The Efficiency of Patent Litigation*

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Abstract

How efficient is the U.S. patent litigation system? Estimating a novel dynamic model, we characterize the litigation system's role in shaping innovation. In our model, heterogeneous firms innovate and may sue each other for patent infringement. In equilibrium, expected future litigation activity impacts firm innovation incentives. Moreover, some firms create positive innovation externalities while others impose negative externalities. Litigation reform can, therefore, improve or harm welfare, depending on how heterogeneous firms endogenously select into lawsuits. Estimating the model, we evaluate historical and recently proposed litigation reforms. Defendant-friendly reforms promote innovation and boost economic growth, improving welfare by up to 3.29%.

Keywords: patent litigation, innovation, firm value, growth, social welfare.

JEL Classification: G30, E22, O30, O40, K40.

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Innovation is essential for economic growth, but inventors' incentives to innovate depend on their ability to profit from costly research and development (R&D). While the U.S. patent system offers inventors protection, the system relies on the courts to enforce patent-holder rights through patent-infringement litigation.

But is this litigation efficient, and does it benefit consumers or spur growth? These questions are inherently quantitative, so to answer them, we develop and estimate a dynamic model of innovation and patent lawsuits. Our model embeds a realistic patent-litigation system in a dynamic general equilibrium model of corporate innovation and economic growth. These features allow us to evaluate the effects of counterfactual reforms to patent litigation. In particular, we find that weakening plaintiff rights by granting fewer injunctions against defendants raises social welfare by 3.29%. Similarly, we show that a reform that makes it twice as costly for a patent-holder plaintiff to file a lawsuit increases social welfare by 2.20%.

These types of quantitative results are important given the intense and enduring debate around patent-litigation reform, which centers around whether plaintiff (patent holder) rights are too strong or too weak. For example, fourteen patent-reform bills were proposed in the 113$^{\text{th}}$ Congress alone, with goals such as increasing plaintiff pleading requirements to strengthen defendant rights (Gugliuzza, 2015). Similarly, a Senate bill introduced in 2023 provided a compromise between tech companies, who believe excessive patent granting has led to frivolous patent lawsuits, and pharmaceutical companies, who believe it is too difficult to protect their innovation with patents.[1]

To flesh out the intuition behind our results, we begin with a simple model that illustrates how the patent litigation system impacts innovation and welfare. In the model, firms choose their level of innovation to maximize profits. This activity leads to better products, creates spillovers for firms with similar technologies, and allows the innovating firm to steal market share from competitors. Some firms underinvest in innovation, relative to the socially efficient benchmark, because they do not internalize spillovers. Other firms overinvest because they inefficiently internalize the transfer they extract from competitors by stealing customers. In this standard setting (Bloom et al., 2013), we introduce a litigation system. When a firm innovates, an incumbent firm might sue to block the innovation. If the court grants an injunction, the innovating firm cannot use its novel technology. We show

[1] See `https://www.reuters.com/legal/litigation/tech-pharma-companies-divided-pto-patent-eligibility-comments-2021-10-19/`.

that a high injunction rate, which is friendly to incumbent firms, leads to less innovation. Importantly, however, this reform has ambiguous effects on welfare. If most litigation occurs in technological fields where firms inefficiently overinvest in innovation, then such a reform improves welfare. Conversely, a high injunction rate harms welfare if most litigators are inefficiently underinvesting.

While this simple model provides helpful intuition, it is too stylized to match empirical facts and provide reliable policy counterfactuals. To overcome this limitation, we build a dynamic equilibrium model of innovation and litigation. In the model, heterogeneous firms compete in product markets. Incumbent firms and new potential entrants spend resources on research and development to innovate a better version of an existing product. After a successful innovation, the owner of the newly improved product enjoys a monopoly on that particular product until a competitor innovates a better version. Firms choose innovation levels and production policies to maximize profits, taking prices as given.

Within this dynamic equilibrium setting, we introduce a patent litigation system. Whenever a firm innovates a better version of a product, there is a chance that the new product infringes on a patent of an existing firm. If infringement occurs, the patent holder observes a random cost of filing a lawsuit. The patent holder sues the innovator if the expected lawsuit payoff exceeds the cost of filing.

In a lawsuit, the defendant (the innovating firm) privately observes its probability of winning a lawsuit. The plaintiff (the patent holder) makes a take-it-or-leave-it offer to settle. The defendant accepts if its continuation value from going to trial is worse than the cost of the proposed settlement payment. Due to the defendant's private information, both settlements and trials occur in equilibrium. If the defendant declines the settlement offer, the lawsuit goes to trial. In the trial, the defendant has an idiosyncratic random probability of winning. If the defendant loses, there is a chance that the plaintiff obtains an injunction, which prevents the defendant from selling its new product. Otherwise, the defendant patents and sells its newly innovated product.

