q}$), and aggregate distortions from market power ($\boldsymbol{B}^{-1}$).

### 3.2.2 Effects on Market Dynamics

These effects at the aggregate level underlie changes in the dynamics of product categories, to which we move next.

Figure 3 shows the evolution of product awareness over time within a product category. Each solid line (respectively, dashed line) represents the share of consumers aware of

Figure 3: Proportion of consumers aware of $n$ firms, $f_n(a)$: Early vs Late calibrations.




**Notes:** This figure plots the proportion of consumers that are aware of $n = 0, 1, \ldots, N$ firms over product category age $a$, for the early calibration (solid lines) and the late calibration (dashed lines).

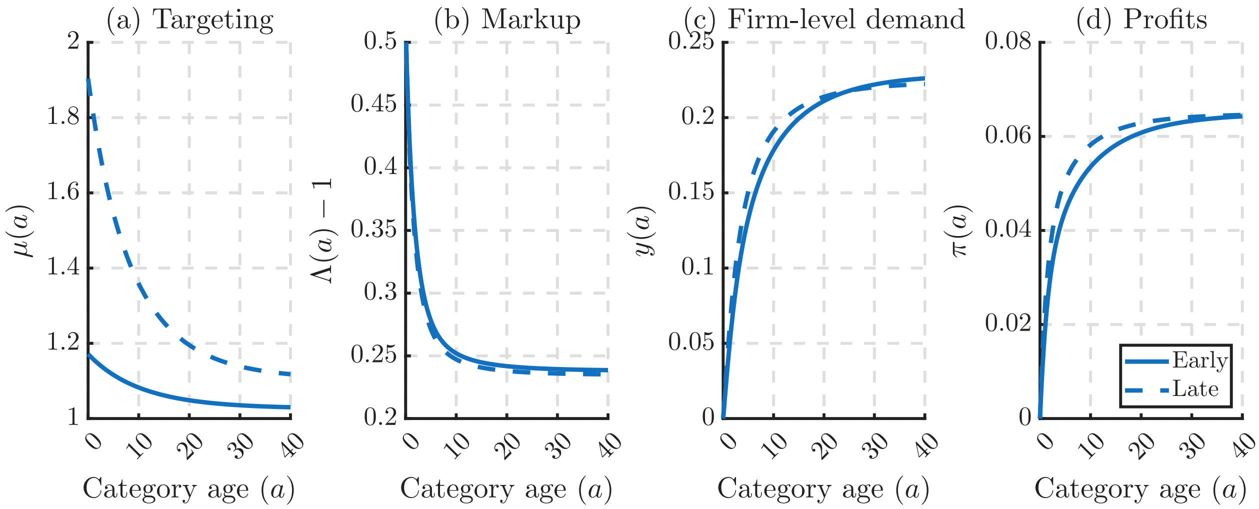
a certain number $n$ of products, $f_n(a)$, in the early (respectively, late) calibration, as a function of the age $a$ of the product category.[51] As time goes by and the product category ages, consumers become gradually aware of the existence of products. Comparing these dynamics in the early and late periods reveals that networks expand faster in the latter. This is because firms in the late period choose a higher contact rate $\theta$, enabling consumers to become aware of more firms earlier on.

Figure 4 shows the consequences of these dynamics on various firm-level outcomes. First, the degree of targeting is higher in the late period, especially in the early stages of the product category, when sorting is low and firms retain a relatively high degree of market power (panel (a)). Second, markups are lower in the late period (panel (b)): as consumers in the late period become aware of more alternatives earlier on, their demand is more price-elastic at every age of the product's life-cycle, and firms' market power is diminished. In spite of lower markups, the late calibration features higher prices at all ages.[52] Thus, firm-level demand (panel (c)) reflects the conflicting effects of higher prices and higher targeting. In the early stages of the product category, higher targeting dominates over price effect, making demand higher in the late period. As consumers become aware of more firms over time, targeting differences between the early and late calibrations narrow, and price

[51] For visual clarity, we only provide the plots for $n \in \{1, 2, 5, 7\}$.

[52] To understand this, recall that $p(a) = \Lambda(a)\mathbf{mc}$, where $\Lambda(a)$ is the markup and $\mathbf{mc} = \boldsymbol{M}^{\frac{1}{\kappa-1}}\boldsymbol{Q}$ is the marginal cost. As argued, markups are lower in the late period. However, aggregate quality $\boldsymbol{Q}$ is much higher due to the improved advertising technologies. As the measure of products is normalized to one in both calibrations ($\boldsymbol{M} = 1$), this implies higher prices.

Figure 4: Firm-level outcomes by product category age.



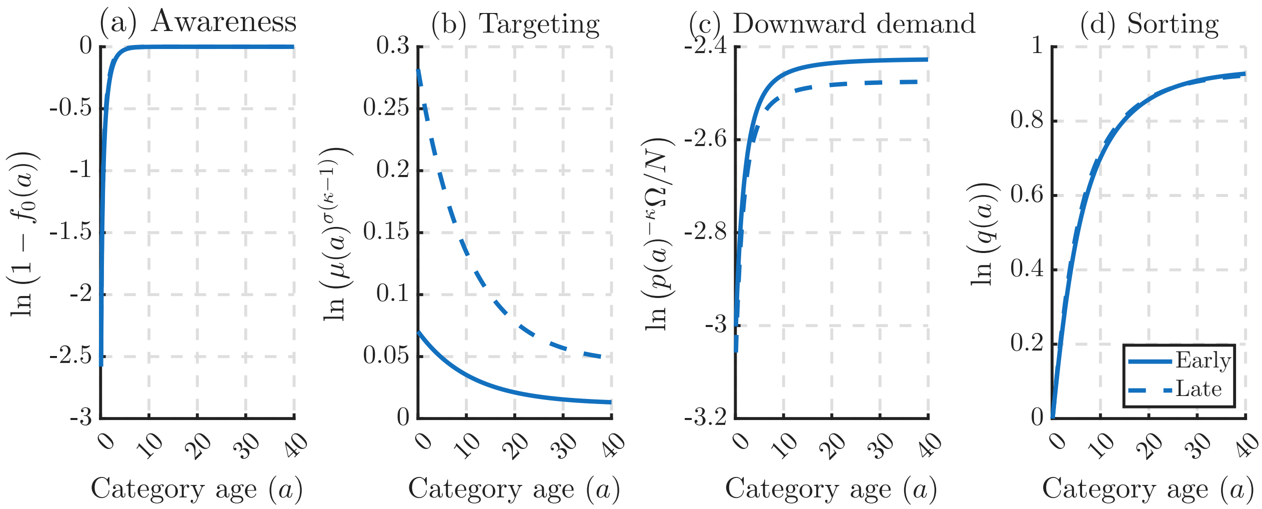

**Notes:** This figure plots various equilibrium outcomes at the firm level as a function of product category age, for the early calibration (solid line) and the late calibration (dashed line). Panel (a) is targeting, $\mu(a)^{\sigma(\kappa-1)}$, a component in firm-level demand (equation (21)); panel (b) shows prices, $p(a)$, defined in Proposition 3; panel (c) shows total firm-level demand, defined in equation (32); panel (d) shows firm-level profits, defined in equation (33).

effect eventually dominates, so firms face higher demand in the later stages of the product life cycle in the early period relative to the late period. A similar pattern is observed for firm-level profits, in panel (d) of Figure 4. All in all, improved advertising technologies helped firms front-load profits along their lifetime.

To better understand the evolution of firm-level demand (panel (c) in Figure 4) over the life cycle, we use the decomposition found in equation (21). When expressed in logs, this equation allows us to write demand as the sum of four components: (i) overall awareness, i.e., the share of consumers who are aware of at least one product; (ii) targeting; (iii) the downward-sloping intensive demand component; and (iv) the extensive-margin sorting component.

We report this log-decomposition for the early and late calibrations in Figure 5. Panel (a) shows that overall product awareness only contributes to demand in the very earliest stages of the product category, with virtually no difference between the early and late calibrations thereafter. Panel (b) shows that the difference in targeting between the two economies plays the most important role in explaining why firm demand is higher in the late period at the early stages of the product category. Panel (c) shows that the downward-sloping component of intensive demand is nevertheless also stronger (i.e., contributes more negatively) in the late period. This is the net result of two conflicting forces. On the one hand, aggregate income is higher in the late period (a calibration target), pushing intensive demand up. On

Figure 5: Components of firm-level demand (in logs).




**Notes:** This figure plots the four components of firm demand identified in equation (21), expressed in logs.

the other hand, prices are also higher in the late period, pushing intensive demand down. In net, the price effect dominates. Finally, the sorting component (panel (d)), which relates to the size of the awareness sets of consumers and thus to the extensive margin of demand, exhibits again small differences between calibrations.

In sum, the higher contact rate in the late calibration implies a higher average number of firms in consumers' awareness sets at any given time, leading to higher demand in the late calibration. However, higher targeting is ultimately the quantitatively dominating force behind the increase in firm demand and profits in the later period.

## 3.3 Counterfactual Experiments

We arrive at the central question of our paper: what are the effects of changes in advertising technologies over time on markups, productivity and welfare, through their effects on product awareness dynamics? To investigate this question, we use our calibration results to construct a series of counterfactual economies for the late period (2014) in which the advertising cost parameters $(\nu, \eta)$ are set back to their (higher) values in the early period (2005), keeping all other parameters fixed at their values for the late calibration.

### 3.3.1 Effects on Match Quality, Sorting and Markups

Table 3 reports the results of these counterfactual exercises on selected equilibrium variables. Columns (3a) and (3b) show the results of our first counterfactual experiment, in which we simultaneously reset the values of both $\nu$ and $\eta$ to their values in 2005, and

Table 3: Counterfactual experiments

| | (1) Early (base) | (2) Late (base) | (3a) Early $\nu$ & $\eta$ | (3b) %-change (wrt late) | (4a) Early $\nu$ only | (4b) %-change (wrt late) | (5a) Early $\eta$ only | (5b) %-change (wrt late) |
|---|---|---|---|---|---|---|---|---|
| **A. Advertising and markups** | | | | | | | | |
| Contact rate ($\theta$) | 0.7573 | 0.8141 | 0.7589 | -6.78% | 0.7140 | -12.30% | 0.8705 | 6.93% |
| Targeting rate ($\mu_0$) | 1.1715 | 1.9036 | 1.1712 | -38.48% | 1.9340 | 1.60% | 1.1634 | -38.88% |
| Average return to targeting | 0.0482 | 0.2127 | 0.0479 | -77.48% | 0.2375 | 11.64% | 0.0420 | -80.27% |
| Average cost-wtd. markup | 0.2470 | 0.2439 | 0.2440 | 0.04% | 0.2454 | 0.64% | 0.2425 | -0.57% |
| Firm value, without adv. cost | 0.6952 | 0.7506 | 0.6977 | -7.05% | 0.7451 | -0.73% | 0.7075 | -5.74% |
| **B. Expenditure shares of GDP** | | | | | | | | |
| Consumption share | 0.7830 | 0.7814 | 0.7830 | 0.20% | 0.7810 | -0.05% | 0.7832 | 0.23% |
| Advertising share | 0.0220 | 0.0225 | 0.0218 | -3.00% | 0.0240 | 6.54% | 0.0206 | -8.48% |
| Category creation inv. share | 0.0258 | 0.0265 | 0.0256 | -3.42% | 0.0256 | -3.35% | 0.0264 | -0.29% |
| Capital investment share | 0.1692 | 0.1696 | 0.1696 | -0.01% | 0.1694 | -0.13% | 0.1698 | 0.11% |
| **C. Income shares of GDP** | | | | | | | | |
| Labor share | 0.5346 | 0.5360 | 0.5359 | -0.01% | 0.5353 | -0.13% | 0.5366 | 0.11% |
| Capital share | 0.2673 | 0.2680 | 0.2680 | -0.01% | 0.2676 | -0.13% | 0.2683 | 0.11% |
| Profit share | 0.1981 | 0.1961 | 0.1961 | 0.03% | 0.1971 | 0.51% | 0.1952 | -0.46% |
| **D. Economic aggregates** | | | | | | | | |
| Mass of product categories | 1 | 1 | 0.9283 | -7.17% | 0.9633 | -3.67% | 0.9663 | -3.37% |
| Wage | 1.5159 | 1.5993 | 1.4268 | -10.79% | 1.5336 | -4.11% | 1.4992 | -6.26% |
| Consumption | 2.2202 | 2.3316 | 2.0845 | -10.60% | 2.2375 | -4.03% | 2.1883 | -6.15% |
| Consumption per category | 2.2202 | 2.3316 | 2.2456 | -3.69% | 2.3228 | -0.38% | 2.2646 | -2.87% |
| Match quality ($\boldsymbol{Q}$) | 1.1913 | 1.2346 | 1.1023 | -10.71% | 1.1783 | -4.56% | 1.1624 | -5.85% |
| Distortion-adjusted quality ($\boldsymbol{QB}^{-1}$) | 1.4856 | 1.5357 | 1.3713 | -10.71% | 1.4675 | -4.44% | 1.4443 | -5.95% |
| **E. Constrained-Efficient Allocation** | | | | | | | | |
| Efficient contact rate ($\theta^*$) | 0.7836 | 0.8411 | 0.7839 | -6.81% | 0.7381 | -12.25% | 0.8983 | 6.80% |
| Efficient targeting rate ($\mu_0^*$) | 1.1871 | 1.9718 | 1.1866 | -39.82% | 2.0068 | 1.78% | 1.1781 | -40.25% |
| Contacting efficiency gap ($\theta/\theta^*$) | 0.9664 | 0.9679 | 0.9682 | 0.03% | 0.9673 | -0.06% | 0.9691 | 0.12% |
| Targeting efficiency gap ($\mu_0/\mu_0^*$) | 0.9869 | 0.9654 | 0.9870 | 2.24% | 0.9637 | -0.18% | 0.9875 | 2.29% |

**Notes:** Results from our counterfactual experiments on selected equilibrium variables. Columns (1) and (2) report baseline results for the early (2005) and late (2014) calibrations, respectively (same as Table 2). Column (3a) reports 2014 results when both $\eta$ and $\nu$ are fixed at their 2005 values, with column (3b) stating the percentage change with respect to the baseline late calibration, i.e., the percentage change of column (3a) relative to column (2). Column (4a) repeats the experiment but re-setting only the contacting cost parameter $\nu$ to its 2005 level, with column (4b) stating the percentage change relative to column (2). Column (5a) does the same except for the targeting cost parameter $\eta$, with column (5b) stating the percentage change relative to column (2).

recompute the model's stationary equilibrium, leaving all other parameters fixed at their late-period calibrated values. The resulting equilibrium shows what the economy would have looked like if advertising technologies had not changed at all from 2005 to 2014.

Our calibrations predicted that advertising costs were higher overall in the early period, both in contacting customers ($\nu_{2005} > \nu_{2014}$), as well as in targeting the high-valuation ones ($\eta_{2005} > \eta_{2014}$). These cost differences explain that firms choose lower advertising investments overall in the counterfactual economy: the advertising share of GDP is 3% lower in the counterfactual compared to its baseline 2014 level, coming from both significantly lower investments in targeting (the targeting rate $\mu_0$ is 38.5% lower) and contacting (the contact rate $\theta$ is 6.8% lower).

A key result from our calibration is that contacting and targeting are net substitutes. To make this point, we recompute the 2014 equilibrium, but separately reset $\nu$ and $\eta$ back to their 2005 levels, respectively. The results are presented in columns (4a)-(4b), for the counterfactual in which only the contacting cost $\nu$ is reset to its value in the early period, and columns (5a)-(5b), for the counterfactual in which only the targeting cost $\eta$ is reset. Raising the cost of contacting new customers, but keeping the cost of targeting fixed (column (4a)), leads to a lower contact rate, but also a *higher* level of targeting relative to the baseline late-period economy. Conversely, raising the cost of targeting, but keeping the cost of contacting new customers fixed (column (5a)), leads to a lower level of targeting, but also to a *higher* contact rate.

This substitutability between contacting and targeting is a quantitative result. Theoretically, there are forces pushing towards both complementarity and substitutability. Intuitively, for given prices, a higher contact rate creates more matches per unit of time, expanding the firm's customer base with new consumers. As targeting pushes up demand at given prices, this is now a more profitable activity. This force makes contacting and targeting complements. At the same time, however, a higher contact rate also makes networks saturate faster, closing the window of opportunity for firms to extract surplus from the high-valuation consumers (recall that the return to targeting is decreasing in network saturation, $\mu(a) = \mu_0^{1-s(a)}$). This pushes towards substitutability. All in all, in our counterfactual we find that this latter force is the quantitatively dominant one.

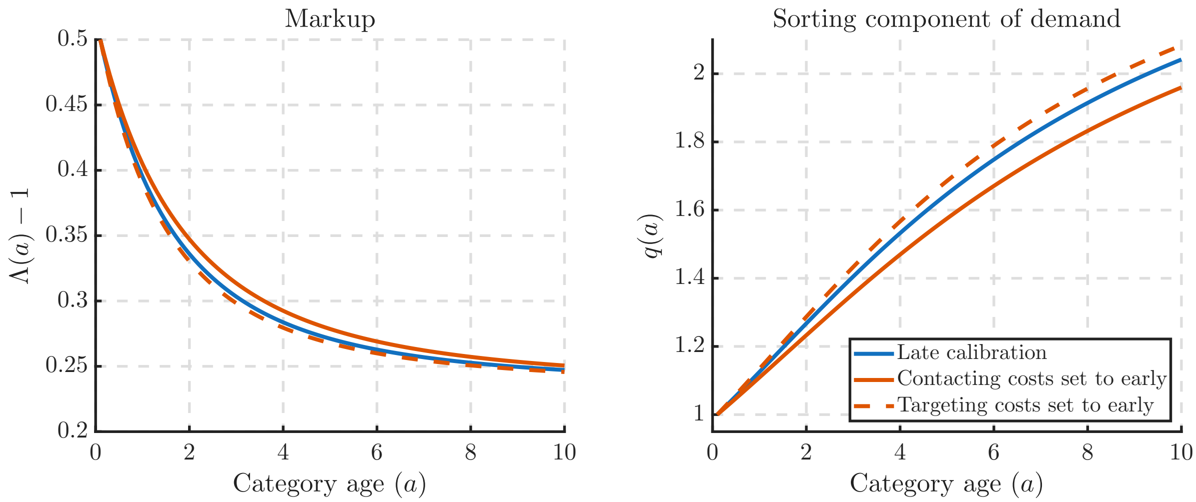
Finally, our counterfactual experiments illustrate various counteracting effects that changes in the contact rate and in the level of targeting can have on the economy. A higher cost of contacting new customers increases the level of targeting in the economy, but reduces the contact rate, implying that customers are on average aware of fewer products. This leads to less competition for firms and an increase in the level of markups: the average markup is 0.64% higher when $\nu$ is returned to its 2005 level (see column (4b)). On the other hand, an increase in the cost of targeting leads firms to invest more in contacting customers, raising competition and lowering markups: the average markup is 0.57% lower when $\eta$ is set to its 2005 level (see column (5b)).[53]

Figure 6 illustrates these two opposing forces over the life-cycle of the firm. The

[53] These numbers should be put in perspective with markups remaining virtually unchanged over this period in our calibrations. For instance, the increase in markups when reverting back the cost of contacting to its early value represents roughly half of the total change in markups over the same period.

effects on markups of changing the contacting and targeting costs separately are significant, changing the entire path of markups over the life-cycle. However, in the aggregate, they are of similar magnitude, making the overall aggregate effect of improved advertising technologies negligible, at 0.04% (column (3)).

Figure 6: Markups and sorting by product category age.




**Notes:** This figure compares the lifecycle of markups and the sorting component of demand in the late calibration (sold blue line) and two counterfactuals: setting the contacting costs $\nu$ to its value in the early calibration (solid red line) and setting the targeting costs $\eta$ to its value in the early calibration (dashed red line).

### 3.3.2 Effects on Welfare

How did the observed decrease in advertising costs and the rise of digital advertising affect consumer welfare? There are several effects to consider in Table 3. First, as just discussed, the changes in the costs of contacting and targeting over time do not generate significant changes in average markups, but this hides heterogeneity between the two sides of advertising.

Second, a lower cost of advertising significantly raises the average quality of a consumer-firm match. Column (3a) of Table 3 shows that returning to the (higher) level for advertising costs from 2005 would imply a 10.7% lower level in both match quality $\boldsymbol{Q}$ and the distortion-adjusted quality $\boldsymbol{Q}\boldsymbol{B}^{-1}$.[54] Our counterfactual experiments of columns (4) and (5) show that most of the change in match quality from higher advertising costs is associated with the cost of targeting. Finally, there are general-equilibrium effects to consider. If the

[54]The reason why the changes in $\boldsymbol{Q}$ and $\boldsymbol{Q}\boldsymbol{B}^{-1}$ are nearly identical is that the profit share, $\boldsymbol{\Pi}/\boldsymbol{Y} = 1 - \boldsymbol{B}$, barely changes over this period. Once again, this can also be seen in the average cost-weighted markup barely moving.

advertising technology had remained unchanged, the category creation investment share ($\boldsymbol{I}^M/\boldsymbol{Y}$) would have been 3.42% lower, implying a lower (by 7.17%) steady-state measure of product categories, translating into lower levels of output and consumption. Unlike for the other two effects, columns (4) and (5) show that most of the increase in $\boldsymbol{I}^M$ is associated with the decrease in the cost of contacting over time.

How do these changes impact welfare? In our economy, consumption-equivalent welfare is measured as the percentage change in aggregate consumption between the counterfactual economy (column (3a)) and the late calibration (column (2)).[55] By this measure, we find that bringing back the cost of advertising to its level in 2005 would decrease welfare by 10.6%. Since the change in the consumption share of output is just 0.2%, virtually all the change in consumption comes from output, which can be further decomposed as follows:

$$\underbrace{\Delta \boldsymbol{C}}_{-10.6\%} \approx \underbrace{\Delta\left(\boldsymbol{M}^{\frac{1}{\kappa-1}}\right)}_{-3.65\%} + \underbrace{\Delta\left(\boldsymbol{Q}\boldsymbol{B}^{-1}\right)}_{-10.71\%} + \underbrace{\Delta\left(\boldsymbol{K}^{\alpha}\right)}_{+3.71\%} \tag{47}$$

where, for a generic variable $x$, we have defined $\Delta x \equiv x^{\mathrm{CF}}/x^{\mathrm{Late}} - 1$. The largest contributor to welfare gains is the change in match quality. When ignoring love-for-variety effects by focusing on consumption per product category, $\boldsymbol{C}/\boldsymbol{M}$, the loss in consumption-equivalent welfare is still significant, at 3.69%.

The overall welfare effect owes more to the lower cost of targeting than to that of contacting. In consumption levels, about two-fifths (4.03% out of 10.6%) of the welfare effects come from the reduction in contacting costs. In terms of normalized consumption, the contribution of contacting is much weaker, at one-tenth (0.38% out of 3.69%) of the overall change. In sum, better targeting technologies brought about by the rise of digital advertising remain the main source of welfare gains over time.[56]

## 3.4 Effects of Improved Technologies in the Efficient Allocation

To conclude our quantitative analysis, we compare the decentralized equilibrium with the solution of the constrained social planner discussed in Section 2.3, both over time (across calibrations), as well as in our counterfactual experiments.

[55] Consumption-equivalent welfare is defined as the share of life-time consumption that a consumer in a benchmark economy (the "late calibration", in our case) would require in order to obtain the same life-time utility as a consumer in the counterfactual economy. Since we focus on the stationary equilibrium in both economies, it is easy to show that this simply equals the percentage change in the aggregate level of consumption between the two allocations.

[56] Appendix C presents a model extension in which, on top of being informative, advertising also plays a persuasive role. Our counterfactual experiments are qualitatively robust to this extension.

Panel E of Table 2 shows the optimal level of the contact rate ($\theta$) and targeting ($\mu_0$) for both early and late periods. The planner's choices exhibit higher contact and targeting rates compared to the decentralized equilibrium in both periods. In the decentralized economy, our calibrations suggested that the costs of both contacting and targeting went down over time (disproportionately so for the cost of targeting), leading firms to raise both their contact and targeting rates (see Table 2). Panel E of Table 2 reveals that the planner reacts in the same way, increasing both the optimal targeting rate (from 1.19 to 1.97), as well as the optimal contacting rate (from 0.78 to 0.84).