In equilibrium, firms have rational expectations about how the patent system shapes the returns to innovating, which are generated by the following tradeoff. On the one hand, firms recognize that a plaintiff-friendly system makes it likely that an incumbent's patent will block their innovation. Plaintiff-friendly litigation reforms can thus discourage innovation. On the other hand, firms also recognize that a plaintiff-friendly system increases the returns to successful innovation. Conditional on an innovation not getting blocked, the innovating firm enjoys a longer period of monopolist profits because it can sue to block new entrants. Thus, plaintiff-friendly reforms can also encourage innovation.

We introduce firm heterogeneity in the model to allow innovation to have differing social values across firms. As in our illustrative model, innovation creates a positive externality through knowledge spillovers for firms using similar technologies. Firms do not internalize this externality, so some underinvest in innovation relative to a socially efficient benchmark. Other firms innovate products that are barely better than existing products. For these firms, innovation has little social value but a large private value because these firms inefficiently internalize the value they extract from incumbents through creative destruction, that is, by stealing their customers. Thus, these firms overinvest in innovation relative to a socially efficient benchmark. By combining this heterogeneity with an endogenous litigation process, we can model which types of firms select into using the patent litigation system.

We estimate the model parameters and use the estimated model to quantify the impact of R&D subsidies and litigation reforms. First, we find that promoting innovation by doubling an existing R&D subsidy raises social welfare by 2.76%. This result implies that aggregate investment in innovation falls below a socially efficient benchmark. However, the degree of over-investment or under-investment varies widely across firms because of differences in research efficiency. R&D subsidies targeting only high research-efficiency firms increase welfare, while those targeting only low research-efficiency firms reduce it.

Second, we examine how changes to the litigation system affect the impact of R&D subsidies. We find that defendant-friendly reforms boost R&D subsidy efficacy, as potential innovators facing fewer lawsuits respond more to subsidies. We conclude that model-based approaches ignoring patent litigation can overestimate R&D subsidy efficacy because of the endogenous linkages between the incentives that shape innovation and litigation.

Third, we examine the impact of historical and recently proposed litigation reforms. In our model, a defendant-friendly reform could encourage or discourage innovation, as firms can both sue and be sued. Our model estimates resolve this tension, as our counterfactuals show that defendant-friendly reforms promote innovation. Moreover, this rise in innovation is concentrated among firms with high research efficiency, so welfare rises. Specifically, we show that a reform modeled after the 2006 “eBay” Supreme Court ruling that lowered injunction rates improves welfare by 3.29%. Similarly, a reform modeled after a recent proposal to increase plaintiff filing costs improves welfare by 2.2%. In each exercise, we show that the improvement in welfare coincides with increased economic growth. Like R&D subsidies, litigation reforms are most effective when they target high-research-efficiency firms.

Fourth, we study targeted policies. Policies that impede litigation directed only at lawsuits aimed at high-efficiency firms improve welfare through enhanced innovation and growth, while the converse reduces welfare. We also examine infringement-specific reforms, finding that policies that impede litigation and that are targeted at cases in which the plaintiff seeks to protect its market share naturally improve incumbent welfare more than policies targeted at cases in which the plaintiff seeks to extract rents.

Finally, we explore two additional scenarios. We find that reducing the risk of infringement raises both innovation and welfare. When we simulate increased patent troll activity, we find less innovation, especially among potential industry entrants, with resources shifting to litigation rather than R&D, ultimately decreasing welfare.

Our paper lies in a body of work in finance and economics that studies the equilibrium consequences of corporate policies through extensions of classic endogenous growth models such as Grossman and Helpman (1991), Aghion and Howitt (1992), or Klette and Kortum (2004). Examples include Lentz and Mortensen (2008), Kung and Schmid (2015), Bena et al. (2016), Akcigit and Kerr (2018), Opp (2019), Cavenaile et al. (2019), Malamud and Zucchi (2019), Bena and Garlappi (2020), Kogan et al. (2020), Acemoglu et al. (2022), and Geelen et al. (2022). Our work extends this literature by addressing a novel topic. We examine how litigation systems shape the externalities that dynamically innovating firms impose on each other. Our framework can be used to answer many questions about the effects of innovation and litigation policies on firm and aggregate outcomes.

Specifically, our work lies in a literature that models innovation, litigation, and their interaction. Innovation models include Lin (2012), Acemoglu et al. (2018), Levine and Warusawitharana (2021), Liu and Ma (2021), Akcigit et al. (2022), Celik (2023), and Celik and Tian (2023). Examples of litigation models include Bessen and Meurer (2006), Marco (2006), Choi and Gerlach (2017), and Antill and Grenadier (2023). Models of the interaction between innovation and litigation include Abrams, Akcigit, Oz, and Pearce (2020) and Rempel (2023). Unlike Abrams, Akcigit, Oz, and Pearce (2020), which focuses on the role of patent trolls, our work addresses the efficiency of potential reforms to this system. This focus also differs from that of Rempel (2023), which studies how patents shape industry characteristics.

Our model also contains two features that separate it from all of this work. First, we model heterogeneous firms that impose both positive and negative externalities through their innovation. Second, firms endogenously choose to litigate depending on their heterogeneous type, shaping equilibrium innovation incentives. This novel combination of model features is important for our main contribution: quantifying the welfare impact of litigation reforms.

Additionally, we contribute to the empirical literature that studies how changes in plaintiff rights impact innovation activity (Sakakibara and Branstetter, 2001; Lerner, 2002; Moser, 2005; Lerner, 2009; Murray and Stern, 2007; Galasso and Schankerman, 2015; Williams, 2013; Cohen, Gurun, and Kominers, 2019; Mezzanotti, 2021; Kempf and Spalt, 2023; Lin, Liu, and Manso, 2021). While these reduced-form studies inform the policy debate around patent litigation reform, a model-based approach provides insights that reduced-form methods cannot. In particular, we quantify the impact of potential counterfactual reforms on both socially beneficial and socially harmful innovation.

1. Institutional Details

1.1. Patents

The U.S. patent system is designed to encourage innovation by giving patent holders the exclusive right to use their patented technology. Following a new discovery, inventors can apply for a patent with the United States Patent and Trademark Office (USPTO). Before granting the patent, the USPTO verifies that the invention is (i) novel; (ii) useful and operable; (iii) a non-obvious improvement relative to prior technology; and (iv) related to a patentable subject matter.[2] To ensure these criteria are met, a patent examiner verifies that the invention is not an obvious extension of an existing patented technology. Once the USPTO grants a patent, it expires 20 years after the application date (35 U.S.C §154).[3] During that period, the patent holder has the right to exclude others from making, using, or selling their patented inventions.

1.2. Patent litigation and injunctions

Patent holders can enforce their patents through patent infringement lawsuits. These lawsuits are typically filed in federal district courts. A patent holder can sue anyone who

[2]See `https://www.justia.com/intellectual-property/patents`

[3]See `https://www.law.cornell.edu/uscode/text/35/154`.

"makes, uses, offers to sell, or sells any patented invention."[4] If the lawsuit proceeds to trial, the plaintiff (the patent holder) and the defendant (the alleged infringer) present evidence to a jury. The infringer's product is compared to the plaintiff's patented invention. To establish infringement, the plaintiff must show the infringing product includes every element of the patented product. This is called the "all elements rule."[5] In some instances, the "doctrine of equivalents" allows a plaintiff to show infringement if some element of the patented product is missing in the infringing product, but the differences are insubstantial.[6]

If the plaintiff wins the lawsuit, the judge will typically grant a permanent injunction against the infringing defendant. The permanent injunction is an order forcing the defendant to stop all activity that infringes on the patent. If the injunction covers any step in the production of the defendant's product, the defendant must entirely shut down that product until a non-infringing process is developed. While patent infringement itself is a tort and not a criminal offense, violating a permanent injunction can lead to criminal penalties.[7]

Before 2006, permanent injunctions were nearly always granted after plaintiff victories. However, in 2006, the U.S. Supreme Court clarified the criteria for granting a permanent injunction. In *eBay Inc v MercExchange L.L.C.* ("eBay"), the Supreme Court ruled that courts must apply a four-factor test to determine whether a permanent injunction is appropriate.[8] In this test, the plaintiff must show (i) it suffered irreparable harm; (ii) other remedies (e.g., monetary damages) are inadequate to compensate the plaintiff; (iii) comparing the resulting hardships for the plaintiff and defendant, equitable relief (e.g., enforcing the patent) is warranted; and (iv) an injunction would not harm the public interest.[9] The rate at which successful plaintiffs obtained injunctions fell from 95% before eBay to 75% (Seaman, 2015).

Anticipating the trial process described above, many plaintiffs and defendants settle patent infringements out of court. Often, a plaintiff files a formal lawsuit that is ultimately settled before trial. In other instances, the plaintiff sends a "demand letter," asking the defendant to pay a license fee to use the patented technology. If the defendant agrees, this process is a form of out-of-court settlement. In our quantitative framework, we assume that

[4] See `https://www.law.cornell.edu/uscode/text/35/271`.

[5] See `https://definitions.uslegal.com/a/all-elements-rule/`.

[6] See `https://www.law.cornell.edu/wex/doctrine_of_equivalents`.

[7] See `https://www.mandourlaw.com/patent-injunction/`.

[8] See `https://www.law.cornell.edu/supct/cert/05-130`.

[9] See `https://content.next.westlaw.com/practical-law/intellectual-property-technology/patent-litigation`.

a plaintiff must hire a legal team to reach a settlement. This assumption is realistic because defendants often view demand letters as non-credible threats when they are not accompanied by formal lawsuits. Similarly, our assumption that a plaintiff must pay the cost of a trial to reach a settlement captures the view that defendants do not take plaintiffs seriously until they hire substantive legal counsel. This is consistent with the empirical observation that many settlements occur immediately before trial (Antill and Grenadier, 2023).