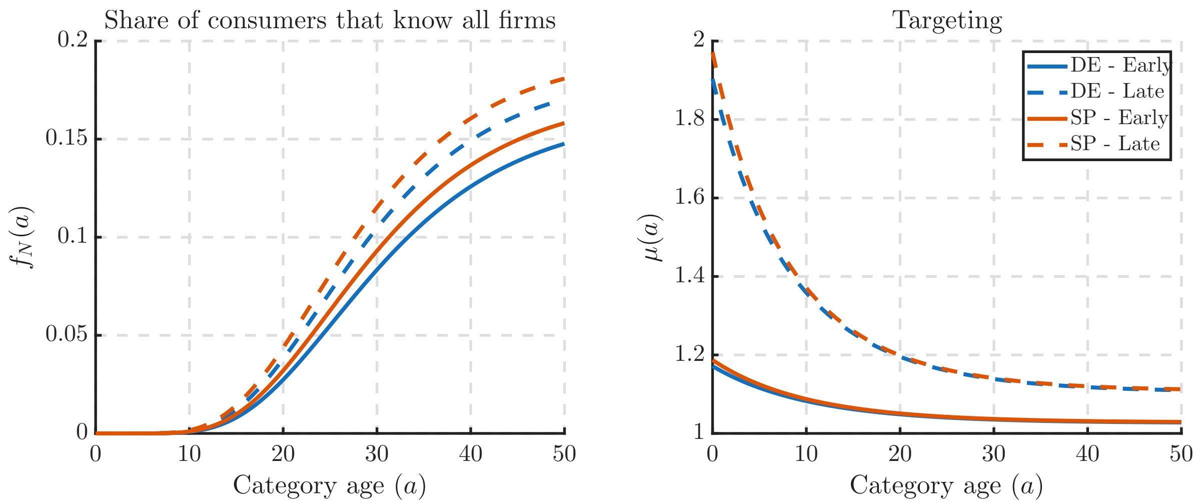
Columns (4) and (5) in Table 3, where we respectively set the cost of contacting and targeting to their higher early value, show that the planner also exhibits substitutability between contacting and targeting.[57] Indeed, the social returns to targeting (encapsulated in aggregate social match quality, which we define in equation (B.1.28) of the appendix) are in part increasing in the choice of contacting (making the two complements) because faster contacting, increasing the number of consumers that are aware of products at a given point, makes targeting more socially beneficial by expanding consumption along the intensive margin. At the same time, the social returns are also partly decreasing in the choice of contacting (making the two substitutes), because faster contacting populates networks faster, which, because targeting is decreasing in saturation, decreases its effectiveness. In our calibrations, we consistently find that substitutability is the stronger force for the planner: the efficient level of targeting increases (by 1.78%) when the contacting cost is set back to its early value (column (4)), and the contact rate increases (by 6.80%) when the targeting cost is set back (column (5)).

The last two rows in Panel E show how the gaps in contacting and targeting between decentralized and constrained-efficient outcomes have changed over time. Notably, while the efficiency gap in contacting remains roughly constant around 3.3%, the efficiency gap in targeting widened, from 1.3% to 3.5%. That is, while firms increased their targeting intensity over time, this increase was insufficient from a socially optimal standpoint.

Finally, Figure 7 compares life-cycle properties of the decentralized equilibrium (in blue) and the social planner allocation (in red) for both early and late periods. It shows the evolution of the share of consumers aware of all varieties (which is directly linked to the contact rate), as well as targeting. As the planner chooses higher contact rates in both periods compared to the decentralized equilibrium, the efficient solution features a faster growth of consumer awareness sets. Compared to the early period, the planner allocation also improves targeting, especially early on in the life of the product category.

[57] Similar forces to those described for the decentralized equilibrium in section 3.3.1 are at play for the planner as well, pushing both towards complementarity and substitutability.

Figure 7: Decentralized and Planner's Solutions: Early vs. Late Calibrations.




**Notes:** Panel (a) plots the proportion of consumers that are aware of all firms ($n = N$) at each product category age $a$, for the early calibration (solid lines) and the late calibration (dashed lines), in the decentralized allocation and the planner's solution. Panel (b) plots the degree of targeting in each case.

# 4 Conclusion

To study the implications of the rise of digital advertising on industry dynamics, competition, aggregate productivity and welfare, we develop a general-equilibrium, heterogeneous-agent model of demand as a network, in which consumers become slowly aware of products. In this framework, advertising can affect both the speed at which buyer-seller networks expand at random ("contacting"), as well as the quality of the matches, i.e., how strongly a firm's product offering is correlated with the customer's idiosyncratic preference for it ("targeting"). In a symmetric equilibrium, in which firms offer best responses to their competitors in advertising and pricing decisions, faster network formation leads to stronger competition and lower markups over the life cycle of products through larger awareness sets —a phenomenon which we call "sorting". Targeting, on the other hand, while not directly affecting markups in the symmetric equilibrium, does allow firms to attract higher-valuation consumers, shifting demand along the intensive margin at given prices. Moreover, in equilibrium, targeting affects the incentives to contact new consumers, thereby indirectly affecting the evolution of awareness and markups.

In our calibrations, the rise of digital advertising in recent years, which we interpret as a strong increase in the return to targeting, yielded an increase in aggregate TFP and welfare through a rise in *aggregate match quality*, an endogenous component of TFP that combines the effects of sorting and targeting. Through counterfactual experiments, we uncover that contacting and targeting act as substitutes at the firm level: a decrease in the cost of targeting not only raises markups directly, but also indirectly as it reduces the contact rate. Solving for the constrained optimal allocation, we find that investment in both contacting and targeting was inefficiently low over this period, but that firms did not sufficiently increase their investment in targeting in response to improved advertising technologies.

Our analysis suggests that policy-makers should take into account the role of digital advertising on industry dynamics in addition to privacy concerns — especially considering that social network, mobile and other forms of online advertising, where targeting is especially prevalent, is becoming the dominant form of advertising nowadays. Our findings suggest that appropriate regulations may need to offer incentives for efficient targeted advertising without excessively restricting consumer awareness. Our paper has left out some considerations, however, such as the potential interactions between informative advertising and firm-level R&D decisions, consumer search, or firm-level entry and exit decisions, which are potentially policy-relevant but beyond the scope of the present study. We leave these as open avenues for future research.

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# A Theory of Dynamic Product Awareness and Targeted Advertising

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*Online Appendix (Not for Publication)*

# A Proofs

## A.1 Proof of Proposition 1

The static problem of the consumer is to allocate resources $y_{imjt}$ to solve:

$$\max_{(y_{imjt}\geq 0)} \underbrace{\left[\int_0^{\boldsymbol{M}_t}\left(\sum_{i\in A_{mjt}}\overline{\Gamma}e^{\sigma\xi_{imj}}y_{imjt}\right)^{\frac{\kappa-1}{\kappa}}dm\right]^{\frac{\kappa}{\kappa-1}}}_{\equiv Y_{jt}},\ \text{s.t.}\ \int_0^{\boldsymbol{M}_t}\sum_{i\in A_{mjt}}\widehat{p}_{imt}y_{imjt}dm\leq P_{jt}\Omega_{jt}. \tag{A.1.1}$$

The Lagrangian of this problem is:

$$\begin{aligned}\mathscr{L}_{jt} = &\left[\int_0^{\boldsymbol{M}_t}\left(\sum_{i\in A_{mjt}}\overline{\Gamma}e^{\sigma\xi_{imj}}y_{imjt}\right)^{\frac{\kappa-1}{\kappa}}dm\right]^{\frac{\kappa}{\kappa-1}}-\lambda_{jt}\left(\int_0^{\boldsymbol{M}_t}\sum_{i\in A_{mjt}}\widehat{p}_{imt}y_{imjt}dm-P_{jt}\Omega_{jt}\right)\\ &+\int_0^{\boldsymbol{M}_t}\sum_{i\in A_{mjt}}\vartheta_{imjt}y_{imjt}dm,\end{aligned} \tag{A.1.2}$$

where $\lambda_{jt}\geq 0$ is the Lagrange multiplier on the budget constraint, and $\left(\{\vartheta_{imjt}\geq 0\}_{i\in A_{mj}}:m\in[0,\boldsymbol{M}_t]\right)$ are the multipliers ensuring weak positivity on every choice of $y_{imjt}$. The first-order condition is:

$$\left(\frac{Y_{jt}}{\sum_{i\in A_{mjt}}\overline{\Gamma}e^{\sigma\xi_{imj}}y_{imjt}}\right)^{\frac{1}{\kappa}}\overline{\Gamma}e^{\sigma\xi_{imj}}=\lambda_{jt}\widehat{p}_{imt}-\vartheta_{imjt}. \tag{A.1.3}$$

By monotonicity of preferences and the complementary slackness condition, we have $\lambda_{jt}>0$, and $\vartheta_{imjt}y_{imjt}=0$, with $\vartheta_{imjt}\geq 0$, $\forall(i,m)$. We conjecture (to be verified later) that if $m\in\mathcal{M}_{jt}\equiv\{m:A_{mjt}\neq\emptyset\}\subseteq[0,\boldsymbol{M}_t]$, the consumer almost surely purchases at most one product from product category $m$. Denote this product by $i(m) \in A_{mjt}$. Then, from equation (A.1.3), we have:

$$\forall m \in \mathcal{M}_{jt}: \quad Y_{jt}^{\frac{1}{\kappa}} \left(\overline{\Gamma} e^{\sigma \xi_{i(m)mj}}\right)^{\frac{\kappa-1}{\kappa}} y_{i(m)mjt}^{-\frac{1}{\kappa}} = \lambda_{jt} \widehat{p}_{i(m)mt}. \tag{A.1.4}$$

Taking the ratio between any two product categories $m, m' \in \mathcal{M}_{jt}$ yields the relative demand function:

$$y_{i(m')m'jt} = y_{i(m)mjt} e^{\sigma(\kappa-1)\left(\xi_{i(m')m'j} - \xi_{i(m)mj}\right)} \left(\frac{\widehat{p}_{i(m)mt}}{\widehat{p}_{i(m')m't}}\right)^{\kappa}. \tag{A.1.5}$$

Multiplying both sides by $\widehat{p}_{i(m')m't}$ and integrating over all product categories with positive purchased quantities (i.e., in the $\mathcal{M}_{jt}$ set), we obtain total nominal purchases for consumer $j$:

$$P_{jt}\Omega_{jt} = \int_{\mathcal{M}_{jt}} \widehat{p}_{i(m')m't} y_{i(m')m'jt} dm' \tag{A.1.6}$$

$$= y_{i(m)mjt} \left(\widehat{p}_{i(m)mt}\right)^{\kappa} e^{\sigma \xi_{i(m)mj}(1-\kappa)} \int_{\mathcal{M}_{jt}} \left(e^{-\sigma \xi_{i(m')m'j}} \widehat{p}_{i(m')m't}\right)^{1-\kappa} dm', \tag{A.1.7}$$

where the first equality comes from the fact that $\lambda_{jt} > 0$ (so that the budget constraint is always binding), and on the right-hand side we have used that product category $m$ is infinitesimal to pull functions of $m$ out of the integral. Next, define the price index $P_{jt}$ as in equation (16), that is:

$$P_{jt} \equiv \overline{\Gamma}^{-1} \left(\int_{\mathcal{M}_{jt}} \left(e^{-\sigma \xi_{i(m')m'j}} \widehat{p}_{i(m')m't}\right)^{1-\kappa} dm'\right)^{\frac{1}{1-\kappa}}. \tag{A.1.8}$$

This allows us to write equation (A.1.7) as:

$$P_{jt}\Omega_{jt} = y_{i(m)mjt} \left(\widehat{p}_{i(m)mt}\right)^{\kappa} \overline{\Gamma}^{1-\kappa} e^{-\sigma(\kappa-1)\xi_{i(m)mj}} P_{jt}^{-(\kappa-1)}. \tag{A.1.9}$$

Rearranging, and defining real prices as $p_{i(m)mjt} \equiv \widehat{p}_{i(m)mt}/P_{jt}$, we find the intensive demand function for product $i$ in product category $m$:

$$y_{i(m)mjt}^{d} = \overline{\Gamma}^{\kappa-1} e^{\sigma(\kappa-1)\xi_{i(m)mj}} p_{i(m)mjt}^{-\kappa} \Omega_{jt}. \tag{A.1.10}$$

This shows part 2 of Proposition 1. To show part 1, and thereby confirm our initial conjecture that the individual consumes at most one product from each product category, fix a product category $m$ for which $A_{mjt} \neq \emptyset$. If $|A_{mjt}| = 1$, then the conjecture is trivially true. Else, take two products, $i$ and $i' \in A_{mjt} \backslash \{i\}$. From our initial conjecture, $y_{i'mjt} = 0$. Then, from (A.1.3), we have that:

$$Y_{jt}^{\frac{1}{\kappa}} \left(\overline{\Gamma} e^{\sigma \xi_{imj}} y_{imjt}\right)^{-\frac{1}{\kappa}} \overline{\Gamma} e^{\sigma \xi_{i'mj}} \leq \lambda_{jt} \widehat{p}_{i'mt}, \tag{A.1.11}$$

where, rearranging from equation (A.1.4), we know:

$$Y_{jt}^{\frac{1}{\kappa}} \left(\overline{\Gamma} e^{\sigma \xi_{imj}} y_{imjt}\right)^{-\frac{1}{\kappa}} = \lambda_{jt} \frac{\widehat{p}_{imt}}{\overline{\Gamma} e^{\sigma \xi_{imj}}}. \tag{A.1.12}$$

Back into equation (A.1.11) and taking logs, we obtain:

$$\ln \left(\frac{\widehat{p}_{i'mt}}{\widehat{p}_{imt}}\right) \geq \sigma(\xi_{i'mj} - \xi_{imj}). \tag{A.1.13}$$

This shows equation (14) in Proposition 1. To finish the proof, we need to confirm our initial conjecture that the individual purchases only one product, almost surely, from each product category in which the consumer is aware of at least one firm. To confirm the conjecture, we show that the set of consumers that choose two or more products per product category is measure zero. Suppose, instead, that there is a non-empty subset $\mathcal{J} \subseteq [0,1]$ such that, for all $j \in \mathcal{J}$, we can find some product category $m \in \mathcal{M}_{jt}$ where $y_{i_1,mjt}, \ldots, y_{i_k,mjt} > 0$ for some subset $\{i_1, \ldots, i_k\} \subseteq A_{mjt}$ with $2 \leq k \leq N$. Then, the optimality conditions for each $n = 1, \ldots, k$ are:

$$\lambda_{jt} \widehat{p}_{i_1,mt} = Y_{jt}^{\frac{1}{\kappa}} \left(\sum_{n=1}^{k} \overline{\Gamma} e^{\sigma \xi_{i_n,mj}} y_{i_n,mjt}\right)^{-\frac{1}{\kappa}} \overline{\Gamma} e^{\sigma \xi_{i_1,mj}}, \tag{A.1.14}$$

$$\lambda_{jt} \widehat{p}_{i_2,mt} = Y_{jt}^{\frac{1}{\kappa}} \left(\sum_{n=1}^{k} \overline{\Gamma} e^{\sigma \xi_{i_n,mj}} y_{i_n,mjt}\right)^{-\frac{1}{\kappa}} \overline{\Gamma} e^{\sigma \xi_{i_2,mj}}, \tag{A.1.15}$$

$$\vdots$$

$$\lambda_{jt} \widehat{p}_{i_k,mt} = Y_{jt}^{\frac{1}{\kappa}} \left(\sum_{n=1}^{k} \overline{\Gamma} e^{\sigma \xi_{i_n,mj}} y_{i_n,mjt}\right)^{-\frac{1}{\kappa}} \overline{\Gamma} e^{\sigma \xi_{i_k,mj}}. \tag{A.1.16}$$

Taking the ratio of any two $r, q \in \{2, \ldots, k\}$ yields:

$$\frac{\widehat{p}_{i_r,mt}}{\widehat{p}_{i_q,mt}} = e^{\sigma(\xi_{i_r,mj} - \xi_{i_q,mj})}. \tag{A.1.17}$$

These $k-1$ conditions require that the preference vector $(\xi_{i_1,mj},\ldots,\xi_{i_k,mj})$ of every consumer $j \in \mathcal{J}$ satisfies

$$\xi_{i_r,mj} - \xi_{i_1,mj} = \frac{1}{\sigma}\ln\left(\frac{\widehat{p}_{i_r,mt}}{\widehat{p}_{i_1,mt}}\right), \quad \text{for } r = 2,\ldots,k. \tag{A.1.18}$$

Note that the right-hand side is a constant independent of $j$. That is, the preference vector of every multi-purchasing consumer must lie on an affine subspace of $\mathbb{R}^k$ of dimension one (only the level $\xi_{i_1,mj}$ is free). By Assumption 1, the $\xi$'s are independently drawn from a continuous (Gumbel) distribution, so the joint distribution of $(\xi_{i_1,mj},\ldots,\xi_{i_k,mj})$ is absolutely continuous with respect to Lebesgue measure on $\mathbb{R}^k$. Since any affine subspace of dimension strictly less than $k$ has $k$-dimensional Lebesgue measure zero, the probability that any given consumer's preferences lie on this subspace is zero. It follows that for almost every consumer $j$, the set of categories $m$ where the consumer multi-purchases has Lebesgue measure zero, contradicting our assumption. Therefore, almost every consumer purchases from at most one firm per product category, and equation (A.1.13) holds as a strict inequality for almost every consumer.

## A.2 Proof of Proposition 2

### A.2.1 Preliminaries

By Proposition 1, each consumer $j$ purchases at most one product from its awareness set $A_{mjt}$ in each category $m$ at time $t$. For any two products $i, i' \in A_{mjt}$, the consumer chooses firm $i$ over firm $i'$ if, and only if, condition (14) is satisfied. In this case, the consumer demands a quantity equal to (15) from firm $i$. Therefore, when setting a price $p$, the total demand for firm $i$ equals the sum of the individual demands of all of those consumers who choose the firm.

Formally, firm $i$'s demand schedule is:

$$y_{imt}(p) = \overline{\Gamma}^{\kappa-1} p^{-\kappa} \int_0^1 e^{\sigma(\kappa-1)\xi_{imj}} \Omega_{jt} \mathbb{1}\left\{ \ln\left(\frac{p_{i'mt}}{p}\right) > \sigma\left(\xi_{i'mj} - \xi_{imj}\right) \middle| \forall i' \in A_{mjt} \backslash \{i\} \right\} dj, \tag{A.2.1}$$

where $\mathbb{1}\{\cdot\}$ is an indicator function. Equation (A.2.1) states that a firm's demand is composed of the sum of demands from all those consumers $j \in [0,1]$ that have the firm's product in their choice set. In turn, a firm is in consumer $j$'s choice set if its product is in the consumer's awareness set and, in addition, the consumer decides to purchase it —the condition inside the indicator function.

Therefore, to compute demand, a firm must integrate across consumers whose awareness sets include the firm. This requires integrating using the joint density over all sets $A \in \mathcal{A}_i$, where recall that $\mathcal{A}_i \equiv \{A \in \mathbb{A} | i \in A\}$ is the set of awareness sets that contain firm $i$, and $\mathbb{A} \equiv \mathbf{2}^{\{1,\dots,N\}}$ is the power-set of firm indices. In particular, among that group of consumers, firm $i$ needs to integrate over the subset of those that choose the firm's product over all other products that they are aware of. This requires integrating over consumer idiosyncratic preferences not only for the firm's product, but also for all the other products in each awareness set.

To make progress, let $\Psi_i(a, A, \vec{\xi}(A))$ be the cumulative density function (cdf) faced by firm $i$ at product category age $a$ corresponding to the joint distribution of (i) awareness sets that contain the firm, $A \in \mathcal{A}_i$, and (ii) preference shifters across all products in the awareness set, $\vec{\xi}(A) \equiv [\xi_1, \dots, \xi_i, \dots, \xi_n]^\top$, where $n \equiv |A| \leq N$. Using Bayes' theorem, we can factor this joint density into the marginal density of awareness sets that contain the firm, which we introduced in the main text as $\widehat{f}(a, A)$, and a conditional density of preferences for each given awareness set, denoted $dH_i(a, \vec{\xi}(A)|A)$.[58] That is,

$$d\Psi_i(a, A, \vec{\xi}(A)) = \widehat{f}(a, A) dH_i(a, \vec{\xi}(A)|A). \tag{A.2.2}$$

By Assumption 1, idiosyncratic preferences are independently and identically distributed and, in particular, unrelated to the evolution of awareness. Hence, we can write the conditional density $dH_i(a, \vec{\xi}(A)|A)$ as the product of marginal densities of preference shifters, for each product $i \in A$. As idiosyncratic preferences are Gumbel-distributed with firm-specific mean $\widetilde{\mu}_i$, we have

$$dH_i(a, \vec{\xi}(A)|A) = dG(\xi_i, \widetilde{\mu}_i) \prod_{i' \in A \setminus \{i\}} dG(\xi_{i'}, \widetilde{\mu}_{i'}), \tag{A.2.3}$$

where $G(\cdot, \widetilde{\mu}_i)$ denotes the cdf of the Gumbel distribution with location parameter $\widetilde{\mu}_i$ and scale parameter equal to one. For analytical convenience, let us re-center this distribution by defining $\mu_i \equiv e^{\widetilde{\mu}_i}$ and, henceforth, let us write the firm's problem in terms of $\mu$'s instead of $\widetilde{\mu}$'s. Using the formula for the cdf of a Gumbel distribution, we can write

$$G(\xi, \mu_i) = e^{-e^{-\xi + \ln(\mu_i)}} = e^{-\mu_i e^{-\xi}}, \tag{A.2.4a}$$

$$G(\xi, \mu_{i'}) = e^{-e^{-\xi + \ln(\mu_{i'})}} = e^{-e^{-\xi}} e^{-(\mu_{i'} - 1)e^{-\xi}}, \quad \forall i' \in A \setminus \{i\}. \tag{A.2.4b}$$

[58] Note that we do not index $\widehat{f}$ by the firm's identity $i$ because all firm dependence is already encoded in the awareness set $A$, which the firm is included in by construction. However, $H_i$ does depend explicitly on $i$ as, conditional on the awareness set, the consumer has heterogeneous preferences over each firm.

### A.2.2 Demand function in the asymmetric case

To make progress toward writing out the total demand of the firm, let us conjecture (a claim that we verify in Proposition 5) that both real income and the price index are identical across consumers, $\Omega_j = \boldsymbol{\Omega}$ and $P_j = \boldsymbol{P}$.[59] This implies that real prices are constant across consumers as well. Let $\vec{p} \in \mathbb{R}^N$ and $\vec{\mu} \in \mathbb{R}^N$ denote the vector of prices and targeting choices, including firm $i$'s. Let us pack the objects that the firm takes as given into $\vec{s} \equiv (\vec{p}\backslash\{p_i\}, \vec{\mu}\backslash\{\mu_i\}, \boldsymbol{\Omega}, \widehat{f})$. Notice that we include the density of awareness sets $\widehat{f} : \mathcal{A}_i \to [0, 1]$ generically among these objects. This measure is a function of age $a$ and of $\vec{\theta} \in \mathbb{R}^N$, the set of contact rates chosen by all the different firms, as we describe below.

Using equation (A.2.1), the demand for firm $i$ when the firm sets price $p \equiv p_i$ is

$$y_i(p; \vec{s}) = \overline{\Gamma}^{\kappa-1} p^{-\kappa} \boldsymbol{\Omega} \textstyle\sum_{A \in \mathcal{A}_i} \widehat{f}(A) \underbrace{\displaystyle\int_{\mathbb{R}^n} e^{\sigma(\kappa-1)\xi_i} \mathbb{1}\left\{ \ln\left(\frac{p_{i'}}{p}\right) > \sigma(\xi_{i'} - \xi_i) \middle| \forall i' \in A\backslash\{i\} \right\} dH_i(\vec{\xi}(A)|A)}_{(*)}, \tag{A.2.5}$$

where by the symbol $\int_{\mathbb{R}^n}$ we mean $\int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty}$, with $n \equiv |A|$ such successive integrals. Using independence (equation (A.2.3)), we can write the term $(*)$ as follows:

$$(*) = \int_{\mathbb{R}^n} e^{\sigma(\kappa-1)\xi_i} \mathbb{1}\left\{ \frac{1}{\sigma} \ln\left(\frac{p_{i'}}{p}\right) + \xi_i > \xi_{i'} \middle| \forall i' \in A\backslash\{i\} \right\} dG(\xi_i, \mu_i) \prod_{i' \in A\backslash\{i\}} dG(\xi_{i'}, \mu_{i'}). \tag{A.2.6}$$

Using Fubini's Theorem, we can separate out the $n$-tuple integral into two iterated integrals:

$$(*) = \int_{-\infty}^{+\infty} e^{\sigma(\kappa-1)\xi_i} \left[ \int_{\mathbb{R}^{n-1}} \mathbb{1}\left\{ \tfrac{1}{\sigma} \ln\left(\tfrac{p_{i'}}{p}\right) + \xi_i > \xi_{i'} \middle| \forall i' \in A\backslash\{i\} \right\} \textstyle\prod_{i' \in A\backslash\{i\}} dG(\xi_{i'}, \mu_{i'}) \right] dG(\xi_i, \mu_i). \tag{A.2.7}$$

Inside the square brackets, we must compute the cdf of the joint distribution of $\vec{\xi}\backslash\{\xi_i\}$ at the point $\frac{1}{\sigma} \ln\left(\frac{p_{i'}}{p}\right) + \xi_i$. For this, we can use the pdf's associated to (A.2.4a)-(A.2.4b),

[59] For the rest of the exposition, we drop time subscripts $t$ because we focus on the static problem of the firm, who takes aggregates equilibrium variables as given. Likewise, to alleviate notation, we drop the age $a$ dependence from all of our expressions.

and write

$$(*) = \int_{-\infty}^{+\infty} e^{\sigma(\kappa-1)\xi_i} \left[ \prod_{i' \in A \setminus \{i\}} \exp\left( -e^{-\left(\frac{1}{\sigma}\ln\left(\frac{p_{i'}}{p}\right)+\xi_i\right)+\ln(\mu_{i'})} \right) \right] e^{-\xi_i} e^{-e^{-\xi_i}} \mu_i e^{-(\mu_i-1)e^{-\xi_i}} d\xi_i$$
$$= \int_{-\infty}^{+\infty} e^{\sigma(\kappa-1)\xi_i} \exp\left( -\sum_{i' \in A \setminus \{i\}} e^{-\left(\frac{1}{\sigma}\ln\left(\frac{p_{i'}}{p}\right)+\xi_i\right)+\ln(\mu_{i'})} \right) e^{-\xi_i} e^{-e^{-\xi_i}} \mu_i e^{-(\mu_i-1)e^{-\xi_i}} d\xi_i. \tag{A.2.8}$$

Next, we can factor the term $e^{-e^{-\xi_i}}$ in (A.2.8) as follows:

$$\begin{aligned}
\exp(-e^{-\xi_i}) &= \exp\left( -e^{-\left(\frac{1}{\sigma}\ln\left(\frac{p}{p}\right)+\xi_i\right)+\ln(\mu_i)-\ln(\mu_i)} \right) \\
&= \exp\left( -\frac{1}{\mu_i} e^{-\left(\frac{1}{\sigma}\ln\left(\frac{p}{p}\right)+\xi_i\right)+\ln(\mu_i)} \right) \\
&= \exp\left( -e^{-\left(\frac{1}{\sigma}\ln\left(\frac{p}{p}\right)+\xi_i\right)+\ln(\mu_i)} \right) \exp\left( \left(1-\frac{1}{\mu_i}\right) e^{-\left(\frac{1}{\sigma}\ln\left(\frac{p}{p}\right)+\xi_i\right)+\ln(\mu_i)} \right) \\
&= \exp\left( -e^{-\left(\frac{1}{\sigma}\ln\left(\frac{p}{p}\right)+\xi_i\right)+\ln(\mu_i)} \right) \exp\left( (\mu_i-1)e^{-\xi_i} \right).
\end{aligned} \tag{A.2.9}$$

Plugging this back into (A.2.8) yields:

$$(*) = \int_{-\infty}^{+\infty} \mu_i e^{(\sigma(\kappa-1)-1)\xi_i} \exp\left( -\sum_{i' \in A} e^{-\left(\frac{1}{\sigma}\ln\left(\frac{p_{i'}}{p}\right)+\xi_i\right)+\ln(\mu_{i'})} \right) d\xi_i. \tag{A.2.10}$$

The sum in the exponent is now over all firms in the awareness set (including firm $i$). Simplifying by factoring exponential terms together, we get:

$$(*) = \int_{-\infty}^{+\infty} \mu_i e^{(\sigma(\kappa-1)-1)\xi_i} \exp\left( -e^{-\xi_i} \left( \mu_i + \sum_{i' \in A \setminus \{i\}} \mu_{i'} \left(\frac{p_{i'}}{p}\right)^{-\frac{1}{\sigma}} \right) \right) d\xi_i. \tag{A.2.11}$$

Next, we can use the fact that $\int_{-\infty}^{+\infty} e^{-a_1 x} e^{-a_2 e^{-x}} dx = a_2^{-a_1}\Gamma(a_1)$, for any two numbers $a_1, a_2 > 0$, where $\Gamma(.)$ is the Gamma function. Using this fact into the last equation then gives us:[60]

$$(*) = \overline{\Gamma}^{1-\kappa} \mu_i^{\sigma(\kappa-1)} \left( 1 + \sum_{i' \in A \setminus \{i\}} \frac{\mu_{i'}}{\mu_i} \left(\frac{p_{i'}}{p}\right)^{-\frac{1}{\sigma}} \right)^{\sigma(\kappa-1)-1}, \tag{A.2.12}$$

[60]Notice that the integral converges because we assumed that $\sigma(\kappa-1) < 1$.

where $\overline{\Gamma} \equiv \Gamma\big(1-\sigma(\kappa-1)\big)^{\frac{1}{1-\kappa}}$. Back into (A.2.5), we obtain the demand for firm $i$:

$$y_i(p,\vec{s}) = \mu_i^{\sigma(\kappa-1)} p^{-\kappa} \mathbf{\Omega} \underbrace{\sum_{A\in\mathcal{A}_i} \widehat{f}(A) \left(1 + \sum_{i'\in A\setminus\{i\}} \frac{\mu_{i'}}{\mu_i} \left(\frac{p_{i'}}{p}\right)^{-\frac{1}{\sigma}}\right)^{\sigma(\kappa-1)-1}}_{\equiv q_i(p,\vec{s})}. \tag{A.2.13}$$

Computing this demand function requires an expression for $\widehat{f}(A)$, the measure of awareness sets that contain the firm. As explained before, this measure changes over age $a$ depending on the time-zero advertising choices of all firms, $\vec{\theta} \equiv \{\theta_{i'}\}_{i'=1}^{N}$, as these determine the evolution of awareness in the product market. While we solve for the symmetric Markov Perfect Equilibrium, we allow for unilateral, off-equilibrium deviations in advertising and pricing choices. First, we show the solution in the symmetric equilibrium. Then, we derive the case for the unilaterally deviating firm.

### A.2.3 Awareness dynamics in the symmetric equilibrium

In the symmetric case in which $\theta$ is common across all $N$ firms, $\widehat{f}(A)$ can be readily obtained. First, by Assumption 2, network connections form independently of existing connections. Since all firms have the same arrival rate of forming ($\theta$) and losing ($\zeta$) connections, then $\widehat{f}(A) = \widehat{f}(A')$ for all sets $A, A' \in \mathcal{A}_i$ with $|A| = |A'|$. This means, in particular, that the probability that firm $i$ is in an awareness set $A$ of size $n \equiv |A|$ comes from urn-like problem *without* replacement: the same firm cannot be drawn again after it has been first introduced into the awareness set, and the ordering in which draws occur is irrelevant for the computation of the probability that a certain firm is drawn. This probability is described by a Hypergeometric distribution: the probability of a "success" event (i.e., firm $i = 1, \ldots, N$ is drawn once and without replacement into a subset of firms of size $n \leq N$) equals $\frac{\binom{n}{1}\binom{N-n}{0}}{\binom{N}{1}} = \frac{n}{N}$. Thus, we can write $\widehat{f}(A)$, the density of awareness sets, over all sets $A \in \mathcal{A}_i$ that contain firm $i$, as the product of the probability of drawing firm $i$, equal to $\frac{n}{N}$, times the proportion of consumers aware of $n \equiv |A|$ products. The latter was called $f_n$ in the main text, and it has law of motion given in equation (4). Therefore, we have:

$$\widehat{f}(A) = \frac{n}{N} f_n. \tag{A.2.14}$$

This result shows that, in the symmetric equilibrium, the probability that a firm can be found in a given awareness set $A$ is only a function of the *size* of the set, $n = |A|$, but not on the *composition* of this set. Conveniently, then, the dynamics of $\widehat{f}(A)$ in the symmetric case are described by $\mathcal{Q}(\theta)$, the generator matrix that was introduced in equation (4).

Replacing $\widehat{f}(A)$ by $\frac{n}{N}f_n$ inside of equation (A.2.13) allows us to have a sum over all possible awareness set sizes rather than over the sets themselves. With symmetry in competitor prices and targeting, i.e., $p_{i'} = p_{-i}$ and $\mu_{i'} = \mu_{-i}$ for all $i' \neq i$, we obtain:

$$y(p, \vec{s}) = \mu^{\sigma(\kappa-1)} p^{-\kappa} \frac{\mathbf{\Omega}}{N} \sum_{n=0}^{N} f_n \underbrace{n \left(1 + (n-1)\frac{\mu_{-i}}{\mu}\left(\frac{p_{-i}}{p}\right)^{-\frac{1}{\sigma}}\right)^{\sigma(\kappa-1)-1}}_{\equiv g(n)}. \tag{A.2.15}$$

To arrive at the expression in Proposition 2, notice

$$\sum_{n=1}^{N} f_n g(n) = (1 - f_0)\mathbb{E}_a[g(\widehat{n})], \tag{A.2.16}$$

where $\mathbb{E}_a[g(\widehat{n})]$ is the expectation (as of time $a$) of $g(\widehat{n})$, with $\widehat{n} \equiv n | n \geq 1$ (recall definition (20)). Putting things together,

$$y(p, \vec{s}) = (1 - f_0)\mu^{\sigma(\kappa-1)} p^{-\kappa} \frac{\mathbf{\Omega}}{N} \mathbb{E}_a \left[ \widehat{n} \left(1 + (\widehat{n}-1)\frac{\mu_{-i}}{\mu}\left(\frac{p_{-i}}{p}\right)^{-\frac{1}{\sigma}}\right)^{\sigma(\kappa-1)-1} \right]. \tag{A.2.17}$$

This is the result we state in Proposition 2.

### A.2.4 Awareness dynamics with unilateral deviations

Suppose now that a single firm, called $i \in \{1, \dots, N\}$, chooses $\theta_i$ and $\mu_{0,i}$ that are potentially different from $\theta$ and $\mu_0$, the (symmetric) equilibrium choices of the $N-1$ competitors. Given that only a single firm is considering an off-equilibrium deviation, we show that a sufficient statistic for two different types of firms is to maintain the joint distribution of the counts of each type.

Denote the vector $f(\theta_i, \theta) \in \mathbb{R}^{2N}$ where the first $N$ indices are the counts of firms when the firm $i$ is *not* in the awareness set (i.e., counts of 0 to $N-1$), and the second part of the vector are the counts when the firm $i$ *is* in the awareness sets, counting the total number of firms including the firm of interest. That is, $f_{N+n-1}$ is measure of awareness sets of size $n$ which include firm $i$, $f_n$ is the measure of awareness sets of size $n$ that do not include firm $i$, and $f_0$ is the measure of awareness sets with no firms.

How does the count of firms in $f(\theta_i, \theta) \in \mathbb{R}^{2N}$ evolve? Following our baseline from

Assumption 2, with all $N$ firms each having a $\frac{\theta}{N}$ probability of being met, the total rate of becoming aware of the first firm is $N\frac{\theta}{N} = \theta$. For any consumers with one firm in their awareness set, the rate of meeting the second firm is the sum of the other $N-1$ arrival rates, i.e., $(N-1)\frac{\theta}{N}$. Given this, if $N-1$ firms each have an arrival rate of $\frac{\theta}{N}$ and the firm of interest has an arrival rate of $\frac{\theta_i}{N}$, then for example:

- Conditional on having no firms in the awareness set, a consumer has a $\frac{\theta_i}{N}$ rate of meeting the deviating firm, and a $(N-1)\frac{\theta}{N}$ of meeting one of the other $N-1$ symmetric firms.
- Conditional on having one firm (but not the deviating firm) in the awareness set, there is a rate $(N-2)\frac{\theta}{N}$ of meeting any of the other $N-2$ firms, and a rate $\frac{\theta_i}{N}$ of meeting the deviating firm.
- Conditional on having the deviating firm and one of the other $N-1$ firms in the awareness set, then there is a rate $(N-2)\frac{\theta}{N}$ of becoming aware of one of the other $N-2$ firms.

And so on. Moreover, recall that there is an intensity $\zeta/N$ of losing each individual firm in the awareness set. Adapting Assumption 2 to the asymmetric case, then, requires defining a new generator matrix $\widetilde{\mathcal{Q}}(\theta_i, \theta)$ of size $2N \times 2N$, so that

$$\partial_a f(\theta_i, \theta) = f(\theta_i, \theta) \cdot \widetilde{\mathcal{Q}}(\theta_i, \theta) \tag{A.2.18}$$

where $\widetilde{\mathcal{Q}}(\theta_i, \theta)$ has the following block structure:

$$\widetilde{\mathcal{Q}}(\theta_i, \theta) \equiv \frac{1}{N} \left[ \begin{array}{c|c} \widetilde{\mathcal{Q}}^{11} & \widetilde{\mathcal{Q}}^{12} \\ \hline \widetilde{\mathcal{Q}}^{21} & \widetilde{\mathcal{Q}}^{22} \end{array} \right] \tag{A.2.19}$$

where the sub-matrices are $N \times N$ each. Of these, $\widetilde{\mathcal{Q}}^{12}$ and $\widetilde{\mathcal{Q}}^{21}$ are the diagonal matrices,

$$\widetilde{\mathcal{Q}}^{12} = \operatorname{diag}(\theta_i, \ldots, \theta_i),$$
$$\widetilde{\mathcal{Q}}^{21} = \operatorname{diag}(\zeta, \ldots, \zeta),$$

and $\widetilde{\mathcal{Q}}^{11}$ and $\widetilde{\mathcal{Q}}^{22}$ are tridiagonal matrices:

$$\widetilde{\mathcal{Q}}^{11} = \begin{bmatrix} -(N-1)\theta-\theta_i & (N-1)\theta & 0 & 0 & \cdots & 0 \\ \zeta & -\zeta-(N-2)\theta-\theta_i & (N-2)\theta & 0 & \cdots & 0 \\ 0 & 2\zeta & -2\zeta-(N-3)\theta-\theta_i & (N-3)\theta & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & -(N-2)\zeta-\theta-\theta_i & \theta \\ 0 & 0 & 0 & \cdots & (N-1)\zeta & -(N-1)\zeta-\theta_i \end{bmatrix},$$

$$\widetilde{\mathcal{Q}}^{22} = \begin{bmatrix} -(N-1)\theta-\zeta & (N-1)\theta & 0 & 0 & \cdots & 0 \\ \zeta & -\zeta-(N-2)\theta-\zeta & (N-2)\theta & 0 & \cdots & 0 \\ 0 & 2\zeta & -2\zeta-(N-3)\theta-\zeta & (N-3)\theta & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & -(N-2)\zeta-\theta-\zeta & \theta \\ 0 & 0 & 0 & \cdots & (N-1)\zeta & -(N-1)\zeta-\zeta \end{bmatrix}.$$

Given this, we can proceed to compute demand via equation (A.2.13). We relegate the analysis of the implications for firm profits to Appendix A.3, where we discuss optimal pricing under unilateral deviations.

## A.3 Proof of Proposition 3

### A.3.1 General solution

A firm $i$ takes $\mathbf{mc}_t$, $\mathbf{\Omega}_t$ and competitor strategies $\{p_{i'}\}_{i'\neq i}$ and $\{\mu_{i'}\}_{i'\neq i}$ as given. The pricing problem of this firm reads

$$\max_p \left\{ \left(p-\mathbf{mc}_t\right) \underbrace{\mu^{\sigma(\kappa-1)} p^{-\kappa} \mathbf{\Omega}_t \underbrace{\sum_{A\in\mathcal{A}_i} \widehat{f}(a,A) \left(1+\sum_{i'\in A\setminus\{i\}} \frac{\mu_{i'}}{\mu}\left(\frac{p_{i'}}{p}\right)^{-\frac{1}{\sigma}}\right)^{\sigma(\kappa-1)-1}}_{=\,q_i(a,\vec{p})}}_{=\,y_{it}(a,\vec{p})} \right\}, \tag{A.3.1}$$

where $\mu$ is firm $i$'s targeting at age $a$. Assuming existence of a solution, the first-order condition can be written as follows:

$$1-\left(1-\frac{\mathbf{mc}_t}{p}\right)\underbrace{\left(-\frac{\partial y_{it}(a,\vec{p})}{\partial p}\frac{p}{y_{it}(a,\vec{p})}\right)}_{\equiv\,\mathcal{E}_i(a,\vec{p})}=0, \tag{A.3.2}$$

Defining $\Lambda \equiv \frac{p}{\mathbf{mc}}$ and solving in (A.3.2) gives immediately that $\Lambda_i(a, \vec{p}) = \frac{\mathcal{E}_i(a,\vec{p})}{\mathcal{E}_i(a,\vec{p})-1}$. By (17), the price-elasticity of demand equals

$$\mathcal{E}_i(a, \vec{p}) = \kappa + \left( -\frac{\partial q_i(a, \vec{p})}{\partial p} \frac{p}{q_i(a, \vec{p})} \right), \tag{A.3.3}$$

where, using definition (18):

$$\frac{\partial q_i(a,\vec{p})}{\partial p}\frac{p}{q_i(a,\vec{p})} = -\frac{1-\sigma(\kappa-1)}{\sigma} \frac{\sum_{A\in\mathcal{A}_i} \widehat{f}(a, A) \left[ \left( 1 + \sum_{i'\in A\setminus\{i\}} \frac{\mu_{i'}}{\mu} \left(\frac{p_{i'}}{p}\right)^{-\frac{1}{\sigma}} \right)^{\sigma(\kappa-1)-2} \sum_{i'\in A\setminus\{i\}} \frac{\mu_{i'}}{\mu} \left(\frac{p_{i'}}{p}\right)^{-\frac{1}{\sigma}} \right]}{\sum_{A\in\mathcal{A}_i} \widehat{f}(a, A) \left( 1 + \sum_{i'\in A\setminus\{i\}} \frac{\mu_{i'}}{\mu} \left(\frac{p_{i'}}{p}\right)^{-\frac{1}{\sigma}} \right)^{\sigma(\kappa-1)-1}}. \tag{A.3.4}$$

As in Appendix A.2, we analyze the symmetric and asymmetric cases separately.

### A.3.2 Markups in the symmetric equilibrium

As argued in Appendix A.2, in the symmetric case in which $\theta$ is common across all $N$ firms, we have that $\widehat{f}(a, A) = \frac{n}{N} f_n(a)$. With symmetry in competitor prices and targeting, i.e., $p_{i'} = p_{-i}$ and $\mu_{i'} = \mu_{-i}$, equation (A.3.4) reads

$$\frac{\partial q_i(a,\vec{p})}{\partial p}\frac{p}{q_i(a,\vec{p})} = -\frac{1-\sigma(\kappa-1)}{\sigma} \frac{\sum_{n=1}^{N} n f_n(a) \left[ \left( 1 + (n-1)\frac{\mu_{-i}}{\mu} \left(\frac{p_{-i}}{p}\right)^{-\frac{1}{\sigma}} \right)^{\sigma(\kappa-1)-2} (n-1)\frac{\mu_{-i}}{\mu} \left(\frac{p_{-i}}{p}\right)^{-\frac{1}{\sigma}} \right]}{\sum_{n=1}^{N} n f_n(a) \left( 1 + (n-1)\frac{\mu_{-i}}{\mu} \left(\frac{p_{-i}}{p}\right)^{-\frac{1}{\sigma}} \right)^{\sigma(\kappa-1)-1}}. \tag{A.3.5}$$

Further specializing this to a symmetric equilibrium with $p_i = p$ and $\mu_i = \mu$ for all $i = 1, \ldots, N$, we readily obtain

$$\frac{\partial q(a)}{\partial p}\frac{p}{q(a)} = -\frac{1-\sigma(\kappa-1)}{\sigma} \left[ 1 - \frac{\mathbb{E}_a\left[\widehat{n}^{\sigma(\kappa-1)-1}\right]}{\mathbb{E}_a\left[\widehat{n}^{\sigma(\kappa-1)}\right]} \right], \tag{A.3.6}$$

Thus, the markup is $\Lambda(a) = \frac{\mathcal{E}(a)}{\mathcal{E}(a)-1}$, with $\mathcal{E}(a) = \kappa + \frac{1-\sigma(\kappa-1)}{\sigma} \left[ 1 - \frac{\mathbb{E}_a\left[\widehat{n}^{\sigma(\kappa-1)-1}\right]}{\mathbb{E}_a\left[\widehat{n}^{\sigma(\kappa-1)}\right]} \right]$. This is the formula that we report in the main text.

### A.3.3 Markups with unilateral deviations

When considering an off-equilibrium $(\mu_i, \theta_i)$ strategy for a single deviating firm, Appendix A.2 argued that $\widehat{f}(A)$, the measure of awareness sets that include the firm in question at age $a$, can still be computed with a counting process.[61] In this case, as we explained above, we must introduce a vector $f(\theta_i, \theta) \in \mathbb{R}^{2N}$ of firm counts, whose law of motion is given by equation (A.2.18).

In addition to the single deviating firm $i$, the pricing game now needs to take into account that any *other* firm may choose to deviate in price. Given that the $N-1$ other are symmetric, we can consider some $j \neq i$ as the deviating firm in prices among the symmetric firms. This leads us to consider 3 prices before applying symmetry in our equilibrium conditions: (i) the price of the firm who deviated from the equilibrium $\mu$ and $\theta$, denoted by $p_i$; (ii) the price for the firms $N-2$ firms that remain symmetric, denoted by $p$; and (iii) the unilateral deviation of firm $j$, who considers choosing $p_j \neq p$.

We start by introducing new notation which will help us find the appropriate awareness sets for the firms making a unilateral deviation:

- $\mathcal{A}_{i\cap j} \equiv \{A \in \mathbb{A} \mid i \in A \text{ and } j \in A\}$, with $p_j$, $p_i$, and $p$ as the relevant prices.
- $\mathcal{A}_{i\setminus j} \equiv \{A \in \mathbb{A} \mid i \in A \text{ and } j \notin A\}$, with $p_i$ and $p$ as the relevant prices.
- $\mathcal{A}_{j\setminus i} \equiv \{A \in \mathbb{A} \mid i \notin A \text{ and } j \in A\}$, with $p_j$ and $p$ as the relevant prices.

Given the awareness count process in equation (A.2.18), we can map the measures of these sets from $\widehat{f}(A)$ to indexing a vector in the $f \in \mathbb{R}^{2N}$ awareness measure. The $f_0$ maps to the no-awareness state, $f_{N-1}$ is the measure of customers with only $i$ in their awareness set, and $f_{2N-1}$ is full awareness including firm $i$.[62] Using this, we can sum up payoff-equivalent sets of particular sizes by:[63]

$$\sum_{A \in \mathcal{A}_{i\cap j}, |A|=n} \widehat{f}(A) = \frac{n-1}{N-1} f_{N-1+n}, \qquad \text{for } n = 2, \ldots, N \tag{A.3.7a}$$

$$\sum_{A \in \mathcal{A}_{j\setminus i}, |A|=n} \widehat{f}(A) = \frac{n}{N-1} f_n, \qquad \text{for } n = 1, \ldots, N-1 \tag{A.3.7b}$$

[61] To alleviate notation, in this section we drop the $a$ dependence from all densities and elasticities.