Following the law literature, we assume that only the defendant has private information (P'ng, 1983; Bebchuk, 1984; Spier, 2007). One-sided information avoids intractable signaling problems and multiple equilibria that arise with two-sided asymmetry in settlement offers. It is also realistic: as Spier (2007) explains, "the defendant may have first-hand knowledge about his degree of involvement in (or liability for) the [tort]" and defendants "may know better the credibility of their own witnesses and the quality and work ethics of their lawyers."

2. Illustrative Model

This section provides a simple model to illustrate the basic intuition behind the quantitative model that follows. We first show that privately optimizing agents can underinvest or overinvest in innovation relative to the socially efficient benchmark. We then show that changing the litigation system to be more plaintiff-friendly can improve or lower welfare, depending on which types of innovating firms use the litigation system.

Firms engage in two activities: producing output and creating new technology. There are two product markets $j = 1, 2$, and two types of technology, called classes, $c = 1, 2$. We refer to a type of firm by its pair (c, j). Within each pair, (c, j), there is a continuum of identical firms with measure one. Every firm starts with one product line. Each firm chooses how much to spend on innovation, with the cost of innovation for each firm in pair (c, j) given by χx_{cj}^2 for a parameter $\chi > 0$, and for a probability of successful innovation, x_{cj}. If a firm in (c, j) successfully innovates, it can steal the product line from a competitor in market j. We call this competitor the "target firm." All firms in pairs $(c = 1, j)$ and $(c = 2, j)$ are equally likely to be targets when a firm in (c, j) innovates.

If the target firm shares the same technology class as the innovator, the target sues to retain its product line with probability α_{cj}. This parameter captures features of the product market and technology class that allow easier legal action. For example, some technology classes might enable the writing of broad patents that relate to many potential innovations.

If the target firm that previously held the product line sues, it wins the lawsuit and obtains an injunction with probability $\zeta \in (0,1)$. If a firm in pair (c,j) innovates, its probability of getting a new product line is

$$\underbrace{\frac{1}{2}}_{\text{Target has c' } \neq c} + \underbrace{\frac{1-\alpha_{cj}\zeta}{2}}_{\text{Target has c=}c} = \frac{2-\alpha_{cj}\zeta}{2}. \tag{1}$$

If a firm keeps its original product line, it gets cash flow π. If a firm with pair (c,j) innovates and steals a product line, it gets cash flow $\pi + \lambda_c + \sigma x_{c,j'}$ from the stolen product line and the target firm losing the product line gets nothing from that line. The parameter λ_c captures the efficacy of innovation in technology class c. The parameter σ captures the technology spillover. If firms in technology class c all innovate more, their innovation has more impact across product lines. Note that the parameter π drives the business stealing incentive, and a social planner does not internalize who gets the cash flow π, but firms do. In contrast, firms do not internalize their technology spillovers. Specifically, fixing the equilibrium strategies of other firms, firm (c,j)'s problem is

$$\max_{x_{cj}} \underbrace{x_{cj}\frac{2-\alpha_{cj}\zeta}{2}(\pi+\lambda_c+\sigma x^*_{cj'})}_{\text{steal product line}} - \chi(x_{cj})^2. \tag{2}$$

Note that each firm cannot take any action to help retain its original product line, so the optimization is only a choice over how much effort to exert to steal another product line. Rearranging the first-order condition, the privately optimal innovation policy is

$$x^*_{cj} = (2-\alpha_{cj}\zeta)\frac{\pi+\lambda_c+\sigma x^*_{cj'}}{4\chi}. \tag{3}$$

The total equilibrium value of firms in the pair (c,j), which includes the value of the original product line, is

$$\text{Value}_{cj}(x^*) = \underbrace{\pi\left(1-\frac{x^*_{c'j}}{2}-\frac{x^*_{cj}(1-\alpha_{cj}\zeta)}{2}\right)}_{\text{Keep original product line}} + \underbrace{x^*_{cj}\frac{2-\alpha_{cj}\zeta}{2}(\pi+\lambda_c+\sigma x^*_{cj'})}_{\text{Steal product line}} - \chi(x^*_{cj})^2. \tag{4}$$

We now compare the privately optimal policies in equation (3) to those of a welfare maximizing social planner. The social planner chooses a vector x^s of innovation policies for all firms to optimize the sum of all firms' values $\sum_{c,j} \text{Value}_{cj}(x^s)$. After canceling terms, this sum can be expressed as

$$\max_{x^s} \sum_{c,j} \text{Value}_{cj}(x^s) = 4\pi + \max_{x^s} \sum_{c,j} x^s_{cj} \frac{2-\alpha_{cj}\zeta}{2}(\lambda_c + \sigma x^s_{cj'}) - \chi(x^s_{cj})^2. \tag{5}$$

In words, the planner cares only about the resources spent on innovation, the gains from successful innovation, λ_c, and the technology spillover. The planner does not care who gets the original product-line cash flow, π, so it does not internalize firms' internalized benefits from business stealing. Rearranging the first-order condition with respect to x^s_{cj}, we obtain

$$x^s_{cj} = \frac{2-\alpha_{cj}\zeta}{4\chi}(\lambda_c + \sigma x^s_{cj'}) + \sigma x^s_{cj'} \frac{2-\alpha_{cj'}\zeta}{4\chi}. \tag{6}$$

2.1. Illustrative model intuition

The following lemma summarizes useful intuition from the illustrative model.