[62] Notice that the symmetric firm $j$ that is considering a unilateral deviation in the price game cannot effect the awareness evolution, and hence we do not need to manage 3 types of firms here.

[63] Note that we have not implemented $\sum_{A \in \mathcal{A}_{i\setminus j}}$, but have added in $\sum_{A \in \mathcal{A}_i}$. The reason is that after applying equilibrium conditions and looking for a symmetric equilibrium for $i' \neq i$, firm $i$ does not consider the individual deviation of firm $j$. This comes from the notion of a Nash equilibrium: deviations only need to be considered unilaterally, taking the equilibrium as given.

$$\sum_{A\in\mathcal{A}_i,\,|A|=n}\widehat{f}(A)=f_{N-1+n},\qquad \text{for } n=1,\ldots,N \tag{A.3.7c}$$

By equation (A.3.4), the optimality condition can be written as

$$\sigma\frac{\mathcal{E}_i(\vec{p})-\kappa}{1-\sigma(\kappa-1)}=\frac{\sum_{A\in\mathcal{A}_i}\widehat{f}(A)\left[\left(1+\sum_{i'\in A\setminus\{i\}}\frac{\mu_{i'}}{\mu}\left(\frac{p_{i'}}{p}\right)^{-\frac{1}{\sigma}}\right)^{\sigma(\kappa-1)-2}\sum_{i'\in A\setminus\{i\}}\frac{\mu_{i'}}{\mu}\left(\frac{p_{i'}}{p}\right)^{-\frac{1}{\sigma}}\right]}{\sum_{A\in\mathcal{A}_i}\widehat{f}(A)\left(1+\sum_{i'\in A\setminus\{i\}}\frac{\mu_{i'}}{\mu}\left(\frac{p_{i'}}{p}\right)^{-\frac{1}{\sigma}}\right)^{\sigma(\kappa-1)-1}} \tag{A.3.8}$$

We can now write (A.3.8) for each one of the 3 different type of firms that we have identified (firm $i$, firm $j$, and the symmetric firms). Notice that the right-hand side of A.3.8 depends only on ratios of prices and targeting. Therefore, we can define $\widetilde{p}\equiv p_i/p$, $\widetilde{\mu}\equiv\mu_i/\mu$, and $\widetilde{x}\equiv\widetilde{\mu}\widetilde{p}^{-\frac{1}{\sigma}}$. In addition, define $\widetilde{p}_j\equiv p_j/p$, and note that $p_j/p_i=\widetilde{p}_j/\widetilde{p}$. In the symmetric equilibrium, $\widetilde{p}=\widetilde{\mu}=\widetilde{x}=1$, but this we can impose only *after* we derive the optimality conditions.

Firm $i$, the firm that unilaterally deviates in $\theta$ and $\mu_0$, needs to take into account that sets with and without firm $j$ would have different price ratios, and hence separately sums densities up over $\mathcal{A}_{i\cap j}$ and $\mathcal{A}_{i\setminus j}$. Specializing the general calculation of the price elasticity in equation (A.3.8) and substituting in the defined ratios above, the price-elasticity depends only on $\widetilde{x}$ and $\widetilde{p}_j$, and it satisfies:[64]

$$\begin{aligned}
&\sigma\frac{\mathcal{E}_i(\widetilde{x},\widetilde{p}_j)-\kappa}{1-\sigma(\kappa-1)}\\
&=\frac{\sum_{A\in\mathcal{A}_{i\cap j}}\widehat{f}(A)\left(1+\left(|A|-2+\widetilde{p}_j^{-\frac{1}{\sigma}}\right)\frac{1}{\widetilde{x}}\right)^{\sigma(\kappa-1)-2}\left(|A|-2+\widetilde{p}_j^{-\frac{1}{\sigma}}\right)\frac{1}{\widetilde{x}}+\sum_{A\in\mathcal{A}_{i\setminus j}}\widehat{f}(A)\left(1+(|A|-1)\frac{1}{\widetilde{x}}\right)^{\sigma(\kappa-1)-2}(|A|-1)\frac{1}{\widetilde{x}}}{\sum_{A\in\mathcal{A}_{i\cap j}}\widehat{f}(A)\left(1+\left(|A|-2+\widetilde{p}_j^{-\frac{1}{\sigma}}\right)\frac{1}{\widetilde{x}}\right)^{\sigma(\kappa-1)-1}+\sum_{A\in\mathcal{A}_{i\setminus j}}\widehat{f}(A)\left(1+(|A|-1)\frac{1}{\widetilde{x}}\right)^{\sigma(\kappa-1)-1}}\\
&=\frac{\sum_{n=2}^{N}\frac{n-1}{N-1}f_{N-1+n}\left(1+\left(n-2+\widetilde{p}_j^{-\frac{1}{\sigma}}\right)\frac{1}{\widetilde{x}}\right)^{\sigma(\kappa-1)-2}\left(n-2+\widetilde{p}_j^{-\frac{1}{\sigma}}\right)\frac{1}{\widetilde{x}}+\sum_{n=1}^{N-1}\frac{n}{N-1}f_n\left(1+\frac{n-1}{\widetilde{x}}\right)^{\sigma(\kappa-1)-2}\frac{n-1}{\widetilde{x}}}{\sum_{n=2}^{N}\frac{n-1}{N-1}f_{N-1+n}\left(1+\left(n-2+\widetilde{p}_j^{-\frac{1}{\sigma}}\right)\frac{1}{\widetilde{x}}\right)^{\sigma(\kappa-1)-1}+\sum_{n=1}^{N-1}\frac{n}{N-1}f_n\left(1+\frac{n-1}{\widetilde{x}}\right)^{\sigma(\kappa-1)-1}}.
\end{aligned}$$

For firm $j$, a firm who follows the symmetric $(\theta,\mu_0)$ strategy but who is considering a one-off, unilateral deviation in the price, we have:

[64] To alleviate notation, we drop the dependence on age $a$ from these equations.

$$\sigma\frac{\mathcal{E}_j(\widetilde{x},\widetilde{p}_j)-\kappa}{1-\sigma(\kappa-1)}$$
$$=\frac{\sum_{A\in\mathcal{A}_{i\cap j}}\widehat{f}(A)\left(1+\left(|A|-2+\widetilde{x}\right)\widetilde{p}_j^{\frac{1}{\sigma}}\right)^{\sigma(\kappa-1)-2}\left(|A|-2+\widetilde{x}\right)\widetilde{p}_j^{\frac{1}{\sigma}}+\sum_{A\in\mathcal{A}_{j\setminus i}}\widehat{f}(A)\left(1+(|A|-1)\widetilde{p}_j^{\frac{1}{\sigma}}\right)^{\sigma(\kappa-1)-2}(|A|-1)\widetilde{p}_j^{\frac{1}{\sigma}}}{\sum_{A\in\mathcal{A}_{i\cap j}}\widehat{f}(A)\left(1+\left(|A|-2+\widetilde{x}\right)\widetilde{p}_j^{\frac{1}{\sigma}}\right)^{\sigma(\kappa-1)-1}+\sum_{A\in\mathcal{A}_{j\setminus i}}\widehat{f}(A)\left(1+(|A|-1)\widetilde{p}_j^{\frac{1}{\sigma}}\right)^{\sigma(\kappa-1)-1}}$$
$$=\frac{\sum_{n=2}^{N}\frac{n-1}{N-1}f_{N-1+n}\left(1+(n-2+\widetilde{x})\widetilde{p}_j^{\frac{1}{\sigma}}\right)^{\sigma(\kappa-1)-2}(n-2+\widetilde{x})\widetilde{p}_j^{\frac{1}{\sigma}}+\sum_{n=1}^{N-1}\frac{n}{N-1}f_n\left(1+(n-1)\widetilde{p}_j^{\frac{1}{\sigma}}\right)^{\sigma(\kappa-1)-2}(n-1)\widetilde{p}_j^{\frac{1}{\sigma}}}{\sum_{n=2}^{N}\frac{n-1}{N-1}f_{N-1+n}\left(1+(n-2+\widetilde{x})\widetilde{p}_j^{\frac{1}{\sigma}}\right)^{\sigma(\kappa-1)-1}+\sum_{n=1}^{N-1}\frac{n}{N-1}f_n\left(1+(n-1)\widetilde{p}_j^{\frac{1}{\sigma}}\right)^{\sigma(\kappa-1)-1}}.$$

At this point, since we are ultimately looking for a symmetric equilibrium, we can solve for the example where $p_j = p$ for all $j \neq i$. In that case, $\widetilde{p}_j = 1$ and we can denote the price elasticity for all $i' \neq i$ as simply $\mathcal{E}(\widetilde{x}) \equiv \mathcal{E}_j(\widetilde{x}, 1)$. Furthermore, given that $p_j = p$ the awareness sets simplify and firm $i$ no longer needs to distinguish between sets with firm $j$ and those without. Substituting into $\mathcal{E}_i(\widetilde{x}, 1) \equiv \mathcal{E}_i(\widetilde{x})$ and $\mathcal{E}_j(\widetilde{x}, 1) \equiv \mathcal{E}(\widetilde{x})$ above, we can simplify the equilibrium elasticities by first applying the symmetry to combine the $\mathcal{A}_{i\cap j}$ and $\mathcal{A}_{i\setminus j}$ sets, and then by using the mapping to the count distribution. This gives

$$\sigma\frac{\mathcal{E}_i(\widetilde{x})-\kappa}{1-\sigma(\kappa-1)}=\frac{\sum_{n=1}^{N}f_{N-1+n}\left(1+\frac{n-1}{\widetilde{x}}\right)^{\sigma(\kappa-1)-2}\frac{n-1}{\widetilde{x}}}{\sum_{n=1}^{N}f_{N-1+n}\left(1+\frac{n-1}{\widetilde{x}}\right)^{\sigma(\kappa-1)-1}}. \tag{A.3.9}$$

With the same transformation for the elasticities of the symmetric firms, we obtain:

$$\sigma\frac{\mathcal{E}(\widetilde{x})-\kappa}{1-\sigma(\kappa-1)}=\frac{\sum_{n=2}^{N}\frac{n-1}{N-1}f_{N-1+n}\,(n-1+\widetilde{x})^{\sigma(\kappa-1)-2}\,(n-2+\widetilde{x})+\sum_{n=1}^{N-1}\frac{n-1}{N-1}f_n\,n^{\sigma(\kappa-1)-1}}{\sum_{n=2}^{N}\frac{n-1}{N-1}f_{N-1+n}\,(n-1+\widetilde{x})^{\sigma(\kappa-1)-1}+\sum_{n=1}^{N-1}\frac{n}{N-1}f_n\,n^{\sigma(\kappa-1)-1}}. \tag{A.3.10}$$

Finally, given these elasticities, we can set up a system of equations to solve for the Bertrand-Nash equilibrium. The two variables to solve for are $\widetilde{x}$ and the associated markups. Define the "numeraire" markup as $\Lambda \equiv p/\mathbf{mc}$, and then define the markup of firm $i$ as $\Lambda_i \equiv p_i/\mathbf{mc} = \widetilde{p}\Lambda$. With this, the Bertrand-Nash equilibrium is the solution $(\widetilde{x}, \Lambda)$ to the system of equations

$$0 = 1-(1-\Lambda^{-1})\mathcal{E}(\widetilde{x}) \tag{A.3.11}$$

$$
\begin{aligned}
0 &= 1 - (1 - \Lambda_i^{-1})\mathcal{E}_i(\widetilde{x}) \\
&= 1 - \left(1 - \Lambda^{-1}\left(\frac{\widetilde{x}}{\widetilde{\mu}}\right)^{\sigma}\right)\mathcal{E}_i(\widetilde{x})
\end{aligned} \tag{A.3.12}
$$

Combining these two equations, we can further reduce the system to only one equation in one unknown ($\widetilde{x}$):

$$
1 - \left(1 - \frac{\mathcal{E}(\widetilde{x}) - 1}{\mathcal{E}(\widetilde{x})}\left(\frac{\widetilde{x}}{\widetilde{\mu}}\right)^{\sigma}\right)\mathcal{E}_i(\widetilde{x}) = 0 \tag{A.3.13}
$$

This is a simple non-linear equation that can be solved numerically. Given the resulting $\widetilde{x}(a)$ and $\Lambda(a)$, we can then map them back to prices using the expressions $\Lambda(a) \equiv p(a)/\mathbf{mc}$, $\Lambda_i(a) \equiv p_i(a)/\mathbf{mc} = \widetilde{p}(a)\Lambda(a)$, $\widetilde{p}(a) = p_i(a)/p(a)$, and $\widetilde{x}(a) \equiv \widetilde{\mu}\widetilde{p}(a)^{-\frac{1}{\sigma}}$.

## A.4 Proof of Proposition 4

The first-order conditions of problem (25) are:

$$
r_t = \alpha \mathbf{mc}_t \left(\frac{y}{k}\right) - \delta_K, \tag{A.4.1}
$$

$$
w_t = (1 - \alpha)\mathbf{mc}_t \left(\frac{y}{l}\right), \tag{A.4.2}
$$

where $\mathbf{mc}_t > 0$ is the Lagrange multiplier, equal to the marginal cost.[65] To find the value of this multiplier, use the production function to write capital as:

$$
k = \left(\frac{y}{zl^{1-\alpha}}\right)^{\frac{1}{\alpha}}, \tag{A.4.3}
$$

and re-write the problem as a choice over labor only:

$$
\min_{l} \left\{ (r_t + \delta_K)\left(\frac{y}{zl^{1-\alpha}}\right)^{\frac{1}{\alpha}} + w_t l \right\}. \tag{A.4.4}
$$

Taking the first-order condition of sub-problem (A.4.4) we find:

$$
l = \left(\frac{1-\alpha}{\alpha}\frac{r_t + \delta_K}{w_t}\right)^{\alpha}\frac{y}{z}, \tag{A.4.5}
$$

[65] To show that the marginal cost coincides with the Lagrange multiplier, substitute (A.4.1)-(A.4.2) back into the objective function of problem (25) to find that $\mathbf{TC}_t(y, a) = \mathbf{mc}_t y$, and therefore $\mathbf{mc}_t = \partial_y \mathbf{TC}_t(y, a)$.

and using this inside (A.4.3) gives:

$$k = \left( \frac{1-\alpha}{\alpha} \frac{r_t + \delta_K}{w_t} \right)^{\alpha - 1} \frac{y}{z}. \tag{A.4.6}$$

Substituting these last two results into the objective function, we find:

$$\mathbf{TC}_t(y) = (r_t + \delta_K)k + w_t l = \frac{1}{z} \left( \frac{r_t + \delta_K}{\alpha} \right)^{\alpha} \left( \frac{w_t}{1-\alpha} \right)^{1-\alpha} y. \tag{A.4.7}$$

Taking the derivative with respect to $y$, we obtain equation (26) in the main text. To find the demands for labor and capital, take the ratio of (A.4.6) and (A.4.5) to find the optimal capital-labor ratio:

$$\widetilde{\boldsymbol{k}}_t \equiv \frac{k}{l} = \frac{\alpha}{1-\alpha} \frac{w_t}{r_t + \delta_K}. \tag{A.4.8}$$

Thus, the capital-labor ratio is not only constant across firms within the product category, but also across product categories. Using equation (26):

$$\mathbf{mc}_t = \frac{1}{1-\alpha} \frac{w_t}{z} \widetilde{\boldsymbol{k}}_t^{-\alpha}. \tag{A.4.9}$$

Thus, the optimal labor input choice is $l_t(y) = \frac{y}{z} \widetilde{\boldsymbol{k}}_t^{-\alpha}$ or, using equation (A.4.9):

$$l_t(y) = (1-\alpha)\mathbf{mc}_t \frac{y}{w_t}. \tag{A.4.10}$$

The optimal capital input choice is then:

$$k_t(y) = \widetilde{\boldsymbol{k}}_t l_t(y). \tag{A.4.11}$$

Finally, in a symmetric equilibrium, from Proposition 2 we know that:

$$y(a) = (1 - f_0(a))\mu(a)^{\sigma(\kappa-1)} p(a)^{-\kappa} \frac{\boldsymbol{\Omega}_t}{N} q(a), \tag{A.4.12}$$

at product category age $a$, with $p(a) = \Lambda(a)\mathbf{mc}_t$ from Proposition 3 and $q(a) = \mathbb{E}_a[\widehat{n}^{\sigma(\kappa-1)}]$ from equation (24). Plugging this into equations (A.4.10) and (A.4.11) gives us the optimal labor and capital input choices in the symmetric equilibrium at product category age $a$.

## A.5 Proof of Proposition 5

To show Part 1 of this proposition, recall that the price index of consumer $j$ at time $t$ is defined by equation (16). Denote the age of a given product category $m$ by $a(m)$. Given $\mathbf{mc}_t$, the equilibrium price of the single product $i(m)$ that the consumer purchases in product category $m$ is only a function of product category age, by Proposition 3. Therefore, we can write equation (16) as

$$P_{jt} = \overline{\Gamma}^{-1} \left( \int_{\mathcal{M}_{jt}} e^{\sigma(\kappa-1)\xi_{i(m)mj}} \widehat{p}(a(m))^{1-\kappa} dm \right)^{\frac{1}{1-\kappa}}, \tag{A.5.1}$$

where recall that $\mathcal{M}_{jt} \equiv \{m : A_{mjt} \neq \emptyset\} \subseteq [0, \boldsymbol{M}_t]$ is the subset of product categories for which consumer $j$ is aware of at least one firm at time $t$.

While a firm's price in the symmetric equilibrium is only a function of age, the integral in (A.5.1) depends on consumer-specific objects: the set of categories in which the consumer is aware of at least one firm, $\mathcal{M}_{jt}$, and the idiosyncratic match value $\xi_{i(m)mj}$ of the chosen product $i(m)$ in each category $m$. However, because there is a continuum of product categories and awareness and preference draws are independent across categories, a law of large numbers applies: for each consumer $j$, the integral over $\mathcal{M}_{jt}$ converges almost surely to its expectation over awareness sets and preference realizations.

To compute this expectation, we proceed in two steps. First, for a product category of age $a$, the probability that consumer $j$ is aware of at least one firm is $1 - f_0(a)$. Second, conditional on being aware of $\widehat{n} \geq 1$ firms, the consumer selects the firm with the highest preference draw. Denote the cdf and pdf of the maximum of $\widehat{n}$ independent Gumbel draws by $G_{(\widehat{n})}(\xi)$ and $g_{(\widehat{n})}(\xi)$, respectively. Then, the expected contribution of a category of age $a$ to the price index integrand is $(1 - f_0(a))\widehat{p}(a)^{1-\kappa}\mathbb{E}_a[\Xi(a)]$, where

$$\Xi(a) \equiv \overline{\Gamma}^{\kappa-1} \int_{-\infty}^{+\infty} e^{\sigma(\kappa-1)\xi} g_{(\widehat{n})}(\xi) d\xi. \tag{A.5.2}$$

Integrating over the age distribution $\phi_t(a)$ and scaling by $\boldsymbol{M}_t$, equation (A.5.1) becomes:

$$\boldsymbol{P}_t = \left( \boldsymbol{M}_t \int_0^{+\infty} (1 - f_0(a))\widehat{p}(a)^{1-\kappa}\mathbb{E}_a[\Xi(a)]\phi_t(a) da \right)^{\frac{1}{1-\kappa}}. \tag{A.5.3}$$

Since the right-hand side is independent of $j$, the price index is common across all consumers: $P_{jt} = \boldsymbol{P}_t$.

The order statistic $G_{(\widehat{n})}(\xi)$ is computed as the distribution of the maximum of $\widehat{n}$ draws from the $G(\xi;\mu)$ re-centered Gumbel distribution introduced in equations (A.2.4a)-(A.2.4b). Therefore, by independence of preferences (Assumption 1), $G_{(\widehat{n})}(\xi)$ is determined by the product of the Gumbel cdf's, or:

$$G_{(\widehat{n})}(\xi) = \prod_{h=1}^{\widehat{n}} G(\xi;\mu_h) = e^{-\mu e^{-\xi}} \left(e^{-\mu_{-i}e^{-\xi}}\right)^{\widehat{n}-1} = e^{-(\mu+(\widehat{n}-1)\mu_{-i})e^{-\xi}}, \tag{A.5.4}$$

where $\mu$ denotes the firm's current level of targeting, and we have used (A.2.4a)-(A.2.4b) and symmetry among competitor's targeting in the second equality. Differentiating to find the pdf:

$$g_{(\widehat{n})}(\xi) = (\mu + (\widehat{n}-1)\mu_{-i})e^{-\xi}e^{-(\mu+(\widehat{n}-1)\mu_{-i})e^{-\xi}}. \tag{A.5.5}$$

This allows us to write:

$$\begin{aligned}
\Xi(a) &= \overline{\Gamma}^{\kappa-1} \int_{-\infty}^{+\infty} e^{\sigma(\kappa-1)\xi} g_{(\widehat{n})}(\xi) d\xi \\
&= (\mu + (\widehat{n}-1)\mu_{-i})\overline{\Gamma}^{\kappa-1} \int_{-\infty}^{+\infty} e^{-(1-\sigma(\kappa-1))\xi} e^{-(\mu+(\widehat{n}-1)\mu_{-i})e^{-\xi}} d\xi \\
&= (\mu + (\widehat{n}-1)\mu_{-i})\overline{\Gamma}^{\kappa-1} (\mu + (\widehat{n}-1)\mu_{-i})^{-(1-\sigma(\kappa-1))} \Gamma\big(1-\sigma(\kappa-1)\big) \\
&= (\mu + (\widehat{n}-1)\mu_{-i})^{\sigma(\kappa-1)},
\end{aligned} \tag{A.5.6}$$

where, to go from the second to the third line, we have used the fact that, for any two numbers $a_1, a_2 > 0$, $\int_{-\infty}^{+\infty} e^{-a_1 x} e^{-a_2 e^{-x}} dx = a_2^{-a_1}\Gamma(a_1)$, where $\Gamma(.)$ is the Gamma function, and recall that $\overline{\Gamma} \equiv \Gamma\big(1-\sigma(\kappa-1)\big)^{\frac{1}{1-\kappa}}$, where $\sigma(\kappa-1) < 1$. Plugging this result back into equation (A.5.3):

$$\boldsymbol{P}_t = \left( \boldsymbol{M}_t \int_0^{+\infty} (1 - f_0(a))\widehat{p}(a)^{1-\kappa} \mu^{\sigma(\kappa-1)} \mathbb{E}_a \left[ \left( 1 + (\widehat{n}-1)\frac{\mu_{-i}}{\mu} \right)^{\sigma(\kappa-1)} \right] \phi_t(a) da \right)^{\frac{1}{1-\kappa}}. \tag{A.5.7}$$

Finally, in a symmetric equilibrium, it must be that $\mu = \mu_{-i} = \mu(a) = \mu_0^{1-s(a)}$, and the price index (A.5.7) becomes:

$$\boldsymbol{P}_t = \left( \boldsymbol{M}_t \int_0^{+\infty} (1 - f_0(a))\widehat{p}(a)^{1-\kappa} \mu(a)^{\sigma(\kappa-1)} q(a) \phi_t(a) da \right)^{\frac{1}{1-\kappa}}, \tag{A.5.8}$$

where $q(a) \equiv \mathbb{E}_a\big[\widehat{n}^{\sigma(\kappa-1)}\big]$ was defined in equation (24). To compute real income, we use the same arguments that allowed us to write equation (A.5.3) from equation (A.5.1) in order to write the composite good in equation (12) as follows:

$$Y_{jt} = \overline{\Gamma} \left( \int_{\mathcal{M}_{jt}} \left( e^{\sigma \xi_{i(m)mj}} y_{i(m)}(a(m)) \right)^{\frac{\kappa-1}{\kappa}} dm \right)^{\frac{\kappa}{\kappa-1}}, \tag{A.5.9}$$

where, once again, $a(m)$ is the age of product category $m$, and $i(m)$ is the product that the consumer purchases in this product category. Developing equation (A.5.9) yields:

$$Y_{jt} = \Omega_{jt} \overline{\Gamma}^{\kappa} \left( \int_{\mathcal{M}_{jt}} e^{\sigma(\kappa-1)\xi_{i(m)mj}} p(a(m))^{1-\kappa} dm \right)^{\frac{\kappa}{\kappa-1}} \tag{A.5.10}$$

$$= \Omega_{jt} \overline{\Gamma}^{\kappa} \boldsymbol{P}_t^{\kappa} \left( \int_{\mathcal{M}_{jt}} e^{\sigma(\kappa-1)\xi_{i(m)mj}} \widehat{p}(a(m))^{1-\kappa} dm \right)^{\frac{\kappa}{\kappa-1}} = \Omega_{jt}, \tag{A.5.11}$$

where in the first equality we have used equation (15), in the second equality we have used $p(a(m)) = \widehat{p}(a(m))/\boldsymbol{P}_t$, and in the third equality we have used (A.5.1) to simplify all the terms. This shows that aggregate real income $\Omega_{jt}$ equals total output from the composite good $Y_{jt}$. To express total output $Y_{jt}$ as a function of aggregate capital and TFP (implying $Y_{jt} = \boldsymbol{Y}_t$), recall that aggregate labor demand is given by:

$$1 = \boldsymbol{L}_t \equiv \boldsymbol{M}_t \int_0^{+\infty} L_t(a) \phi_t(a) da, \tag{A.5.12}$$

where $L_t(a) = N l_t(a)$ is the product category's labor demand, equal to:

$$L_t(a) = (1-\alpha)(1-f_0(a)) \mu(a)^{\sigma(\kappa-1)} \mathbf{mc}_t^{1-\kappa} \Lambda(a)^{-\kappa} q(a) \frac{\boldsymbol{Y}_t}{w_t} \tag{A.5.13}$$

by equations (21) and (31a), and using that $\boldsymbol{\Omega}_t = \boldsymbol{Y}_t$. Next, divide both sides of equation (A.5.8) by $\boldsymbol{P}_t$, use $\widehat{p}(a) = p(a)\boldsymbol{P}_t$ and $p(a) = \Lambda(a)\mathbf{mc}_t$, and solve for $\mathbf{mc}_t$ to find:

$$\mathbf{mc}_t = \boldsymbol{M}_t^{\frac{1}{\kappa-1}} \boldsymbol{Q}_t, \tag{A.5.14}$$

where $\boldsymbol{Q}_t$ is defined in equation (35a). Using (A.5.14) in (A.5.12) and (A.5.13):

$$\begin{aligned} w_t \boldsymbol{L}_t &= (1-\alpha) \boldsymbol{Q}_t^{1-\kappa} \boldsymbol{Y}_t \int_0^{+\infty} (1-f_0(a)) \Lambda(a)^{-\kappa} \mu(a)^{\sigma(\kappa-1)} q(a) \phi_t(a) da \\ &= (1-\alpha) \boldsymbol{Q}_t^{1-\kappa} \boldsymbol{Y}_t \boldsymbol{B}_t \boldsymbol{Q}_t^{\kappa-1} \\ &= (1-\alpha) \boldsymbol{B}_t \boldsymbol{Y}_t, \end{aligned} \tag{A.5.15}$$

where $\boldsymbol{B}_t$ is defined as in equation (35b). On the other hand, recall by equation (A.4.9) that $w_t = (1-\alpha)z\mathbf{mc}_t\widetilde{\boldsymbol{k}}_t^{\alpha}$, where $\widetilde{\boldsymbol{k}}_t$ is the capital-labor ratio in a product category. By equation (A.4.8), the capital-labor ratio depends only on input prices and is, therefore, constant across product categories. Thus, aggregate capital equals $\boldsymbol{K}_t = \boldsymbol{L}_t\widetilde{\boldsymbol{k}}_t$, and thus:

$$w_t = (1-\alpha)z\boldsymbol{M}_t^{\frac{1}{\kappa-1}}\boldsymbol{Q}_t\boldsymbol{K}_t^{\alpha}\boldsymbol{L}_t^{-\alpha}. \tag{A.5.16}$$

Putting (A.5.15) and (A.5.16) together, we get:

$$\boldsymbol{Y}_t = \boldsymbol{Z}_t\boldsymbol{K}_t^{\alpha}\boldsymbol{L}_t^{1-\alpha}, \tag{A.5.17}$$

with $\boldsymbol{Z}_t \equiv z\boldsymbol{M}_t^{\frac{1}{\kappa-1}}\boldsymbol{Q}_t\boldsymbol{B}_t^{-1}$. This proves Part 1 of the proposition.[66]

To prove Part 2, we must show that the labor, capital and profit shares of total income $\boldsymbol{Y}_t$ are given by $(1-\alpha)\boldsymbol{B}_t$, $\alpha\boldsymbol{B}_t$, and $1-\boldsymbol{B}_t$, respectively. In (A.5.15) we already obtained that $\frac{w_t\boldsymbol{L}_t}{\boldsymbol{Y}_t} = (1-\alpha)\boldsymbol{B}_t$. From (A.4.8), recall $\boldsymbol{K}_t = \frac{\alpha}{1-\alpha}\frac{w_t}{r_t+\delta_K}$, from which it follows that $\frac{(r_t+\delta_K)\boldsymbol{K}_t}{\boldsymbol{Y}_t} = \alpha\boldsymbol{B}_t$. Finally, use equations (32), (33), and (A.5.14) to write firm profits as:

$$\pi(a) = (1-f_0(a))\boldsymbol{M}_t^{-1}\boldsymbol{Q}_t^{1-\kappa}\mu(a)^{\sigma(\kappa-1)}q(a)(\Lambda(a)-1)\Lambda(a)^{-\kappa}\frac{\boldsymbol{Y}_t}{N}. \tag{A.5.18}$$

Therefore, using $\boldsymbol{\Pi}_t \equiv \boldsymbol{M}_t\int_0^{+\infty} N\pi(a)\phi_t(a)da$, we get:

$$\frac{\boldsymbol{\Pi}_t}{\boldsymbol{Y}_t} = \boldsymbol{Q}_t^{1-\kappa}\int_0^{+\infty}(1-f_0(a))\mu(a)^{\sigma(\kappa-1)}q(a)(\Lambda(a)-1)\Lambda(a)^{-\kappa}\phi_t(a)da = 1-\boldsymbol{B}_t, \tag{A.5.19}$$

where the second equality uses the definitions of $\boldsymbol{Q}_t$ and $\boldsymbol{B}_t$ from (35a) and (35b). In sum, we have found that real income pays for labor, capital and profit income, so that $\boldsymbol{Y}_t = w_t\boldsymbol{L}_t + (r_t+\delta_K)\boldsymbol{K}_t + \boldsymbol{\Pi}_t$, as we wanted to show.

## A.6 Proof of Proposition 6

The current-value Hamiltonian corresponding to problem (43a)-(43b)-(43c) reads:

$$\mathscr{H}_t = \frac{\boldsymbol{C}_t^{1-\gamma}}{1-\gamma} + \varrho_t^A\left(r_t\boldsymbol{A}_t + w_t + (r_t+\delta_K)\boldsymbol{K}_t - \boldsymbol{C}_t - \boldsymbol{I}_t^K - \left(\boldsymbol{I}_t^M\right)^2 + z_M\boldsymbol{I}_t^M\boldsymbol{V}_t^0\right) + \varrho_t^K\left(\boldsymbol{I}_t^K - \delta_K\boldsymbol{K}_t\right), \tag{A.6.1}$$

[66]Notice, moreover, that if we substitute equation (21) at $p = p_{-i} = p$ and $\mu = \mu_{-i} = \mu(a)$ into (A.5.7) and solve for $\boldsymbol{P}_t\boldsymbol{Y}_t$, we obtain $\boldsymbol{P}_t\boldsymbol{Y}_t = \boldsymbol{M}_t\int_0^{+\infty} N\widehat{p}(a)y(a)\phi_t(a)da$. In words, aggregate nominal income can be expressed as the product of the ideal price and output indices.

where $\boldsymbol{C}_t$, $\boldsymbol{I}_t^M$ and $\boldsymbol{I}_t^K$ are the control variables, $\boldsymbol{A}_t$ and $\boldsymbol{K}_t$ are the states variables, and $\varrho_t^A, \varrho_t^K \geq 0$ are the corresponding multipliers. The necessary conditions for optimality are:

$$\partial_{\boldsymbol{C}}\mathscr{H}_t = 0 \quad \Leftrightarrow \quad \boldsymbol{C}_t^{-\gamma} = \varrho_t^A, \tag{A.6.2a}$$

$$\partial_{\boldsymbol{I}^M}\mathscr{H}_t = 0 \quad \Leftrightarrow \quad \boldsymbol{I}_t^M = \frac{1}{2} z_M \boldsymbol{V}_t^0, \tag{A.6.2b}$$

$$\partial_{\boldsymbol{I}^K}\mathscr{H}_t = 0 \quad \Leftrightarrow \quad \varrho_t^A = \varrho_t^K, \tag{A.6.2c}$$

$$\partial_{\boldsymbol{A}}\mathscr{H}_t = \rho\varrho_t^A - \partial_t\varrho_t^A \quad \Leftrightarrow \quad \partial_t\varrho_t^A = -(r_t - \rho)\varrho_t^A, \tag{A.6.2d}$$

$$\partial_{\boldsymbol{K}}\mathscr{H}_t = \rho\varrho_t^K - \partial_t\varrho_t^K \quad \Leftrightarrow \quad \varrho_t^A(r_t + \delta_K) - \varrho_t^K\delta_K = \rho\varrho_t^K - \partial_t\varrho_t^K. \tag{A.6.2e}$$

with the transversality conditions $\lim_{t\to+\infty} e^{-\rho t}\varrho_t^A \boldsymbol{A}_t = \lim_{t\to+\infty} e^{-\rho t}\varrho_t^K \boldsymbol{K}_t = 0$. From (A.6.2a), note that $\frac{\partial_t \varrho_t^A}{\varrho_t^A} = -\gamma\frac{\partial_t \boldsymbol{C}_t}{\boldsymbol{C}_t}$. Combining this with (A.6.2d), we obtain the Euler equation:

$$\frac{\partial_t \boldsymbol{C}_t}{\boldsymbol{C}_t} = \frac{r_t - \rho}{\gamma}. \tag{A.6.3}$$

In levels, this implies $\boldsymbol{C}_t = \boldsymbol{C}_0 e^{\frac{\bar{r}_t - \rho}{\gamma}t}$, where $\bar{r}_t \equiv \frac{1}{t}\int_0^t r_s ds$, and it allows us to write the transversality conditions as $\lim_{t\to+\infty} e^{-\bar{r}_t t}\boldsymbol{A}_t = \lim_{t\to+\infty} e^{-\bar{r}_t t}\boldsymbol{K}_t = 0$. In a stationary equilibrium, $\frac{\partial_t \boldsymbol{C}_t}{\boldsymbol{C}_t} = 0$ and thus $r_t = \rho$.

## A.7 Proof of Proposition 7

Recall that the law of motion for the measure of product categories is:

$$\frac{\partial_t \boldsymbol{M}_t}{\boldsymbol{M}_t} + \delta_M = \frac{z_M \boldsymbol{I}_t^M}{\boldsymbol{M}_t}. \tag{A.7.1}$$

The law of motion for the age distribution is given by:

$$\partial_t\widehat{\Phi}_t(a) = -\partial_a\widehat{\Phi}_t(a) - \delta_M\widehat{\Phi}_t(a) + z_M \boldsymbol{I}_t^M, \tag{A.7.2}$$

where $\widehat{\Phi}_t(a) \equiv \boldsymbol{M}_t\Phi_t(a)$. Computing the derivatives $\partial_t\widehat{\Phi}_t(a)$ and $\partial_a\widehat{\Phi}_t(a)$ yields:

$$\partial_t\widehat{\Phi}_t(a) = \boldsymbol{M}_t\partial_t\Phi_t(a) + \Phi_t(a)\partial_t\boldsymbol{M}_t \tag{A.7.3}$$

$$\partial_a\widehat{\Phi}_t(a) = \boldsymbol{M}_t\partial_a\Phi_t(a) \tag{A.7.4}$$

Dividing (A.7.2) by $\boldsymbol{M}_t$, and using (A.7.1), (A.7.3) and (A.7.4) gives:

$$\partial_t \Phi_t(a) = -\partial_a \Phi_t(a) + \left(\delta_M + \frac{\partial_t \boldsymbol{M}_t}{\boldsymbol{M}_t}\right)\big(1 - \Phi_t(a)\big). \tag{A.7.5}$$

In a stationary equilibrium, $\partial_t \Phi_t(a) = 0$, $\forall a \geq 0$, and $\partial_t \boldsymbol{M}_t = 0$, so that $\Phi_t(a) = \Phi(a)$ and $\boldsymbol{M}_t = \boldsymbol{M}$. Imposing this on (A.7.5) gives:

$$\partial_a \Phi(a) - \delta_M\big(1 - \Phi(a)\big) = 0. \tag{A.7.6}$$

This is a first-order, ordinary, autonomous and linear differential equation with boundary conditions $\Phi(0) = 0$ and $\lim_{a \to +\infty} \Phi(a) = 1$, which can be solved with simple methods. The solution is:

$$\Phi(a) = 1 - e^{-\delta_M a}. \tag{A.7.7}$$

The corresponding pdf is $\phi(a) = \partial_a \Phi(a) = \delta_M e^{-\delta_M a}$.

# B Social Planner

In this section, we solve the problem of a social planner that is constrained by the same product market frictions as the ones that consumers are subject to in the market economy.

The social planner's problem can be split into two parts: a static part (Section B.1), in which the planner allocates expenditures to consumers, as well as labor and capital input choices across firms and product categories, for a given distribution of awareness sets; and a dynamic part (Section B.2), in which given this static allocation, the planner makes consumption and investment choices (on capital and advertising), which in turn determine the distribution of awareness sets.

## B.1 The Static Problem

The static problem of the planner is to allocate resources $y_{imjt}$ and capital and labor inputs $k_{imt}$ and $l_{imt}$ across consumers $j \in [0, 1]$, product categories $m \in [0, \boldsymbol{M}_t]$, and firms $i = 1, \ldots, N$, at each time $t$, in order to maximize static output given the resource constraints of the economy. The planner takes the distribution of awareness sets $(A_{mjt})_{j \in [0,1], m \in [0, \boldsymbol{M}_t]}$ as given. The problem is:

$$\max_{\substack{\left((y_{imjt})_{j=0}^{1}, l_{imt}, k_{imt}\right)_{m\in[0,\boldsymbol{M}_t]} \\ i=1,\dots,N}} \overbrace{\int_0^1 \underbrace{\left[\int_0^{\boldsymbol{M}_t} \left(\sum_{i\in A_{mjt}} \overline{\Gamma} e^{\sigma\xi_{imj}} y_{imjt}\right)^{\frac{\kappa-1}{\kappa}} dm\right]^{\frac{\kappa}{\kappa-1}}}_{\equiv Y_{jt}} dj}^{\equiv \boldsymbol{Y}_t} \tag{B.1.1}$$

subject to:

$$\int_0^1 y_{imjt} dj \leq z k_{imt}^{\alpha} l_{imt}^{1-\alpha}, \quad \forall (i, m) \tag{B.1.2a}$$

$$0 \leq y_{imjt}, \qquad \forall (i, m), j \tag{B.1.2b}$$

$$\int_0^{\boldsymbol{M}_t} \left(\sum_{i=1}^{N} l_{imt}\right) dm \leq 1, \tag{B.1.2c}$$

$$\int_0^{\boldsymbol{M}_t} \left(\sum_{i=1}^{N} k_{imt}\right) dm \leq \boldsymbol{K}_t. \tag{B.1.2d}$$

where $\Phi_t(a)$, $\boldsymbol{M}_t > 0$ and $\boldsymbol{K}_t > 0$ are taken as given by the planner.

### B.1.1 Constrained-efficient static allocation

The Lagrangian is:

$$\begin{aligned}
\mathscr{L}_t = & \int_0^1 \left[\int_0^{\boldsymbol{M}_t} \left(\sum_{i\in A_{mjt}} \overline{\Gamma} e^{\sigma\xi_{imj}} y_{imjt}\right)^{\frac{\kappa-1}{\kappa}} dm\right]^{\frac{\kappa}{\kappa-1}} dj \\
& - \int_0^{\boldsymbol{M}_t} \sum_{i=1}^{N} \lambda_{imt} \left(\int_0^1 y_{imjt} dj - z k_{imt}^{\alpha} l_{imt}^{1-\alpha}\right) dm + \int_0^1 \int_0^{\boldsymbol{M}_t} \sum_{i\in A_{mjt}} \vartheta_{imjt} y_{imjt} dm dj \\
& - \varsigma_t \left(\int_0^{\boldsymbol{M}_t} \left(\sum_{i=1}^{N} l_{imt}\right) dm - 1\right) - \varrho_t \left(\int_0^{\boldsymbol{M}_t} \left(\sum_{i=1}^{N} k_{imt}\right) dm - \boldsymbol{K}_t\right)
\end{aligned}$$

where $\lambda_{imt}, \vartheta_{imjt}, \varsigma_t, \varrho_t \geq 0$ are Lagrange multipliers. The first-order condition with respect to $y_{imjt}$ is:

$$\left(\frac{Y_{jt}}{\sum_{i\in A_{mjt}} \overline{\Gamma} e^{\sigma\xi_{imj}} y_{imjt}}\right)^{\frac{1}{\kappa}} \overline{\Gamma} e^{\sigma\xi_{imj}} = \lambda_{imt} - \vartheta_{imjt}. \tag{B.1.3}$$

Following a similar logic as in the characterization of the decentralized equilibrium (recall Appendix A.1), we conjecture that if $m \in \mathcal{M}_{jt} \equiv \{m : A_{mjt} \neq \emptyset\} \subseteq [0, \boldsymbol{M}_t]$, the planner assigns at most one product from product category $m$ to each consumer. Denote this product by $i^*(m) \in A_{mjt}$. Then, from equation (B.1.3), we have:

$$\forall m \in \mathcal{M}_{jt}: \quad Y_{jt}^{\frac{1}{\kappa}} \left(\overline{\Gamma} e^{\sigma \xi_{i^*(m)mj}}\right)^{\frac{\kappa-1}{\kappa}} y_{i^*(m)mjt}^{-\frac{1}{\kappa}} = \lambda_{i^*(m)mt} \tag{B.1.4}$$

or, solving for $y_{i^*(m)mjt}$:

$$y_{i^*(m)mjt} = \lambda_{i^*(m)mt}^{-\kappa} \left(\overline{\Gamma} e^{\sigma \xi_{i^*(m)mj}}\right)^{\kappa-1} Y_{jt}. \tag{B.1.5}$$

Aggregating across product categories and using the definition for $Y_{jt}$ yields:

$$\overline{\Gamma}^{-1} \left( \int_{\mathcal{M}_{jt}} \left(\lambda_{i^*(m')m't} e^{-\sigma \xi_{i^*(m')m'j}}\right)^{1-\kappa} dm' \right)^{\frac{1}{1-\kappa}} = 1. \tag{B.1.6}$$

The first-order conditions of problem (B.1.1) with respect to $l_{imt}$ and $k_{imt}$ are:

$$\lambda_{imt} z (1-\alpha) \left(\frac{k_{imt}}{l_{imt}}\right)^{\alpha} = \varsigma_t, \tag{B.1.7}$$

$$\lambda_{imt} z \alpha \left(\frac{k_{imt}}{l_{imt}}\right)^{\alpha-1} = \varrho_t. \tag{B.1.8}$$

Taking the ratio of equations (B.1.7) and (B.1.8), we obtain:

$$\frac{k_{imt}}{l_{imt}} = \frac{\alpha}{1-\alpha} \frac{\varsigma_t}{\varrho_t}, \tag{B.1.9}$$

so the capital-labor ratio is the same across all firms and industries. Plugging this back into (B.1.2d), we get the aggregate capital-labor ratio (equal to the aggregate capital stock, as the labor supply is measure-one):

$$\boldsymbol{K}_t = \widetilde{\boldsymbol{k}}_t \equiv \frac{\alpha}{1-\alpha} \frac{\varsigma_t}{\varrho_t}. \tag{B.1.10}$$

Moreover, plugging (B.1.9) into (B.1.7) yields:

$$\lambda_{imt} z (1-\alpha) \left(\frac{\alpha}{1-\alpha} \frac{\varsigma_t}{\varrho_t}\right)^{\alpha} = \varsigma_t, \tag{B.1.11}$$

which shows that $\lambda_{imt} = \boldsymbol{\lambda}_t$, for all $(i, m)$. In particular:

$$\boldsymbol{\lambda}_t = \frac{1}{z}\left(\frac{\varrho_t}{\alpha}\right)^{\alpha}\left(\frac{\varsigma_t}{1-\alpha}\right)^{1-\alpha}. \tag{B.1.12}$$

By equations (B.1.10) and (B.1.11), we have:

$$\boldsymbol{\lambda}_t z(1-\alpha)\widetilde{\boldsymbol{k}}_t^{\alpha} = \varsigma_t, \tag{B.1.13}$$

$$\boldsymbol{\lambda}_t z\alpha\widetilde{\boldsymbol{k}}_t^{\alpha-1} = \varrho_t. \tag{B.1.14}$$

Since $\lambda_{imt} = \boldsymbol{\lambda}_t$, then by equation (B.1.6) we obtain $\boldsymbol{\lambda}_t = \overline{\Gamma}/\widetilde{\boldsymbol{Q}}_{jt}^*$, where we have denoted

$$\widetilde{\boldsymbol{Q}}_{jt}^* \equiv \left(\int_{\mathcal{M}_{jt}} e^{\sigma(\kappa-1)\xi_{i^*(m)mj}}dm\right)^{\frac{1}{1-\kappa}}. \tag{B.1.15}$$

Therefore, $\widetilde{\boldsymbol{Q}}_{jt}^* = \widetilde{\boldsymbol{Q}}_{j't}^* = \widetilde{\boldsymbol{Q}}_t^*$ for all $j, j' \in [0,1]$. Using (B.1.5), we have:

$$y_{i^*(m)mjt} = \overline{\Gamma}^{-1}(\widetilde{\boldsymbol{Q}}_t^*)^{\kappa} e^{\sigma(\kappa-1)\xi_{i^*(m)mj}} Y_{jt} \tag{B.1.16}$$

Finally, we must confirm the conjecture that the planner wants individual $j$ to indeed consume at most one product from each product category. If $|A_{mjt}| = 1$, then the conjecture is trivially true. Else, take two products, $i$ and $i' \in A_{mjt}\setminus\{i\}$. From our initial conjecture, $y_{i'mjt} = 0$. Then, from (B.1.3), we have that:

$$Y_{jt}^{\frac{1}{\kappa}}\left(\overline{\Gamma}e^{\sigma\xi_{imj}}y_{imjt}\right)^{-\frac{1}{\kappa}}\overline{\Gamma}e^{\sigma\xi_{i'mj}} \leq \lambda_{i'mt}, \tag{B.1.17}$$

where, rearranging from equation (B.1.4), we know:

$$Y_{jt}^{\frac{1}{\kappa}}\left(\overline{\Gamma}e^{\sigma\xi_{imj}}y_{imjt}\right)^{-\frac{1}{\kappa}} = \frac{\lambda_{imt}}{\overline{\Gamma}e^{\sigma\xi_{imj}}}. \tag{B.1.18}$$

Back into equation (B.1.17) and taking logs, we obtain:

$$\ln\left(\frac{\lambda_{i'mt}}{\lambda_{imt}}\right) \geq \sigma(\xi_{i'mj} - \xi_{imj}). \tag{B.1.19}$$

However, as $\lambda_{i'mt} = \lambda_{imt}$ and $\sigma > 0$, then

$$\xi_{i'mj} \leq \xi_{imj}.$$

By Assumption 1, however, the event $\xi_{i'mj} = \xi_{imj}$ is probability zero because preferences are i.i.d. (Gumbel) continuous with respect to the Lebesgue measure. Thus, the planner allocates firm $i$, and not firm $i'$, to consumer $j$ almost surely.