Lemma 1. *Assume that $\pi > 0$, $\lambda_c > 0$, and $0 < \sigma < \chi$. Then:*

1. *As $\sigma \to 0$, privately optimizing agents overinvest in innovation: $x^*_{cj} > x^s_{cj}$.*
2. *As $\pi \to 0$, privately optimizing agents underinvest in innovation: $x^*_{cj} < x^s_{cj}$. As $\pi \to \infty$, privately optimizing agents overinvest in innovation: $x^*_{cj} > x^s_{cj}$.*
3. *As $\lambda_c \to 0$, privately optimizing agents overinvest in innovation: $x^*_{cj} > x^s_{cj}$.*

The proof is in Internet Appendix B. Intuitively, private agents inefficiently internalize the transfer π they extract from other agents when innovating to steal product lines. When this transfer gets large, firms spend too much on innovation. Conversely, private agents do not internalize the positive externality their innovation creates through technology spillovers. These technology spillovers grow with the parameter σ. As $\sigma \to 0$, there is no positive externality from innovation, so business stealing leads to overinvestment in innovation. As $\pi \to 0$, the business stealing incentive disappears, there is no reason to overinvest, so the private agents underinvest in innovation because they don't internalize technology spillovers.

Finally, we see that λ_c matters for the social value of innovation. As $\lambda_c \to 0$, the main incentive to innovate is to steal business from others, as there is little marginal improvement in technology. This condition means the level of innovation exceeds the socially efficient benchmark. In summary, the parameters π, σ, λ_c determine whether the level of innovation exceeds or falls short of the socially efficient benchmark.

2.2. The role of litigation

Equation (3) implies that injunctions (higher ζ) reduce innovation, with the effect varying by a firm's litigation propensity, α_{cj}. If firms that overinvest in innovation are more likely to litigate, discouraging their innovation through injunctions improves welfare. Conversely, if firms that underinvest in innovation are more likely to litigate, injunctions harm welfare. From Lemma 1, firms with low λ_c are more likely to overinvest in innovation. Thus, raising the injunction rate improves welfare when litigation is dominated by low λ_c firms, but reduces it when high λ_c firms dominate.

We now formalize this intuition with a numerical example. We use equations (3) and (6) to solve the model, with the parameter values in Panel A of Table 1. We assume technology classes 1 and 2 have $\lambda_1 = 3$ and $\lambda_2 = 0.1$. In Panel B, we consider a case in which only efficient innovators ($c = 1, \lambda_1 = 3$) use litigation, and in Panel C, we consider the case in which only inefficient innovators ($c = 2, \lambda_2 = 0.1$) use litigation. Comparing the third line of Panels B and C in Table 1, we find that technology class 1 firms ($c = 1$) underinvest in innovation, relative to the socially efficient benchmark, while class 2 firms ($c = 2$) overinvest, consistent with Lemma 1. In Panel B, increasing ζ leads to less innovation for firms in technology class 1, while firms in class 2 are unaffected. Class 1 firms underinvest more than before, harming welfare. In Panel C, we find that increasing ζ leads to less innovation for firms in technology class 2, which ameliorates their overinvestment and improves welfare.

2.3. Illustrative model limitations

This illustrative model demonstrates the key intuition behind the equilibrium quantitative model that follows. The tradeoff between positive technology spillovers and negative business stealing incentives implies that firms can innovate too much or too little relative to the socially efficient benchmark. Changes in the litigation system can encourage or discourage innovation, and the welfare effects of such changes depend on whether litigating firms are over-innovators or under-innovators.

However, the illustrative model has many limitations. First, the litigation system does not allow for trial outcomes such as injunctions to shape the incentives of firms to settle out of court. This feature is critical because most lawsuits are settled out of court. Second, the model does not allow incumbent innovation to crowd out new entrants. Because patents create temporary monopolies, to quantify welfare, we must model the effects of these barriers to entry. Third, the model is static. This feature severely limits its quantitative usefulness. For example, in the illustrative model, potential innovators always expect to be defendants in lawsuits, so a defendant-friendly system always encourages innovation. However, in practice, firms know when they innovate, they might end up using the litigation system to defend their patents in the future. Capturing the potential for an innovator to be a current defendant and a future plaintiff is essential for understanding how litigation shapes innovation.

To overcome these limitations, we develop a dynamic general equilibrium model with a realistic litigation system featuring asymmetric information and endogenous settlements. The model captures firms' dual roles as potential future defendants and plaintiffs, along with incumbents' and entrants' incentives. This realism allows us to match key features of our data, so we can quantify how changing the litigation system shapes innovation.