### B.1.2 Aggregation

Since the planner is subject to the same product awareness frictions as the consumers, the planner also uses a measure $\widehat{f} : \mathcal{A}_i \to [0,1]$ for each firm $i$, which maps awareness sets in $\mathcal{A}_i \equiv \{A \in \mathbf{2}^{\{1,\dots,N\}} | i \in A\}$ into a density, at each product category age $a$.[67]

As we argued in Appendix A.2, by Bayes' theorem we can write the joint density of awareness sets $A$ that contain a given firm $i$ and the preference shifters $\vec{\xi}(A)$ across all products contained within these sets as the product of a marginal and a conditional density, or $d\Psi_i(A, \vec{\xi}(A)) = \widehat{f}(A) dH_i(\vec{\xi}(A)|A)$ for each $A \in \mathcal{A}_i$. The conditional density $dH_i(\vec{\xi}(A)|A)$ can, in turn, be written as a product of Gumbel densities, denoted $dG$ below, because preferences are i.i.d. (Assumption 1). These measures are functions of the planner's advertising choices, which determine the rate at which awareness sets evolve. For now, we take these choices as given.

Let us conjecture that, in the planner's solution, $Y_j = \boldsymbol{Y}$. Under this conjecture (which we confirm later on), we can write the total output of firm $i$ in the planner's solution by equation (B.1.16) as:

$$y_i = \overline{\Gamma}^{-1} (\widetilde{\boldsymbol{Q}}^*)^{\kappa} \boldsymbol{Y} \sum_{A \in \mathcal{A}_i} \widehat{f}(A) \underbrace{\int_{\mathbb{R}^n} e^{\sigma(\kappa-1)\xi_i} \mathbb{1}\left\{ \xi_i > \xi_{i'} \middle| \forall i' \in A \backslash \{i\} \right\} dG(\xi_i; \mu) \prod_{i' \in A \backslash \{i\}} dG(\xi_{i'}; \mu_{i'})}_{(*)}, \tag{B.1.20}$$

where $\{\mu_i\}_{i=1}^N$, with $\mu \equiv \mu_i$, are the planner's choice for targeting within the product category, corresponding to the means of the re-centered Gumbel draws. Following the same logic as in Appendix A.2, the term $(*)$ can be written as follows:

$$(*) = \overline{\Gamma}^{1-\kappa} \mu^{\sigma(\kappa-1)} \left( 1 + \sum_{i' \in A \backslash \{i\}} \frac{\mu_{i'}}{\mu} \right)^{\sigma(\kappa-1)-1}. \tag{B.1.21}$$

[67] We drop the age $a$ and time $t$ dependence everywhere to alleviate notation.

Back into (A.2.5), we obtain the demand for firm $i$:

$$y_i = \overline{\Gamma}^{-\kappa}(\widetilde{Q}^*)^{\kappa}\mu^{\sigma(\kappa-1)}Y\sum_{A\in\mathcal{A}_i}\widehat{f}(A)\left(1+\sum_{i'\in A\setminus\{i\}}\frac{\mu_{i'}}{\mu}\right)^{\sigma(\kappa-1)-1}. \tag{B.1.22}$$

In an equilibrium with symmetric competitor strategies, $\theta_{i'} = \theta_{-i}$ and $\mu_{i'} = \mu_{-i}$, for all $i' \neq i$. As argued in Appendix A.2, this implies that $\widehat{f}(A) = \frac{n}{N}f_n$, where $n \equiv |A|$ is the size of awareness set $A$, and $f_n$ is the share of consumers that are aware of $n = 0, 1, 2, \ldots, N$ firms (at a given product category age $a$). Replacing inside of (B.1.22) yields:

$$y_i = \overline{\Gamma}^{-\kappa}(\widetilde{Q}^*)^{\kappa}\mu^{\sigma(\kappa-1)}\frac{Y}{N}\sum_{n=0}^{N}nf_n\left(1+(n-1)\frac{\mu_{-i}}{\mu}\right)^{\sigma(\kappa-1)-1}, \tag{B.1.23}$$

or, using the expectation operator introduced in equation (20):

$$y_i = (1-f_0)\overline{\Gamma}^{-\kappa}(\widetilde{Q}^*)^{\kappa}\mu^{\sigma(\kappa-1)}\frac{Y}{N}\mathbb{E}_a\left[\widehat{n}\left(1+(\widehat{n}-1)\frac{\mu_{-i}}{\mu}\right)^{\sigma(\kappa-1)-1}\right], \tag{B.1.24}$$

where $\widehat{n} \equiv n|n \geq 1$. Further, imposing full symmetry ($\theta_{-i} = \theta$ and $\mu_{-i} = \mu$) and re-introducing the $a$ and $t$ dependence into our expressions, we obtain:

$$y_t(a) = (1-f_0(a))\overline{\Gamma}^{-\kappa}(\widetilde{Q}_t^*)^{\kappa}\big(\mu(a)\big)^{\sigma(\kappa-1)}\frac{Y_t}{N}q(a), \tag{B.1.25}$$

where $q(a) \equiv \mathbb{E}_a\left[\widehat{n}^{\sigma(\kappa-1)}\right]$.

We can now derive aggregate output in the planning economy. First, we develop $\widetilde{Q}_t^*$, defined in equation (B.1.15). In this equation, aggregate quality depends on consumer-specific objects: the set of categories in which the consumer is aware of at least one firm, $\mathcal{M}_{jt}$, and the idiosyncratic match value $\xi_{i(m)mj}$ of the chosen product $i(m)$ in category $m$. Since there is a continuum of product categories and awareness and preference draws are independent across categories, then by the law of large numbers the integral over $\mathcal{M}_{jt}$ in equation (B.1.15) converges almost surely to its expectation over awareness sets and preference realizations. We compute this expectation, in turn, as the product of the probability that the consumer $j$ is aware of at least one firm at some age $a$, equal to $(1-f_0(a))$, and an expectation conditional on the consumer knowing $\widehat{n} \geq 1$ firms, among which the consumer chooses the one with the highest preference draw. The expected contribution of a category of age $a$ to the quality index (B.1.15) is

$$\Xi(a) \equiv \overline{\Gamma}^{\kappa-1} \int_{-\infty}^{+\infty} e^{\sigma(\kappa-1)\xi} g_{(\widehat{n})}(\xi) d\xi, \tag{B.1.26}$$

where $g_{(\widehat{n})}(\xi)$ is the pdf associated to $\xi \equiv \max\{\xi_i : i \in A, |A| = \widehat{n}\}$, the maximum of $\widehat{n}$ independent Gumbel draws. Integrating over the unnormalized age distribution, we can then write (B.1.15) as follows:

$$\widetilde{\boldsymbol{Q}}_t^* = \overline{\Gamma}\left[\boldsymbol{M}_t \int_0^{+\infty} (1 - f_0(a))\mathbb{E}_a[\Xi(a)]\phi_t(a)da\right]^{\frac{1}{1-\kappa}} \tag{B.1.27}$$

In a symmetric equilibrium, $\Xi(a) = (\widehat{n}\mu(a))^{\sigma(\kappa-1)}$, as shown in the derivation that we used to obtain equation (A.5.6). Plugging things back into (B.1.27) gives $\widetilde{\boldsymbol{Q}}_t^* = \overline{\Gamma}\boldsymbol{M}_t^{\frac{1}{1-\kappa}}(\boldsymbol{Q}_t^*)^{-1}$, where

$$\boldsymbol{Q}_t^* \equiv \left[\int_0^{+\infty} (1 - f_0(a))(\mu(a))^{\sigma(\kappa-1)} q(a)\phi_t(a)da\right]^{\frac{1}{\kappa-1}} \tag{B.1.28}$$

and $q(a) = \mathbb{E}_a[\widehat{n}^{\sigma(\kappa-1)}]$. Recalling that $\boldsymbol{\lambda}_t = \overline{\Gamma}/\widetilde{\boldsymbol{Q}}_t^*$, we have found

$$\boldsymbol{\lambda}_t = \boldsymbol{M}_t^{\frac{1}{\kappa-1}} \boldsymbol{Q}_t^* = \frac{1}{z}\left(\frac{\varrho_t}{\alpha}\right)^{\alpha}\left(\frac{\varsigma_t}{1-\alpha}\right)^{1-\alpha}. \tag{B.1.29}$$

With these findings, we can now write the output of an individual firm in a product category of age $a$ and time $t$ (equation (B.1.25)) as

$$y_t(a) = (1 - f_0(a))\boldsymbol{\lambda}_t^{-\kappa}\left(\mu(a)\right)^{\sigma(\kappa-1)}\frac{\boldsymbol{Y}_t}{N} q(a), \tag{B.1.30}$$

To find aggregate income, recall that the resources allocated to consumer $j$ can be written as

$$Y_{jt} = \left[\int_0^{\boldsymbol{M}_t} \left(\sum_{i \in A_{mjt}} \overline{\Gamma} e^{\sigma\xi_{imj}} y_{imjt}\right)^{\frac{\kappa-1}{\kappa}} dm\right]^{\frac{\kappa}{\kappa-1}} = \overline{\Gamma}\left[\int_{\mathcal{M}_{jt}} \left(e^{\sigma\xi_{i^*(m)mj}} y_{i^*(m)}(a(m))\right)^{\frac{\kappa-1}{\kappa}} dm\right]^{\frac{\kappa}{\kappa-1}} \tag{B.1.31}$$

Once again, to compute the integral over product categories, we can invoke a law of large numbers and integrate, instead, over the age distribution of product categories. This allows us to write the last equation as:

$$Y_{jt} = \overline{\Gamma}\left[\boldsymbol{M}_t \int_0^{+\infty} (1-f_0(a))(y(a))^{\frac{\kappa-1}{\kappa}} \mathbb{E}_a\left[\Xi^*(a)\right]\phi_t(a)da\right]^{\frac{\kappa}{\kappa-1}}, \tag{B.1.32}$$

where we have

$$\Xi^*(a) \equiv \int_{-\infty}^{+\infty} e^{\sigma\left(\frac{\kappa-1}{\kappa}\right)\xi} g_{(\widehat{n})}(\xi)d\xi = \Gamma\left(1-\sigma\left(\frac{\kappa-1}{\kappa}\right)\right)\left(\widehat{n}\mu(a)\right)^{\sigma\left(\frac{\kappa-1}{\kappa}\right)}. \tag{B.1.33}$$

Notice that the right-hand side of equation (B.1.32) is independent of $j$, which confirms our conjecture, invoked at the beginning of Section B.1.2, that $Y_{jt} = \int_0^1 Y_{jt}dj = \boldsymbol{Y}_t$, $\forall j \in [0,1]$.

By equation (B.1.2a), $y_t(a) = zk_t(a)^\alpha l_t(a)^{1-\alpha} = z\boldsymbol{K}_t^\alpha l_t(a)$, where recall that $\boldsymbol{K}_t = \widetilde{\boldsymbol{k}}_t$, the capital-labor ratio (which is constant within and across product categories). By feasibility, $1 = \boldsymbol{L}_t = \boldsymbol{M}_t \int_0^{+\infty} Nl_t(a)\phi_t(a)da$, so

$$\boldsymbol{M}_t \int_0^{+\infty} Ny_t(a)\phi_t(a)da = z\boldsymbol{K}_t^\alpha \tag{B.1.34}$$

or, using equation (B.1.30),

$$\boldsymbol{Y}_t\boldsymbol{M}_t\boldsymbol{\lambda}_t^{-\kappa} \underbrace{\int_0^{+\infty}(1-f_0(a))\left(\mu(a)\right)^{\sigma(\kappa-1)}q(a)\phi_t(a)da}_{=(\boldsymbol{Q}_t^*)^{\kappa-1}} = z\boldsymbol{K}_t^\alpha \tag{B.1.35}$$

Solving for $\boldsymbol{Y}_t$ and using $\boldsymbol{\lambda}_t = \boldsymbol{M}_t^{\frac{1}{\kappa-1}}\boldsymbol{Q}_t^*$, we finally obtain:

$$\boldsymbol{Y}_t = \boldsymbol{Z}_t^*\boldsymbol{K}_t^\alpha, \tag{B.1.36}$$

where the planner's TFP is

$$\boldsymbol{Z}_t^* \equiv z\boldsymbol{M}_t^{\frac{1}{\kappa-1}}\boldsymbol{Q}_t^*, \tag{B.1.37}$$

and $\boldsymbol{Q}_t^*$ is defined in equation (B.1.28).

### B.1.3 Comparison between the planner's and the market's static allocations

How do these conditions compare to those obtained in the decentralized equilibrium? Starting from the individual choices, in the decentralized equilibrium (equations (15) and (16)) we had:

$$y_{imjt} = \overline{\Gamma}^{\kappa-1}e^{\sigma(\kappa-1)\xi_{imj}}p_{imjt}^{-\kappa}Y_{jt}, \tag{B.1.38}$$

where $p_{imjt} \equiv \widehat{p}_{imt}/P_{jt}$ denotes the real price paid by consumer $j$,

$$P_{jt} = \overline{\Gamma}^{-1} \left( \int_{\mathcal{M}_{jt}} \left( e^{-\sigma \xi_{i(m)mj}} \widehat{p}_{i(m)mt} \right)^{1-\kappa} dm \right)^{\frac{1}{1-\kappa}} \tag{B.1.39}$$

is the price index of individual $j$, and $i(m)$ is the firm that consumer $j$ chooses from her awareness set in product category $m$.

We claim that the planner's static allocation derived above would be attained by this decentralized solution if nominal prices in the decentralized solution could be written as $\widehat{p}_{imt} = \Lambda \mathbf{mc}_t$ for some number $\Lambda \geq 1$ that is constant both within and across product categories, where $\mathbf{mc}_t = \boldsymbol{M}_t^{\frac{1}{\kappa-1}} \boldsymbol{Q}_t$ is the marginal cost in the decentralized economy. Noticing that $\mathbf{mc}_t = \Lambda^{-1} \boldsymbol{\lambda}_t$, in this case the price index in (B.1.39) would become:

$$P_{jt} = \Lambda \mathbf{mc}_t \overline{\Gamma}^{-1} \left( \int_{\mathcal{M}_{jt}} \left( e^{-\sigma \xi_{i(m)mj}} \right)^{1-\kappa} dm \right)^{\frac{1}{1-\kappa}} = \Lambda \mathbf{mc}_t \overline{\Gamma}^{-1} \widetilde{\boldsymbol{Q}}_t^* = \Lambda \frac{\mathbf{mc}_t}{\boldsymbol{\lambda}_t} = 1, \tag{B.1.40}$$

Plugging real prices $p_{imjt} = \Lambda \mathbf{mc}_t$ back into (B.1.38), we obtain:

$$y_{imjt} = \overline{\Gamma}^{-1} (\widetilde{\boldsymbol{Q}}_t^*)^{\kappa} e^{\sigma(\kappa-1)\xi_{imj}} Y_{jt}, \tag{B.1.41}$$

which coincides with the planner's choice, in equation (B.1.16). Furthermore, comparing equations (B.1.9) and (B.1.12) with their counterparts in the market economy, equations (A.4.8) and (26), it is clear that $\boldsymbol{\varsigma}_t$ (the shadow value of labor for the planner) corresponds to $w_t$ (the price of labor), and $\boldsymbol{\varrho}_t$ (the shadow value of capital for the planner) corresponds to $r_t + \delta_K$ (the price of capital).

In words, at the individual level, *the static allocation in the decentralized equilibrium would be efficient if markup dispersion was zero both within and across industries*.[68] As long as there is no dispersion, the overall level of the markup is irrelevant from the point of view of static allocative efficiency.[69]

At the aggregate level, recall that we could write aggregate income in the decentralized

[68]To be precise, this statement is true up to the level of marginal costs. Indeed, the planner's choice for $\boldsymbol{M}_t$ does not, in general, coincide with the market allocation (where there is underinvestment in product category creation). Moreover, the planner makes different advertising choices, $(\theta^*, \mu_0^*)$, which imply a different aggregate match quality, $\boldsymbol{Q}_t^*$, as in the market economy. Therefore, the marginal cost in the planning economy, $\mathbf{mc}_t^* = (\boldsymbol{M}_t^*)^{\frac{1}{\kappa-1}} \boldsymbol{Q}_t^*$, does not coincide in general with the marginal cost in the market allocation, $\mathbf{mc}_t = \boldsymbol{M}_t^{\frac{1}{\kappa-1}} \boldsymbol{Q}_t$.

[69]Dynamically, because the supply of capital is elastic, the presence of markups depresses output even without markup dispersion.

economy as $\boldsymbol{Y}_t = \boldsymbol{Z}_t \boldsymbol{K}_t^\alpha$ (as $\boldsymbol{L}_t = 1$), with TFP equal to:

$$\boldsymbol{Z}_t = z \boldsymbol{M}_t^{\frac{1}{\kappa-1}} \boldsymbol{Q}_t \boldsymbol{B}_t^{-1} \tag{B.1.42}$$

where $\boldsymbol{Q}_t$ is defined in equation (35a) and $\boldsymbol{B}_t$ is defined in equation (35b). We have just established that the decentralized equilibrium attains the efficient static allocation if $\Lambda(a) = \Lambda$, for some constant $\Lambda \geq 1$. Imposing $\Lambda(a) = \Lambda$ inside (35a) and (35b) yields $\boldsymbol{B}_t = \Lambda^{-1}$ and

$$\boldsymbol{Q}_t = \Lambda \left[ \int_0^{+\infty} (1 - f_0(a))(\mu(a))^{\sigma(\kappa-1)} q(a) \phi_t(a) da \right]^{\frac{1}{\kappa-1}}, \tag{B.1.43}$$

Thus, in this case, $\boldsymbol{Q}_t \boldsymbol{B}_t^{-1}$ has the same functional form as $\boldsymbol{Q}_t^*$ in equation (B.1.28). In sum, without markup dispersion, there are no static TFP losses relative to the planner's solution (up to differences in $\boldsymbol{M}$ between the two allocations).

## B.2 The Dynamic Problem

Given the static allocation derived above, the planner chooses aggregate consumption $\boldsymbol{C}_t$, investment into physical capital $\boldsymbol{I}_t^K$, and investment into new product category creation $\boldsymbol{I}_t^M$, at each instant $t$. In addition, for each new cohort of product categories born at time $\tau$, the planner selects a contact rate $\theta_\tau$ and a targeting rate $\mu_{0,\tau}$ which remains constant for the lifetime of the category. These advertising choices are made at the cohort's inception and remain fixed for its lifetime, determining the cohort's awareness dynamics through the generator matrix $\mathcal{Q}(\theta_\tau)$.

At time $t$, aggregate match quality $\boldsymbol{Q}_t^*$, defined in equation (B.1.28), depends on the advertising choices of all surviving cohorts of firms. Specifically, a cohort born at time $\tau \leq t$ is currently at age $a = t - \tau$ and carries advertising parameters $(\theta_\tau, \mu_{0,\tau})$. Define the quality contribution of a cohort of age $a$ with advertising parameters $(\theta, \mu_0)$ as

$$\begin{aligned} Q^{\text{cohort}}(a; \theta, \mu_0) &\equiv (1 - f_0(a))(\mu(a))^{\sigma(\kappa-1)} q(a) \\ &= \mu_0^{\sigma(\kappa-1)\left(1 - \frac{1}{N}\sum_{n=1}^N n f_n(a;\theta)\right)} \left( \sum_{n=1}^N f_n(a;\theta)\, n^{\sigma(\kappa-1)} \right), \end{aligned} \tag{B.2.1}$$

where, to go from the first to the second line, we have used that $\mu(a) = \mu_0^{1-s(a)}$ with $s(a) = \frac{1}{N}\sum_{n=1}^N n f_n(a)$, and that $q(a) = \mathbb{E}_a\left[\widehat{n}^{\sigma(\kappa-1)}\right] = \frac{1}{1-f_0(a)} \sum_{n=1}^N f_n(a) n^{\sigma(\kappa-1)}$. Then, the aggregate quality index in equation (B.1.28) can be written as

$$\boldsymbol{Q}_t^* = \left[\int_0^{+\infty} Q^{\text{cohort}}\big(a;\, \theta_{t-a},\, \mu_{0,t-a}\big)\, \phi_t(a)\, da\right]^{\frac{1}{\kappa-1}}, \tag{B.2.2}$$

where $\phi_t(a)$ is the age density of product categories at time $t$. Taking initial conditions $\boldsymbol{K}_0$, $\boldsymbol{M}_0$, and $\Phi_0(a)$ as given, the planner's problem is:

$$\max_{\substack{(\boldsymbol{C}_t, \boldsymbol{I}_t^K, \boldsymbol{I}_t^M)_{t\geq 0} \\ (\theta_\tau, \mu_{0,\tau})_{\tau\geq 0}}} \int_0^{+\infty} e^{-\rho t} \frac{\boldsymbol{C}_t^{1-\gamma}}{1-\gamma}\, dt, \tag{B.2.3}$$

subject to

$$\boldsymbol{C}_t + \boldsymbol{I}_t^K + (\boldsymbol{I}_t^M)^2 = z\boldsymbol{M}_t^{\frac{1}{\kappa-1}} \boldsymbol{Q}_t^* \boldsymbol{K}_t^\alpha - z_M \boldsymbol{I}_t^M N\Big(\nu\theta_t^2 + \eta(\mu_{0,t}-1)^2\Big) \tag{B.2.4a}$$

$$\partial_t \boldsymbol{K}_t = \boldsymbol{I}_t^K - \delta_K \boldsymbol{K}_t \tag{B.2.4b}$$

$$\partial_t \boldsymbol{M}_t = z_M \boldsymbol{I}_t^M - \delta_M \boldsymbol{M}_t \tag{B.2.4c}$$

Note that the advertising cost $z_M \boldsymbol{I}_t^M N\big(\nu\theta_t^2 + \eta(\mu_{0,t}-1)^2\big)$ pertains to the cohort born at time $t$: each of the $z_M \boldsymbol{I}_t^M$ new product categories that are created at time $t$ contains $N$ firms, and each of these firms incurs the advertising cost $\nu\theta_t^2 + \eta(\mu_{0,t}-1)^2$.

### B.2.1 Optimality conditions for flow controls

The flow controls $(\boldsymbol{C}_t, \boldsymbol{I}_t^K, \boldsymbol{I}_t^M)$ and the state variables $(\boldsymbol{K}_t, \boldsymbol{M}_t)$ are governed by a current-value Hamiltonian, taking the advertising policy $\{(\theta_\tau, \mu_{0,\tau})\}_{\tau\geq 0}$ as given:

$$\mathscr{H}_t = \frac{1}{1-\gamma}\Bigg(\overbrace{z\boldsymbol{M}_t^{\frac{1}{\kappa-1}} \boldsymbol{Q}_t^* \boldsymbol{K}_t^\alpha - \boldsymbol{I}_t^K - (\boldsymbol{I}_t^M)^2 - z_M \boldsymbol{I}_t^M N\Big(\nu\theta_t^2 + \eta(\mu_{0,t}-1)^2\Big)}^{=\boldsymbol{C}_t}\Bigg)^{1-\gamma} + \lambda_t^K\Big(\boldsymbol{I}_t^K - \delta_K \boldsymbol{K}_t\Big) + \lambda_t^M\Big(z_M \boldsymbol{I}_t^M - \delta_M \boldsymbol{M}_t\Big) \tag{B.2.5}$$

The optimality conditions for $\boldsymbol{I}_t^K$ and $\boldsymbol{I}_t^M$, and the costate equations for $\boldsymbol{K}_t$ and $\boldsymbol{M}_t$, are:

$$\partial_{I^K}\mathscr{H}_t = 0 \quad\Leftrightarrow\quad \boldsymbol{C}_t^{-\gamma} = \lambda_t^K \tag{B.2.6a}$$

$$\partial_{I^M}\mathscr{H}_t = 0 \quad\Leftrightarrow\quad \boldsymbol{C}_t^{-\gamma}\left(\frac{2\boldsymbol{I}_t^M}{z_M} + N\Big(\nu\theta_t^2 + \eta(\mu_{0,t}-1)^2\Big)\right) = \lambda_t^M \tag{B.2.6b}$$

$$\partial_K\mathscr{H}_t = \rho\lambda_t^K - \partial_t\lambda_t^K \quad\Leftrightarrow\quad \boldsymbol{C}_t^{-\gamma}\alpha\frac{\boldsymbol{Y}_t}{\boldsymbol{K}_t} = (\rho+\delta_K)\lambda_t^K - \partial_t\lambda_t^K \tag{B.2.6c}$$

$$\partial_M \mathscr{H}_t = \rho\lambda_t^M - \partial_t\lambda_t^M \quad \Leftrightarrow \quad \boldsymbol{C}_t^{-\gamma}\frac{1}{\kappa-1}\frac{\boldsymbol{Y}_t}{\boldsymbol{M}_t} = (\rho+\delta_M)\lambda_t^M - \partial_t\lambda_t^M \tag{B.2.6d}$$

plus the transversality conditions $\lim_{t\to+\infty} e^{-\rho t}\lambda_t^K \boldsymbol{K}_t = 0$ and $\lim_{t\to+\infty} e^{-\rho t}\lambda_t^M \boldsymbol{M}_t = 0$. Combining (B.2.6a) and (B.2.6c) yields the Euler equation for the planner:

$$\frac{\partial_t \boldsymbol{C}_t}{\boldsymbol{C}_t} = \frac{\alpha \boldsymbol{Z}_t^* \boldsymbol{K}_t^{\alpha-1} - \delta_K - \rho}{\gamma}, \tag{B.2.7}$$

where $\boldsymbol{Z}_t^* \equiv z\boldsymbol{M}_t^{\frac{1}{\kappa-1}}\boldsymbol{Q}_t^*$ is aggregate TFP.