3. Model Setup

3.1. Environment and preferences

Time is continuous and denoted by $t \geq 0$. An infinitely-lived representative household has lifetime preferences given by

$$\int_0^\infty e^{-\rho t} \ln C_t dt, \tag{7}$$

where $\rho > 0$ is the discount rate, and C_t denotes consumption of the final good at time t. The household owns all assets A_t in the economy, which deliver a rate of return equal to r_t. It supplies labor $L = 1$ inelastically to firms at the real wage rate w_t.

3.2. Final good production

The final consumption good Y_t is produced competitively using differentiated goods from different industries indexed by $j \in \{1, \ldots, J\}$. The production function is expressed as

$$\ln Y_t = \sum_{j=1}^{J} \omega_j \ln Y_{jt}, \tag{8}$$

where $\omega_j \in (0,1)$ denotes the Cobb-Douglas weight of industry j's output Y_{jt} in production, with $\sum_{j=1}^{J} \omega_j = 1$. The output of each industry j, in turn, is produced by combining a continuum of differentiated goods in said industry according to the production function

$$\ln Y_{jt} = \int_0^1 \ln y_{ijt} di, \tag{9}$$

where y_{ijt} denotes the quantity of differentiated good $i \in [0,1]$ in industry j at time t. The price of the final consumption good is set as the numeraire, and the price of good i in industry j at time t is denoted as p_{ijt}.

3.3. Differentiated good production

As in Klette and Kortum (2004), in an industry, j, each firm owns a portfolio of blueprints to produce various differentiated goods, and multiple firms own blueprints for each differentiated good, i. A blueprint gives a firm the potential to produce. If it produces, it uses labor as an input, with productivity q_{ijt}. Following the Schumpeterian growth literature, we assume Bertrand competition between firms, so only the productivity leader produces any single good in equilibrium. We refer to each good a firm produces as a "product line," which it produces using the production function

$$y_{ijt} = q_{ijt} l_{ijt}, \tag{10}$$

where $l_{ijt} \geq 0$ is the labor that the leader hires for production.

3.4. Firms, technology classes, and product markets

Ignoring the effects of litigation, a firm can become the leader in a new product line by innovating to discover a better technology than the incumbent's. Likewise, a firm can lose its status as the leader if a competitor discovers a better technology. Without legal intervention, this creative destruction leads the prior leader to cede its product line to the innovating competitor, and a firm with no product lines exits.

Departing from the existing Schumpeterian growth literature, we introduce two further dimensions of firm heterogeneity. First, firms fundamentally differ from each other in terms of their innovation process, which we call a technology class. Specifically, each firm has a technology class $c \in \{1,\ldots,C\}$ that determines the productivity improvement from its successful innovations. As discussed in more detail below, the class c affects the knowledge base for developing new blueprints, and this knowledge base shapes the spillovers that enhance the firm's innovation activities. The technology class also determines whether new innovations can infringe upon the intellectual property of other firms.

Although an industry can contain firms with multiple technology classes, firms compete only within their own industry, so they can only obtain the product lines of firms in the same industry. As such, a firm's industry determines the returns to successful innovation from taking over new product lines and the risk of creative destruction from competitors in the same product market.

3.5. Incumbent innovation

Incumbent firms can engage in risky innovation to improve upon existing blueprints and thus potentially expand into new product lines. Each owned product line provides the firm with a lab to generate a Poisson arrival rate of successful innovation $x_{ijt} \geq 0$. Conditional on success, the firm improves upon one of the existing technology leaders' blueprints to produce a differentiated good, chosen randomly among all possible goods in the innovating firm's product market. The product line might or might not be in the same technology class. The productivity of the improved blueprint is given by

$$q_{ijt}^{new} = (1+\lambda_c) q_{ijt}^{old}, \tag{11}$$

where q_{ijt}^{old} is the productivity of the existing leader, and $\lambda_c > 0$ is the step size by which the new innovation improves upon the previous one. The size of λ_c is determined by the technology class c of the innovating firm.

The innovation process is costly. To generate the arrival rate, x_{ijt}, the firm must spend on R&D according to the cost function

$$C_c(x_{ijt}) = \frac{(1-s_{cj})\chi_c x_{ijt}^{\psi} Y_t}{1+\sigma M_{ct}}, \tag{12}$$

where $\chi_c > 0$ is a scale parameter, $\psi > 1$ introduces convexity, $s_{cj} \in [0,1]$ is an industry- and technology-class specific incumbent R&D subsidy rate, and Y_t ensures the R&D costs scale up with aggregate output along a balanced growth path (BGP) equilibrium. The last term,

σM_{ct}, $\sigma \geq 0$, captures the knowledge spillovers from other firms in the same technology class c. To define M_{ct}, we let I_{cjt} denote the set of goods i in industry j for which the leader has technology class c, and $\mu_{cjt} \in [0,1]$ denote the measure of the set I_{cjt}. Then $M_{ct} \in [0,1]$ is

$$M_{ct} = \sum_{j=1}^{J} \omega_j \mu_{cjt}. \tag{13}$$

This expression is the fraction of all product lines in the economy currently owned by firms with technology class c, where different industries receive weight in proportion to their Cobb-Douglas share in final good production. The higher the value of M_{ct} is, the cheaper it is for all firms in technology class c to discover new ideas, so past successful innovation by other firms in the same technology class increases a firm's research efficiency. The strength of this technology-class-specific knowledge spillover is governed by the parameter $\sigma \geq 0$, with a higher value of σ indicating stronger knowledge spillovers within the same technology class.