### B.2.2 Optimality conditions for advertising

The advertising choice $(\theta_\tau, \mu_{0,\tau})$ for the cohort born at time $\tau$ is qualitatively different from the flow controls: it is an investment made once-and-for-all at time $\tau$ whose benefits accrue over the cohort's remaining lifetime. We derive the optimality conditions using the calculus of variations.

Consider a perturbation $\theta_\tau \to \theta_\tau + d\theta$ from the optimal choice $\theta_\tau$ made by the cohort born at time $\tau$, holding all other choices fixed. The marginal cost, borne at time $\tau$, is

$$\mathrm{MC}_\tau \equiv z_M \boldsymbol{I}_\tau^M N 2\nu\theta_\tau d\theta. \tag{B.2.8}$$

The marginal benefit accrues at all future times $\tau + a$ (for all ages $a > 0$), as the cohort's improved quality contribution $Q^{\text{cohort}}(a;\theta_\tau,\mu_{0,\tau})$ raises aggregate match quality (recall equation (B.2.2)) and thereby output. The marginal benefit is

$$\mathrm{MB}_\tau \equiv \int_0^{+\infty} \mathbf{m}_{\tau,\tau+a}\left(\frac{\boldsymbol{Y}_{\tau+a}}{(\kappa-1)(\boldsymbol{Q}_{\tau+a}^*)^{\kappa-1}}\,\partial_\theta Q^{\text{cohort}}(a;\theta_\tau,\mu_{0,\tau})\right)\phi_{\tau+a}(a)\,da d\theta. \tag{B.2.9}$$

where $\mathbf{m}_{\tau,\tau+a} \equiv e^{-\rho a}\left(\boldsymbol{C}_{\tau+a}/\boldsymbol{C}_\tau\right)^{-\gamma}$ denotes the stochastic discount factor between dates $\tau$ and $\tau + a$.

Setting $\mathrm{MB}_\tau = \mathrm{MC}_\tau$ and canceling $d\theta$, we obtain the first-order condition for $\theta_\tau$:

$$\int_0^{+\infty} \mathbf{m}_{\tau,\tau+a}\left(\frac{\boldsymbol{Y}_{\tau+a}}{(\kappa-1)(\boldsymbol{Q}_{\tau+a}^*)^{\kappa-1}}\,\partial_\theta Q^{\text{cohort}}(a;\theta_\tau,\mu_{0,\tau})\right)\phi_{\tau+a}(a)\,da = z_M \boldsymbol{I}_\tau^M N 2\nu\theta_\tau. \tag{B.2.10}$$

Similarly, for $\mu_{0,\tau}$:

$$\int_0^{+\infty} \mathbf{m}_{\tau,\tau+a} \left( \frac{\boldsymbol{Y}_{\tau+a}}{(\kappa-1)(\boldsymbol{Q}^*_{\tau+a})^{\kappa-1}} \, \partial_{\mu_0} Q^{\text{cohort}}(a;\theta_\tau,\mu_{0,\tau}) \right) \phi_{\tau+a}(a) \, da = z_M \boldsymbol{I}^M_\tau N 2\eta(\mu_{0,\tau}-1). \tag{B.2.11}$$

In words, the planner invests in advertising up to the point at which the present discounted value of the marginal increase in aggregate output, evaluated using the stochastic discount factor $\mathbf{m}_{\tau,\tau+a}$, equals the marginal advertising cost. Note that the discount rate reflects both time preference ($\rho$) and the cohort's survival probability (embedded in $\phi_{\tau+a}(a)$). This is the natural counterpart to the firm's advertising optimality condition in the decentralized equilibrium (described in Section 2.2.4), where the firm discounts future profits at rate $\rho+\delta_M$.

### B.2.3 Stationary equilibrium

In a stationary equilibrium, $\boldsymbol{K}_t = \boldsymbol{K}$, $\boldsymbol{M}_t = \boldsymbol{M}$, $\boldsymbol{Q}^*_t = \boldsymbol{Q}^*$, $\boldsymbol{Y}_t = \boldsymbol{Y}$, and the planner chooses the same advertising strategy $(\theta,\mu_0)$ for all cohorts. Moreover, $\mathbf{m}_{\tau,\tau+a} = e^{-\rho a}$, for all $\tau > 0$, and the stationary age density is $\phi(a) = \delta_M e^{-\delta_M a}$.

From the Euler equation (B.2.7), the steady-state level of capital satisfies $\alpha \boldsymbol{Z}^* \boldsymbol{K}^{\alpha-1} = \delta_K + \rho$. The stationarity conditions $z_M \boldsymbol{I}^M = \delta_M \boldsymbol{M}$ and $\boldsymbol{I}^K = \delta_K \boldsymbol{K}$ hold. Steady-state consumption is $\boldsymbol{C} = z\boldsymbol{M}^{\frac{1}{\kappa-1}} \boldsymbol{Q}^* \boldsymbol{K}^\alpha - \delta_K \boldsymbol{K} - \left(\frac{\delta_M \boldsymbol{M}}{z_M}\right)^2 - \delta_M \boldsymbol{M} N \big(\nu\theta^2 + \eta(\mu_0-1)^2\big)$.

Evaluating the advertising optimality conditions (B.2.10)–(B.2.11) in the stationary equilibrium yields:

$$\frac{1}{(\kappa-1)(\boldsymbol{Q}^*)^{\kappa-1}} \int_0^{+\infty} \delta_M e^{-(\rho+\delta_M)a} \, \frac{\partial Q^{\text{cohort}}(a;\theta,\mu_0)}{\partial \theta} \, da = z_M \boldsymbol{I}^M N \frac{2\nu\theta}{\boldsymbol{Y}} \tag{B.2.12a}$$

$$\frac{1}{(\kappa-1)(\boldsymbol{Q}^*)^{\kappa-1}} \int_0^{+\infty} \delta_M e^{-(\rho+\delta_M)a} \, \frac{\partial Q^{\text{cohort}}(a;\theta,\mu_0)}{\partial \mu_0} \, da = z_M \boldsymbol{I}^M N \frac{2\eta(\mu_0-1)}{\boldsymbol{Y}} \tag{B.2.12b}$$

In words, the planner invests in advertising until the elasticity of aggregate match quality to advertising (left-hand side) equals the marginal cost (right-hand side). The elasticity of aggregate match quality is computed as the present-discounted sum of the elasticities of the quality contributions of different cohorts to the advertising choices. The relevant rate of discounting to compute this sum is $\rho+\delta_M$, taking into account both utility discounting ($\rho$) and the fact that older cohorts are less likely to survive ($\delta_M$). The marginal cost, in turn, is computed as the per-firm marginal cost, times the measure of firms $z_M \boldsymbol{I}^M N$ that are born each time period.

# C Model Extension: Persuasive Advertising

In this appendix, we modify the baseline model to introduce a new role for advertising expenditures: *persuasive advertising*. We think of persuasive advertising as directly shifting consumer preferences at the individual product level. That is, in this extension, firms use advertising to persuade consumers to purchase products they are already aware of.

## C.1 Assumptions and Equilibrium Conditions

The extended model's assumptions are nearly identical to the baseline's, with a few differences. First, an individual's consumption bundle is now given by

$$C_{jt} = \left[ \int_0^{M_t} \left( \sum_{i \in A_{mjt}} \underbrace{\varpi_{imt}}_{\substack{\text{Perceived} \\ \text{quality}}} \overline{\Gamma} e^{\sigma \xi_{imj}} c_{imjt} \right)^{\frac{\kappa-1}{\kappa}} dm \right]^{\frac{\kappa}{\kappa-1}}, \tag{C.1.1}$$

Compared to equation (2), there is now a new term $\varpi_{imt}$, which we call the *perceived quality* of product $(i, m)$ at time $t$. We assume $\varpi_{imt}$ is constant across all consumers that have this product in their awareness set (that is, like with prices, firms cannot discriminate).

Perceived quality is chosen by firm $i$ every instant $t$ by spending $\chi(\varpi_{imt} - 1)^2$ units of its own output, where $\chi > 0$ is a scale parameter in the persuasive advertising cost function.[70] The baseline model is a special case for which $\chi \to +\infty$ (in which case $\varpi_{imt} = 1$).

Every period, firms choose advertising and prices in a static simultaneous game, taking the price and advertising choices of competitors as given. As in the baseline model, we solve for the symmetric equilibrium but consider the possibility of unilateral, off-equilibrium deviations in advertising choices $(\theta, \mu_0)$, period prices $p$ and, new to this extension, also in persuasive advertising choices.

Under this specification, consumer $j$'s relative demand between any two product categories $m, m' \in \mathcal{M}_{jt}$ is:[71]

$$y_{i(m')m'jt} = y_{i(m)mjt} e^{\sigma(\kappa-1)\left(\xi_{i(m')m'j} - \xi_{i(m)mj}\right)} \left( \frac{\widehat{p}_{i(m)mt}}{\widehat{p}_{i(m')m't}} \right)^{\kappa} \left( \frac{\varpi_{i(m)mt}}{\varpi_{i(m')m't}} \right)^{1-\kappa}. \tag{C.1.2}$$

[70] Notice that this functional form mirrors the one we assumed for the other types of advertising in equation (8).

[71] Both the notation and the derivations follow those of Appendix A.1.

The nominal income of consumer $j$ can be written as follows:

$$P_{jt}\Omega_{jt} = y_{i(m)mjt}\big(\widehat{p}_{i(m)mt}\big)^{\kappa}\varpi^{1-\kappa}_{i(m)mt}\overline{\Gamma}^{1-\kappa}e^{-\sigma(\kappa-1)\xi_{i(m)mj}}P_{jt}^{-(\kappa-1)}. \tag{C.1.3}$$

where consumer $j$'s price index is now defined by

$$P_{jt} \equiv \overline{\Gamma}^{-1}\left(\int_{\mathcal{M}_{jt}}\left(e^{-\sigma\xi_{i(m')m'j}}\frac{\widehat{p}_{i(m')m't}}{\varpi_{i(m')m't}}\right)^{1-\kappa}dm'\right)^{\frac{1}{1-\kappa}}. \tag{C.1.4}$$

Defining real prices by $p_{i(m)mjt} \equiv \widehat{p}_{i(m)mt}/P_{jt}$, the intensive demand function for product $i$ in product category $m$ is now:

$$y^{d}_{i(m)mjt} = \overline{\Gamma}^{\kappa-1}e^{\sigma(\kappa-1)\xi_{i(m)mj}}\varpi^{\kappa-1}_{i(m)mt}p^{-\kappa}_{i(m)mjt}\Omega_{jt}. \tag{C.1.5}$$

On the extensive margin, it is easy to show that the individual still consumes at most one product from each product category. In particular, for any given $i \in \mathcal{M}_{jt}$ and all $i' \in \mathcal{M}_{jt}\backslash\{i\}$, consumer $j$ chooses good $i$ over $i'$ if:

$$\ln\left(\frac{\widehat{p}_{i'mt}}{\widehat{p}_{imt}}\right) - \ln\left(\frac{\varpi_{i'mt}}{\varpi_{imt}}\right) \geq \sigma(\xi_{i'mj} - \xi_{imj}). \tag{C.1.6}$$

That is, when choosing which product to purchase, the consumer now compares not just nominal prices, $\widehat{p}_{i'}$ and $\widehat{p}_i$, but *quality-adjusted* prices, $\widehat{p}_{i'}/\varpi_{i'}$ and $\widehat{p}_i/\varpi_i$.

At any point in time, firm $i$ with targeting $\mu$ (determined by its age-zero choice), chooses quality $\varpi$ and price $p$. This firm takes as given all of its rival's choices for prices $\{p_{i'}\}_{i'\neq i}$, perceived qualities $\{\varpi_{i'}\}_{i'\neq i}$, and targeting $\{\mu_{i'}\}_{i'\neq i}$, as well as aggregate income $\mathbf{\Omega}_t$ and marginal costs $\mathbf{mc}_t$, and the density of awareness sets that, as of age $a$, include the firm: $\widehat{f}(a,\cdot) : \mathcal{A}_i \to [0,1]$.

The outcome of the Bertrand game in prices and qualities is the solution to:

$$\max_{p,\varpi}\left\{\Big(p - \mathbf{mc}_t\Big)y_i(a,\vec{p},\vec{\varpi}) - \underbrace{\chi(\varpi-1)^2}_{\substack{\text{Persuasive}\\ \text{adv. cost}}}\right\}. \tag{C.1.7}$$

where

$$y_i(a,\vec{p},\vec{\varpi}) = \mu^{\sigma(\kappa-1)}\varpi^{\kappa-1}p^{-\kappa}\mathbf{\Omega}\underbrace{\sum_{A\in\mathcal{A}_i}\widehat{f}(a,A)\left(1+\sum_{i'\in A\setminus\{i\}}\frac{\mu_{i'}}{\mu}\left(\frac{\varpi}{\varpi_{i'}}\frac{p_{i'}}{p}\right)^{-\frac{1}{\sigma}}\right)^{\sigma(\kappa-1)-1}}_{=\,q_i(a,\vec{p},\vec{\varpi})} \tag{C.1.8}$$

Assuming existence of a solution, the first-order condition of problem (C.1.7) with respect to $p$ still gives rise to a markup $\Lambda \equiv p/\mathbf{mc}_t$ equal to:

$$\Lambda_i(a,\vec{p},\vec{\varpi}) = \frac{\mathcal{E}_i(a,\vec{p},\vec{\varpi})}{\mathcal{E}_i(a,\vec{p},\vec{\varpi})-1}, \tag{C.1.9}$$

where the total price-elasticity of demand is still given by the sum of the intensive- and extensive-margin elasticities:

$$\mathcal{E}_i(a,\vec{p},\vec{\varpi}) \equiv \kappa + \left(-\frac{\partial q_i(a,\vec{p},\vec{\varpi})}{\partial p}\frac{p}{q_i(a,\vec{p},\vec{\varpi})}\right) \tag{C.1.10}$$

To calculate the extensive-margin elasticity, we can still use equation (A.3.4) for the general case and equation (A.3.6) for the fully symmetric case.

The first-order condition of problem (C.1.7) with respect to $\varpi$ can be written as follows:

$$\underbrace{py_i(a,\vec{p},\vec{\varpi})\left(1-\frac{1}{\Lambda_i(a,\vec{p},\vec{\varpi})}\right)}_{\substack{\text{Profit before persuasive}\\ \text{advertising spending}}}\mathcal{W}_i(a,\vec{p},\vec{\varpi}) = 2\chi(\varpi-1)\varpi \tag{C.1.11}$$

where we have defined

$$\mathcal{W}_i(a,\vec{p},\vec{\varpi}) \equiv \underbrace{\kappa-1}_{\substack{\text{Intensive-margin}\\ \text{elasticity}}} + \underbrace{\frac{\partial q_i(a,\vec{p},\vec{\varpi})}{\partial\varpi}\frac{\varpi}{q_i(a,\vec{p},\vec{\varpi})}}_{\substack{\text{Extensive-margin}\\ \text{elasticity}}} \tag{C.1.12}$$

as the *total elasticity of demand with respect to persuasive advertising*.

Like the price-elasticity $\mathcal{E}_i$, the advertising-elasticity $\mathcal{W}_i$ has an intensive-margin component and an extensive-margin component. Intuitively, when a firm increases its persuasive advertising choice in the margin, it increases demand from its existing customers because the extra advertising spending persuades them to purchase a higher quantity of the firm's product at given nominal prices (an increase in intensive-margin demand). Additionally, the increase in advertising spending also decreases the firm's quality-adjusted price, thereby persuading customers from competitor firms to switch into the firm (an increase in extensive-margin demand).

The extensive-margin elasticity with respect to $\varpi$ and $p$ coincide in absolute value:

$$\frac{\partial q_i(a,\vec{p},\vec{\varpi})}{\partial \varpi}\frac{\varpi}{q_i(a,\vec{p},\vec{\varpi})} = -\frac{\partial q_i(a,\vec{p},\vec{\varpi})}{\partial p}\frac{p}{q_i(a,\vec{p},\vec{\varpi})} \tag{C.1.13}$$

This is intuitive: along the extensive margin, marginally increasing perceived quality (through advertising) is akin to marginally decreasing the nominal price, because the consumer's purchasing choice is dictated by the price-to-quality ratio (equation (C.1.6)).

To solve for the equilibrium markups and persuasive advertising choices while still allowing for unilateral deviations in both when a firm unilaterally deviates from the symmetric $(\theta, \mu_0)$ strategy, we follow the work that we presented in Appendix A.3.3. As in that appendix, let us define $\widetilde{p} = p_i/p$, $\widetilde{\mu} = \mu_i/\mu$, and $\widetilde{x} = \widetilde{\mu}\widetilde{p}^{-\frac{1}{\sigma}}$, where $i$ is the firm who is unilaterally deviating from the symmetric $(\theta, \mu_0)$ strategy. Additionally, we now define $\widetilde{\varpi} = \varpi_i/\varpi$.

Then, the pricing problem can still be written in the form of equation (A.3.13):

$$1 - \left[1 - \frac{\mathcal{E}(\widetilde{x},\widetilde{\varpi}) - 1}{\mathcal{E}(\widetilde{x},\widetilde{\varpi})}\left(\frac{\widetilde{x}}{\widetilde{\mu}}\right)^{\sigma}\right]\mathcal{E}_i(\widetilde{x},\widetilde{\varpi}) = 0 \tag{C.1.14}$$

where $\mathcal{E}_i(\widetilde{x},\widetilde{\varpi})$ and $\mathcal{E}(\widetilde{x},\widetilde{\varpi})$ are now defined implicitly by

$$\sigma\frac{\mathcal{E}_i(\widetilde{x},\widetilde{\varpi}) - \kappa}{1-\sigma(\kappa-1)} = \frac{\sum_{n=1}^{N} f_{N-1+n}\left(1+\frac{n-1}{\widetilde{x}\widetilde{\varpi}^{1/\sigma}}\right)^{\sigma(\kappa-1)-2}\frac{n-1}{\widetilde{x}\widetilde{\varpi}^{1/\sigma}}}{\sum_{n=1}^{N} f_{N-1+n}\left(1+\frac{n-1}{\widetilde{x}\widetilde{\varpi}^{1/\sigma}}\right)^{\sigma(\kappa-1)-1}}. \tag{C.1.15}$$

and

$$\sigma\frac{\mathcal{E}(\widetilde{x},\widetilde{\varpi}) - \kappa}{1-\sigma(\kappa-1)} = \tag{C.1.16}$$
$$\frac{\sum_{n=2}^{N}\frac{n-1}{N-1}f_{N-1+n}\left(n-1+\widetilde{x}\widetilde{\varpi}^{1/\sigma}\right)^{\sigma(\kappa-1)-2}(n-2+\widetilde{x}\widetilde{\varpi}^{1/\sigma}) + \sum_{n=1}^{N-1}\frac{n-1}{N-1}f_n\, n^{\sigma(\kappa-1)-1}}{\sum_{n=2}^{N}\frac{n-1}{N-1}f_{N-1+n}\left(n-1+\widetilde{x}\widetilde{\varpi}^{1/\sigma}\right)^{\sigma(\kappa-1)-1} + \sum_{n=1}^{N-1}\frac{n}{N-1}f_n\, n^{\sigma(\kappa-1)-1}}.$$

The differences relative to equations (A.3.9) and (A.3.10) have been highlighted in blue.

Recalling that the advertising-elasticity and the price-elasticity are related by $\mathcal{W} = \mathcal{E} - 1$, then using equation (C.1.11) the advertising choices solve:

$$p_i y_i(\widetilde{x},\varpi_i)\overbrace{\left(1-\Lambda_i^{-1}\right)}^{=\Lambda_i^{-1}}\left(\mathcal{E}_i(\widetilde{x},\widetilde{\varpi})-1\right) = 2\chi(\varpi_i-1)\varpi_i, \tag{C.1.17a}$$

$$p y(\widetilde{x},\varpi)\underbrace{\left(1-\Lambda^{-1}\right)\left(\mathcal{E}(\widetilde{x},\widetilde{\varpi})-1\right)}_{=\Lambda^{-1}} = 2\chi(\varpi-1)\varpi, \tag{C.1.17b}$$

for the deviating and the symmetric firms, respectively. Taking the ratio of (C.1.17a) and (C.1.17b), and noting that $p_i\Lambda_i^{-1} = p\Lambda^{-1} = \mathbf{mc}$, we get:

$$\frac{y_i(\widetilde{x},\varpi_i)}{y(\widetilde{x},\varpi)} = \frac{\varpi_i-1}{\varpi-1}\widetilde{\varpi} \tag{C.1.18}$$

where $y_i(\widetilde{x},\varpi_i) = \mu_i^{\sigma(\kappa-1)}\varpi_i^{\kappa-1}p_i^{-\kappa}q_i(\widetilde{x},\widetilde{\varpi})\mathbf{\Omega}$ and $y(\widetilde{x},\varpi) = \mu^{\sigma(\kappa-1)}\varpi^{\kappa-1}p^{-\kappa}q(\widetilde{x},\widetilde{\varpi})\mathbf{\Omega}$, with

$$q_i(\widetilde{x},\widetilde{\varpi}) = \sum_{n=1}^{N} f_{N-1+n}\left(1+\frac{n-1}{\widetilde{x}\widetilde{\varpi}^{1/\sigma}}\right)^{\sigma(\kappa-1)-1} \tag{C.1.19a}$$

$$q(\widetilde{x},\widetilde{\varpi}) = \frac{1}{N-1}\left(\sum_{n=2}^{N}(n-1)f_{N-1+n}\left(n-1+\widetilde{x}\widetilde{\varpi}^{1/\sigma}\right)^{\sigma(\kappa-1)-1} + \sum_{n=1}^{N-1} n f_n\, n^{\sigma(\kappa-1)-1}\right) \tag{C.1.19b}$$

Therefore:

$$\frac{y_i(\widetilde{x},\varpi_i)}{y(\widetilde{x},\varpi)} = \widetilde{\mu}^{\sigma(\kappa-1)}\widetilde{\varpi}^{\kappa-1}\widetilde{p}^{-\kappa}\widetilde{q}(\widetilde{x},\widetilde{\varpi}) = \widetilde{\mu}^{-\sigma}\widetilde{x}^{\sigma\kappa}\widetilde{\varpi}^{\kappa-1}\widetilde{q}(\widetilde{x},\widetilde{\varpi}) \tag{C.1.20}$$

where $\widetilde{q}(\widetilde{x},\widetilde{\varpi}) \equiv \frac{q_i(\widetilde{x},\widetilde{\varpi})}{q(\widetilde{x},\widetilde{\varpi})}$. Back into (C.1.18), we finally obtain:

$$\widetilde{\mu}^{-\sigma}\widetilde{x}^{\sigma\kappa}\widetilde{q}(\widetilde{x},\widetilde{\varpi}) = \frac{\varpi_i-1}{\varpi-1}\widetilde{\varpi}^{-(\kappa-2)} \tag{C.1.21}$$

Moreover, from (C.1.17a), we get:

$$y_i(\widetilde{x},\varpi_i)\mathbf{mc}_t - 2\chi(\varpi_i-1)\varpi_i = 0 \tag{C.1.22}$$

Equations (C.1.21)-(C.1.22) constitute a system of two equations in two unknowns, $\widetilde{x}$ and $\varpi_i$, which can be solved with a non-linear solver.

## C.2 Quantitative Results

We calibrate the extended model following the same calibration strategy as in the baseline model, which we outlined in Section 3.1. That is, we set $\rho = 0.04$, $\gamma = 2$, $\alpha = 1/3$, $\kappa = 3$, $z = 1$, $\zeta = 0.15$, $N = 10$, $\delta_K = 0.069$, $\delta_M = 0.03$, and assume as initial condition for awareness density that $f_0(0) = 1$ and $f_n(0) = 0$ for all $n = 1, \ldots, N$. The parameters $(\sigma, z_M, \nu, \eta)$, as well as the new parameter $\chi$, are all calibrated internally twice, to a set of moments for an "early" (2005) period, and to the same set of moments for a "late" (2014) period. The set of calibrated moments and their values in both periods is the same as in the baseline (see Table 1). Additionally, we calibrate $\chi$ to hit a certain share of total advertising expenditure that is spent in persuasive advertising. As there is no systematic nor reliable empirical evidence on this number, we choose the relatively conservative value of 10% for both periods.

Table D.1 in Appendix D shows the fit of the model on both calibrations. The quantitative results from the baseline model are overall robust to this extension. Comparing early and late model fits, we see that most of the change in the composition of advertising in the data is captured by the advertising-related parameters. As in the baseline model, both contacting and targeting are cheaper in the late period: the contacting cost drops from $\nu = 0.1848$ to $\nu = 0.1375$ (a 25.6% decline) and the targeting cost drops from $\eta = 0.5053$ to $\eta = 0.0604$ (a 88.1% decline), very similar to the baseline. The cost of persuasive advertising, by contrast, increases slightly, from $\chi = 22.93$ to $\chi = 23.36$ (a 1.88% increase).

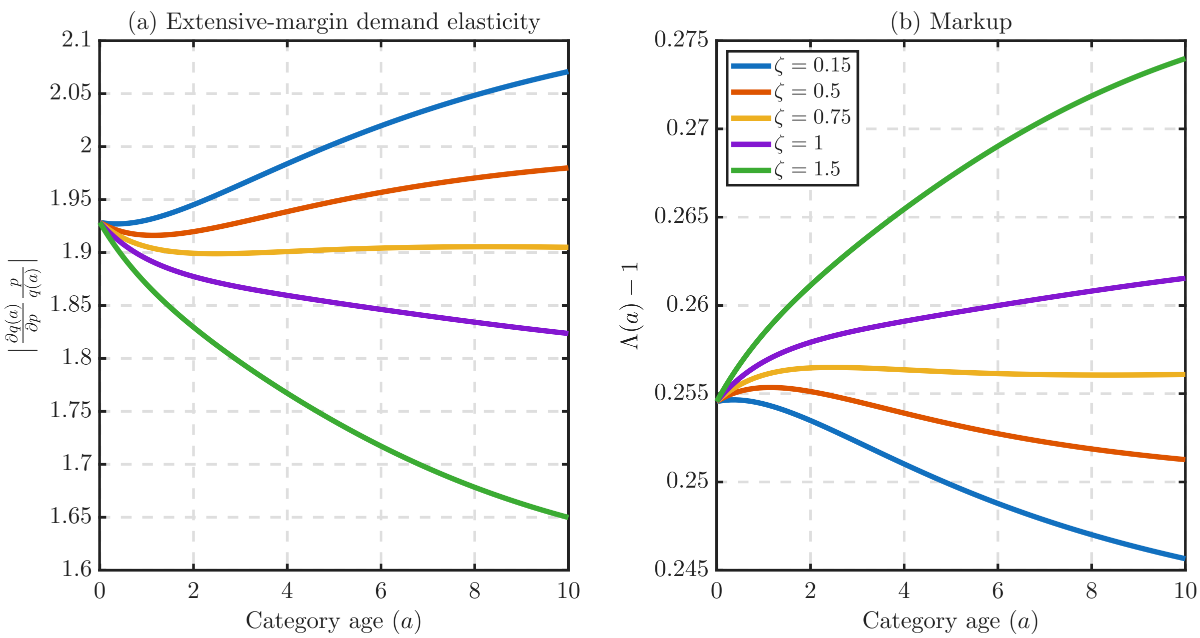
Table D.2 shows the results of the counterfactuals in which we fix all parameters to their late calibration values, except for $\eta$ and $\nu$, which we reset (together, in column (3a), and separately, in columns (4a) and (5a)) back to their early values. First, in columns (1) and (2) we see that both $\theta$ and $\mu_0$ increase from early to late, as was the case in the baseline model. The contact rate increases from $\theta = 0.9745$ to $\theta = 1.0122$ (a 3.87% increase) and the targeting rate increases from $\mu_0 = 1.2020$ to $\mu_0 = 2.0717$ (a 72.3% increase). Relative to the baseline model, the increase in contacting is less pronounced, and the one in targeting is more pronounced.

In terms of counterfactuals, qualitatively our results are very similar to those of the baseline model, and most of the effects of improved advertising technologies go in similar directions. Quantitatively, there are some differences. The overall welfare effects are slightly smaller, equal to 9.85% is consumption-equivalent terms (versus 10.6% in the baseline model), coming from the fact that the impact on aggregate distortion-adjusted match quality is slightly weaker at 10.14% (versus 10.71% in the baseline model).

Moreover, as in the baseline model, firms substitute across different types of advertising. When only the contacting cost is set back to its initial (higher) level (column (4a)), investment in targeting goes up (by 1.69%, versus 1.60% in the baseline). Similarly, when only the targeting cost is set back to its initial level (column (5a)), investment in contacting goes up (by 10.10%, versus 6.93% in the baseline). Interestingly, the share of total advertising spending that is devoted to persuasive advertising goes down (by 5.94%, column (4b)) when targeting becomes relatively cheaper, and up (by 5.41%, column (5b)) when contacting becomes relatively cheaper.

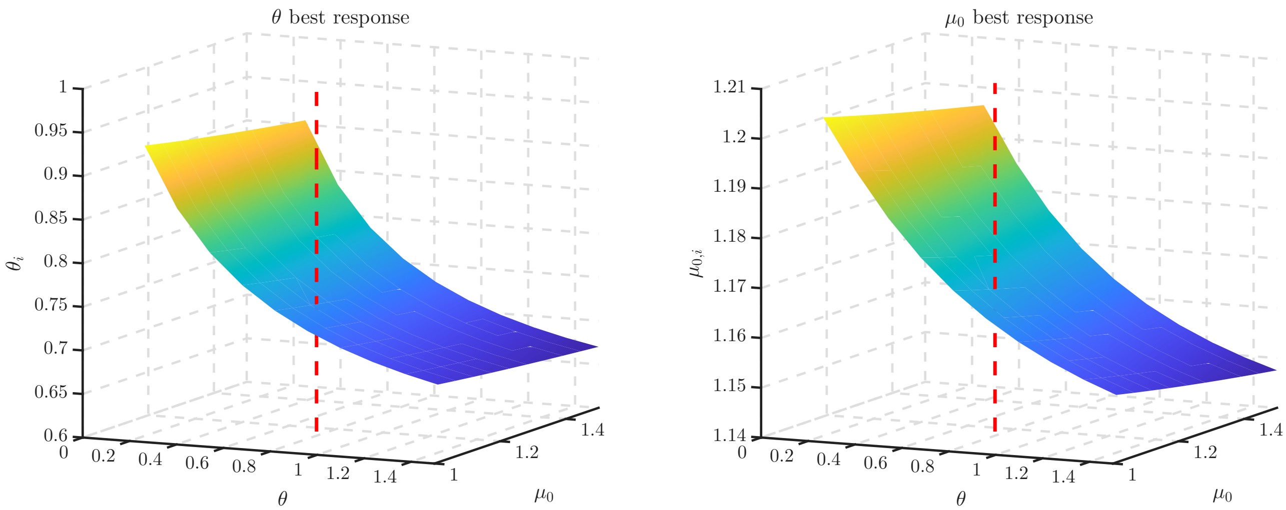
# D Additional Tables and Figures

Figure D.1: Examples of life-cycle dynamics.




**Notes:** This figure shows the age profiles of the extensive-margin price-elasticity of demand (left) and of markups (right) for different levels of the link destruction rate $\zeta$, assuming that the initial distribution of awareness across firms is uniform, i.e., $f_n(0) = \frac{1}{N+1}$ for every $n = 0, 1, \ldots, N$. The remaining parameters are set to their values in the early calibration (see Section 3.1 for details).

Figure D.2: Best response functions for the contact rate ($\theta$) and targeting ($\mu_0$).




**Notes:** This figure plots the best response functions for the contact rate ($\theta_i$) and time-zero targeting ($\mu_{0,i}$), as a function of the symmetric strategies $(\theta, \mu_0)$, in the early calibration. Best responses are downward-sloping in both $\theta$ and $\mu_0$, showing that advertising choices are strategic substitutes. The interior symmetric equilibrium levels $\theta_i = \theta$ and $\mu_{0,i} = \mu_0$ are marked with a red dashed vertical line.

Table D.1: Persuasive advertising extension: Calibrated parameters and model fit

| **Parameter** | | **Value** | **Moment** | **Data** | **Model** |
|---|---|---|---|---|---|
| *A. Early calibration (2005)* | | | | | |
| Product differentiation | $\sigma$ | 0.2246 | Cost-weighted average markup | 0.247 | 0.247 |
| Product category creation efficiency | $z_M$ | 0.1053 | Mass of categories (normalization) | 1 | 1 |
| Contact rate cost | $\nu$ | 0.1848 | Advertising share of GDP | 0.0220 | 0.0220 |
| Targeting cost | $\eta$ | 0.5053 | Return to targeting | 0.0482 | 0.0482 |
| Persuasive advertising cost | $\chi$ | 22.93 | Persuasive share of advertising | 0.10 | 0.10 |
| *B. Late calibration (2014)* | | | | | |
| Product differentiation | $\sigma$ | 0.2241 | Cost-weighted average markup | 0.246 | 0.247 |
| Product category creation efficiency | $z_M$ | 0.1007 | Mass of categories (normalization) | 1 | 1 |
| Contact rate cost | $\nu$ | 0.1375 | Advertising share of GDP | 0.0224 | 0.0224 |
| Targeting cost | $\eta$ | 0.0604 | Return to targeting | 0.2129 | 0.2130 |
| Persuasive advertising cost | $\chi$ | 23.36 | Persuasive share of advertising | 0.10 | 0.10 |
| | | | Real GDP per capita growth | 0.0523 | 0.0523 |

**Notes:** This table reports parameter values and model fit for the early calibration, corresponding to data moments from 2005, and the late calibration, corresponding to 2014, in the extended model with persuasive advertising.

Table D.2: Persuasive advertising extension: Counterfactual experiments

| | (1) Early (base) | (2) Late (base) | (3a) Early $\nu$ & $\eta$ | (3b) %-change (wrt late) | (4a) Early $\nu$ only | (4b) %-change (wrt late) | (5a) Early $\eta$ only | (5b) %-change (wrt late) |
|---|---|---|---|---|---|---|---|---|
| **A. Advertising and markups** | | | | | | | | |
| Contact rate ($\theta$) | 0.9745 | 1.0122 | 0.9853 | -2.65% | 0.9051 | -10.58% | 1.1144 | 10.10% |
| Targeting rate ($\mu_0$) | 1.2020 | 2.0717 | 1.2030 | -41.93% | 2.1067 | 1.69% | 1.1929 | -42.42% |
| Average return to targeting | 0.0482 | 0.2130 | 0.0480 | -77.44% | 0.2362 | 10.92% | 0.0419 | -80.31% |
| Average cost-wtd. markup | 1.2470 | 1.2469 | 1.2464 | -0.04% | 1.2479 | 0.09% | 1.2454 | -0.12% |
| **B. Expenditure shares of GDP** | | | | | | | | |
| Consumption share | 0.7811 | 0.7796 | 0.7810 | 0.18% | 0.7791 | -0.06% | 0.7813 | 0.21% |
| Advertising share | 0.0220 | 0.0224 | 0.0221 | -1.59% | 0.0239 | 6.56% | 0.0209 | -6.76% |
| Persuasive share of adv. | 0.1000 | 0.1000 | 0.0998 | -0.25% | 0.0941 | -5.94% | 0.1054 | 5.41% |
| Category creation inv. share | 0.0273 | 0.0284 | 0.0273 | -3.95% | 0.0275 | -3.02% | 0.0280 | -1.18% |
| Capital investment share | 0.1696 | 0.1696 | 0.1697 | 0.03% | 0.1695 | -0.09% | 0.1698 | 0.12% |
| **C. Income shares of GDP** | | | | | | | | |
| Labor share | 0.5358 | 0.5359 | 0.5360 | 0.03% | 0.5354 | -0.09% | 0.5365 | 0.12% |
| Capital share | 0.2679 | 0.2679 | 0.2680 | 0.03% | 0.2677 | -0.09% | 0.2682 | 0.12% |
| Profit share | 0.1963 | 0.1962 | 0.1959 | -0.13% | 0.1969 | 0.35% | 0.1953 | -0.47% |
| **D. Economic aggregates** | | | | | | | | |
| Mass of product categories | 1 | 1 | 0.9297 | -7.03% | 0.9694 | -3.06% | 0.9614 | -3.86% |
| Wage | 1.5923 | 1.6758 | 1.5085 | -9.98% | 1.6226 | -3.18% | 1.5692 | -6.36% |
| Consumption | 2.3213 | 2.4380 | 2.1979 | -9.85% | 2.3612 | -3.15% | 2.2852 | -6.27% |
| Consumption per category | 2.3213 | 2.4380 | 2.3641 | -3.03% | 2.4356 | -0.10% | 2.3770 | -2.51% |
| Match quality ($Q$) | 1.2310 | 1.2737 | 1.1449 | -10.11% | 1.2274 | -3.64% | 1.1953 | -6.15% |
| Distortion-adjusted quality ($QB^{-1}$) | 1.5317 | 1.5846 | 1.4239 | -10.14% | 1.5282 | -3.55% | 1.4854 | -6.26% |

**Notes:** Results from our counterfactual experiments on selected equilibrium variables for the extended model with persuasive advertising. *Notes:* See Table 3.

Table D.3: Set of internally identified parameters and model fit: product category destruction ($\delta_M = 0.06$)

| Parameter | | Value | Moment | Data | Model |
|---|---|---|---|---|---|
| *A. Early calibration (2005)* | | | | | |
| Product differentiation | $\sigma$ | 0.2267 | Cost-weighted average markup | 0.247 | 0.247 |
| Product category creation efficiency | $z_M$ | 0.161 | Mass of categories (normalization) | 1 | 1 |
| Contact rate cost | $\nu$ | 0.0194 | Advertising share of GDP | 0.022 | 0.022 |
| Targeting cost | $\eta$ | 0.1924 | Return to targeting | 0.0482 | 0.0482 |
| *B. Late calibration (2014)* | | | | | |
| Product differentiation | $\sigma$ | 0.231 | Cost-weighted average markup | 0.246 | 0.2524 |
| Product category creation efficiency | $z_M$ | 0.1539 | Mass of categories (normalization) | 1 | 1 |
| Contact rate cost | $\nu$ | 0.0165 | Advertising share of GDP | 0.0224 | 0.0224 |
| Targeting cost | $\eta$ | 0.0249 | Return to targeting | 0.2129 | 0.213 |
| | | | Real GDP per capita growth | 0.0523 | 0.0523 |

**Notes:** This table reports parameter values and model fit for the early calibration, corresponding to data moments from 2005, and the late calibration, corresponding to 2014, in the robustness check for the rate of product category destruction.

Table D.4: Counterfactual experiments: product category destruction ($\delta_M = 0.06$)

| | (1) Early (base) | (2) Late (base) | (3a) Early $\nu$ & $\eta$ | (3b) %-change (wrt late) | (4a) Early $\nu$ only | (4b) %-change (wrt late) | (5a) Early $\eta$ only | (5b) %-change (wrt late) |
|---|---|---|---|---|---|---|---|---|
| **A. Advertising and markups** | | | | | | | | |
| Contact rate ($\theta$) | 2.2582 | 2.1410 | 2.2871 | 6.82% | 2.0116 | -6.04% | 2.4385 | 13.90% |
| Targeting rate ($\mu_0$) | 1.2733 | 2.3537 | 1.2794 | -45.64% | 2.3861 | 1.38% | 1.2699 | -46.05% |
| Average return to targeting | 0.0482 | 0.2130 | 0.0490 | -77.02% | 0.2277 | 6.89% | 0.0449 | -78.91% |
| Average cost-wtd. markup | 0.2470 | 0.2524 | 0.2511 | -0.51% | 0.2529 | 0.20% | 0.2507 | -0.68% |
| **B. Expenditure shares of GDP** | | | | | | | | |
| Consumption share | 0.7638 | 0.7623 | 0.7636 | 0.16% | 0.7619 | -0.05% | 0.7639 | 0.21% |
| Advertising share | 0.0220 | 0.0224 | 0.0221 | -1.41% | 0.0233 | 4.28% | 0.0212 | -5.37% |
| Category creation inv. share | 0.0450 | 0.0468 | 0.0457 | -2.38% | 0.0463 | -1.04% | 0.0462 | -1.29% |
| Capital investment share | 0.1692 | 0.1685 | 0.1687 | 0.10% | 0.1684 | -0.04% | 0.1687 | 0.14% |
| **C. Income shares of GDP** | | | | | | | | |
| Labor share | 0.5346 | 0.5323 | 0.5329 | 0.10% | 0.5321 | -0.04% | 0.5330 | 0.14% |
| Capital share | 0.2673 | 0.2662 | 0.2664 | 0.10% | 0.2660 | -0.04% | 0.2665 | 0.14% |
| Profit share | 0.1981 | 0.2015 | 0.2007 | -0.41% | 0.2019 | 0.16% | 0.2004 | -0.55% |
| **D. Economic aggregates** | | | | | | | | |
| Mass of product categories | 1 | 1 | 0.9518 | -4.82% | 0.9891 | -1.09% | 0.9645 | -3.55% |
| Wage | 1.6491 | 1.7278 | 1.6048 | -7.12% | 1.7075 | -1.17% | 1.6305 | -5.63% |
| Consumption | 2.3560 | 2.4744 | 2.2997 | -7.06% | 2.4451 | -1.19% | 2.3367 | -5.56% |
| Consumption per category | 2.3560 | 2.4744 | 2.4163 | -2.35% | 2.4720 | -0.10% | 2.4227 | -2.09% |
| Match quality ($\boldsymbol{Q}$) | 1.2601 | 1.2999 | 1.2072 | -7.13% | 1.2826 | -1.33% | 1.2282 | -5.51% |
| Distortion-adjusted quality ($\boldsymbol{QB}^{-1}$) | 1.5713 | 1.6280 | 1.5104 | -7.22% | 1.6071 | -1.29% | 1.5362 | -5.64% |

**Notes:** Results from our counterfactual experiments on selected equilibrium variables for the robustness check on the rate of product category destruction.

Table D.5: Set of internally identified parameters and model fit: return to targeting robustness check

| Parameter | | Value | Moment | Data | Model |
|---|---|---|---|---|---|
| *A. Early calibration (2005)* | | | | | |
| Product differentiation | $\sigma$ | 0.2215 | Cost-weighted average markup | 0.247 | 0.247 |
| Product category creation efficiency | $z_M$ | 0.1127 | Mass of categories (normalization) | 1 | 1 |
| Contact rate cost | $\nu$ | 0.3791 | Advertising share of GDP | 0.0220 | 0.0220 |
| Targeting cost | $\eta$ | 1.4275 | Return to targeting | 0.0241 | 0.0241 |
| *B. Late calibration (2014)* | | | | | |
| Product differentiation | $\sigma$ | 0.2060 | Cost-weighted average markup | 0.246 | 0.2321 |
| Product category creation efficiency | $z_M$ | 0.1097 | Mass of categories (normalization) | 1 | 1 |
| Contact rate cost | $\nu$ | 0.2440 | Advertising share of GDP | 0.0224 | 0.0226 |
| Targeting cost | $\eta$ | 0.0394 | Return to targeting | 0.31935 | 0.3193 |
| | | | Real GDP per capita growth | 0.0523 | 0.0523 |

**Notes:** This table reports parameter values and model fit for the early calibration, corresponding to data moments from 2005, and the late calibration, corresponding to 2014, in the robustness check for the return to targeting.

Table D.6: Counterfactual experiments: return to targeting robustness check

| | (1) Early (base) | (2) Late (base) | (3a) Early $\nu$ & $\eta$ | (3b) %-change (wrt late) | (4a) Early $\nu$ only | (4b) %-change (wrt late) | (5a) Early $\eta$ only | (5b) %-change (wrt late) |
|---|---|---|---|---|---|---|---|---|
| **A. Advertising and markups** | | | | | | | | |
| Contact rate ($\theta$) | 0.7158 | 0.7595 | 0.7021 | -7.57% | 0.6368 | -16.16% | 0.8494 | 11.83% |
| Targeting rate ($\mu_0$) | 1.0836 | 2.4242 | 1.0792 | -55.48% | 2.4811 | 2.35% | 1.0743 | -55.68% |
| Average return to targeting | 0.0241 | 0.3193 | 0.0227 | -92.89% | 0.3669 | 14.89% | 0.0188 | -94.10% |
| Average cost-wtd. markup | 0.2470 | 0.2321 | 0.2318 | -0.10% | 0.2344 | 1.01% | 0.2297 | -1.03% |
| **B. Expenditure shares of GDP** | | | | | | | | |
| Consumption share | 0.7833 | 0.7806 | 0.7831 | 0.31% | 0.7801 | -0.07% | 0.7833 | 0.34% |
| Advertising share | 0.0220 | 0.0226 | 0.0214 | -5.48% | 0.0246 | 9.08% | 0.0198 | -12.36% |
| Category creation inv. share | 0.0254 | 0.0255 | 0.0243 | -4.90% | 0.0243 | -4.55% | 0.0253 | -0.88% |
| Capital investment share | 0.1692 | 0.1713 | 0.1713 | 0.02% | 0.1709 | -0.19% | 0.1716 | 0.19% |
| **C. Income shares of GDP** | | | | | | | | |
| Labor share | 0.5346 | 0.5411 | 0.5412 | 0.02% | 0.5401 | -0.19% | 0.5421 | 0.19% |
| Capital share | 0.2673 | 0.2705 | 0.2706 | 0.02% | 0.2700 | -0.19% | 0.2711 | 0.19% |
| Profit share | 0.1981 | 0.1884 | 0.1882 | -0.08% | 0.1899 | 0.82% | 0.1868 | -0.84% |
| **D. Economic aggregates** | | | | | | | | |
| Mass of product categories | 1 | 1 | 0.8927 | -10.73% | 0.9524 | -4.76% | 0.9426 | -5.74% |
| Wage | 1.4887 | 1.5855 | 1.3288 | -16.19% | 1.5037 | -5.16% | 1.4240 | -10.19% |
| Consumption | 2.1813 | 2.2874 | 1.9227 | -15.94% | 2.1719 | -5.05% | 2.0574 | -10.05% |
| Consumption per category | 2.1813 | 2.2874 | 2.1539 | -5.84% | 2.2806 | -0.30% | 2.1828 | -4.58% |
| Match quality ($\boldsymbol{Q}$) | 1.1770 | 1.2275 | 1.0309 | -16.02% | 1.1563 | -5.80% | 1.1094 | -9.62% |
| Distortion-adjusted quality ($\boldsymbol{QB}^{-1}$) | 1.4677 | 1.5124 | 1.2699 | -16.03% | 1.4274 | -5.62% | 1.3642 | -9.80% |

**Notes:** Results from our counterfactual experiments on selected equilibrium variables for the robustness check on the return to targeting. Columns (1) and (2) report baseline results for the early (2005) and late (2014) calibrations, respectively. Column (3a) reports 2014 results when both $\eta$ and $\nu$ are fixed at their 2005 values, with column (3b) stating the percentage change with respect to the baseline late calibration, i.e., the percentage change of column (3a) relative to column (2). Column (4a) repeats the experiment but re-setting only the contacting cost parameter $\nu$ to its 2005 level, with column (4b) stating the percentage change relative to column (2). Column (5a) does the same except for the targeting cost parameter $\eta$, with column (5b) stating the percentage change relative to column (2).