This technology-class-specific knowledge spillover complements the inherent Schumpeterian knowledge spillovers, which arise from enhancing the productivity of the current leader, as in equation (11). As such, our model includes *within-industry* knowledge spillovers that occur both *within* and *across* technology classes, as well as *within-technology-class* spillovers that span both *within* and *across* industries.

3.6. Entrant innovation

There is a measure-one continuum of identical entrepreneurs that can found new businesses through successful innovation. We use "entrepreneur" and "entrant" interchangeably. Entrants spend on R&D, which allows them to generate a Poisson arrival rate of successful innovation $z_t \geq 0$. The R&D cost function is

$$C_e(z_t) = (1 - s_e)\nu z_t^{\psi} Y_t, \tag{14}$$

where $\nu > 0$ is a scale parameter and $s_e \in [0,1]$ is the entrant R&D subsidy rate. As is the case for incumbent innovation, $\psi > 1$ is the convexity parameter, and the term Y_t ensures that R&D costs scale up with aggregate output along a BGP equilibrium.

If the entrepreneur's innovation is successful, it forms a new firm. With probability $\eta_{cj} \in [0,1]$, the new firm is in technology class c and industry j. For all c and j, the probabilities η_{cj} are exogenous parameters that satisfy $\sum_{c=1}^{C}\sum_{j=1}^{J}\eta_{cj} = 1$. The new firm with a single product line is immediately sold off at fair market value by the successful entrepreneur, who remains an entrepreneur and continues to found new businesses.

3.7. Patent infringement and litigation

When either an existing firm or an entrant successfully innovates, it creates litigation risk because its new innovation might infringe upon the intellectual property of existing firms in the same technology class. We consider two types of potential patent infringement.

Type-1 infringement occurs when an innovator (either an entrepreneur or an existing firm) attempts to take a product line from an incumbent firm and infringes on the incumbent's patent in the process. We assume that the incumbent must have (i) the same technology class as the innovator, so that the patent overlaps with the newly innovated technology, and (ii) the same industry as the innovator, because innovating firms can only take product lines from firms in the same industry. If the incumbent successfully sues the innovating firm, the incumbent can avoid losing its product line to the innovating firm. Conditional on successful innovation and the incumbent and innovator sharing a technology class, type-1 infringement occurs with exogenous probability $\kappa_1 \in [0,1]$.

An example of type-1 infringement occurred in 2010 when Motorola filed several patent-infringement lawsuits against Apple. The patents related to technologies used in smartphones, which both Motorola and Apple produced. Motorola sought injunctions to prevent Apple from using these technologies to produce the iPhone and similar products. This example is type-1 because Motorola and Apple directly competed in the market for smartphones.[10]

Type-2 infringement occurs when an innovator tries to take a product line from an incumbent firm and infringes on a *third party*'s patent in the process. The patent holder shares a technology class with the innovator, so the patent overlaps with the new invention. The exact infringed patent is randomly chosen from all product lines in the innovator's technology class, including those in other industries. In contrast to type-1 infringement, the plaintiff in a type-2 lawsuit is not the incumbent who owns the product line being taken. The type-2 plaintiff has no direct stake in whether the innovator or the incumbent owns the product line. However, the plaintiff can extract rents by suing to obtain a possible settlement,

[10] See, for example, 1:10-cv-23580 in Florida southern district court and https://www.wsj.com/articles/SB10001424052748703735804575536230822496028.

as a successful lawsuit can block the innovator's product line capture. Conditional on successful innovation and the incumbent and innovator having different technology classes, type-2 infringement occurs with exogenous probability $\kappa_2 \in [0,1]$.

An example of type-2 infringement occurred in 2003, when AT&T sued eBay for patent infringement. AT&T, a telecommunications company, had patented a system for secure online payment. eBay, an ecommerce company, owned the Paypal payment system that it used for processing payments. AT&T sought an injunction, claiming that eBay's Paypal system infringed on AT&T's patent. This example is type-2 because eBay and AT&T operated in distinct industries but had sufficient technological overlap for patent infringement to occur.[11]

After either type of infringement, litigation potentially ensues. We model the litigation subgame as follows, with its timeline illustrated in Figure 1. First, the plaintiff decides whether to hire a legal team. The cost of hiring a legal team is γY_t, where $\gamma > 0$ is drawn randomly, and Y_t ensures that litigation costs grow at the same rate as output in a BGP equilibrium. If the plaintiff chooses not to hire a legal team, the lawsuit is dropped, and the defendant gets to take over the product line. However, if the plaintiff hires a legal team, it makes a take-it-or-leave-it out-of-court settlement offer to the defendant.

The defendant has private information about its probability of winning the trial, τ, which is drawn from a uniform distribution with endpoints (τ_1^l, τ_1^h) or (τ_2^l, τ_2^h), with $\tau_1^l, \tau_1^h, \tau_2^l, \tau_2^h \in [0,1]$, for type-1 and type-2 infringements, respectively. Based on its private information τ, the defendant can accept the settlement or refuse. Refusal leads to a trial.

With probability τ, the defendant wins the trial and takes over the product line. With probability $1-\tau$, the defendant loses and the court decides whether to grant an injunction. With probability $\zeta_1 \in [0,1]$, an injunction is granted for a type-1 infringement, thus blocking the product line takeover. With probability $1-\zeta_1$ no injunction occurs and the defendant takes over the product line. The equivalent probability is denoted $\zeta_2 \in [0,1]$ for type-2 infringements. Below we examine changes to ζ_1 and ζ_2, as these parameters capture the inclination of a court to grant an injunction in the case of a proven patent infringement.

4. Model Solution

To solve the model, we calculate a BGP equilibrium with the following features:

[11] See 1:03-cv-01051 in Delaware district court and https://www.wired.com/2003/11/att-sues-ebay-in-patent-dispute/.

1. All agents act optimally given the equilibrium behavior of other agents, the constant real interest rate r, and the constant rate g at which the aggregate economy grows.
2. The solution to the household's consumption-saving problem determines r.
3. Each firm's policies solve a dynamic optimization in which the key state variable is the number of product lines that the firm produces.
4. Whenever patent infringement occurs, a subgame perfect equilibrium of a litigation subgame determines the outcome of the infringement.
5. The equilibrium policies of incumbents, entrants, and the household determine the growth rate g.

Features 3 and 4 imply that each firm chooses its innovation policy based on its rational expectation of future litigation activity. We now summarize each piece of the equilibrium. For ease of exposition, we delegate formal derivations to Internet Appendix C.

4.1. Household's problem

The representative household solves a consumption-savings problem. Specifically, given initial assets A_0, the representative household chooses its consumption C_t in each instant to maximize its lifetime utility

$$\max_{[C_t, A_t]_{t=0}^{\infty}} \left\{ \int_0^{\infty} e^{-\rho t} \ln C_t dt \right\}, \quad \text{subject to} \tag{15}$$

$$\dot{A}_t = r_t A_t + w_t - C_t, \quad \forall t \geq 0, \tag{16}$$

where $\dot{A}_t$ denotes the asset growth rate dA_t/dt. The household takes the wage rate, w_t, and the real interest rate, r_t, as given and faces a standard tradeoff. It is impatient and prefers to consume early, but doing so hinders the growth of its assets A_t. Given this tradeoff, the household chooses a consumption process. The household's condition for optimality delivers the Euler equation $\frac{\dot{C}_t}{C_t} = r_t - \rho$, which implies that the growth rate of C_t equals $r_t - \rho$. In a BGP equilibrium, C_t must grow at the constant equilibrium rate g, so $r_t = r = \rho + g$.

4.2. Final good producer's problem

Given equations (8) and (9), a competitive final good producer solves a static profit maximization problem at each instant t:

$$\max_{\left\{[y_{ijt}]_{i=0}^{1}\right\}_{j=1}^{J}} \left\{ \exp\left(\sum_{j=1}^{J} \omega_j \left(\int_0^1 \ln y_{ijt} di \right) \right) - \sum_{j=1}^{J} \left(\int_0^1 p_{ijt} y_{ijt} di \right) \right\}. \tag{17}$$

The final good producer chooses the quantity of each input y_{ijt} trading off (i) the marginal output it can produce by using another unit of y_{ijt} against (ii) the marginal cost p_{ijt} of purchasing the additional unit. For any good i in industry j, the first-order condition delivers $y_{ijt} = \left(\omega_j Y_t / \, p_{ijt}\right)$. This expression pins down the demand for y_{ijt} as a function of the price p_{ijt} charged by the owner of product line i.

4.3. Product-line owner's static pricing problem

Each incumbent firm solves a dynamic optimization described in the next section, but its pricing decisions are static and can be solved independently at each instant t. Under Bertrand competition, only the technology leader produces a positive quantity of good i. This leader has productivity q_{ijt}, while the second-most-productive firm has productivity $q^{old} = q_{ijt}/(1+\lambda_c)$, where c is the leader's technology class. This productivity gap exists because the leader improved upon the second-most-productive firm's technology by factor $1+\lambda_c$. In Bertrand competition, the product-line leader charges a price, p_{ijt}, that would leave the second-most-productive firm with zero profit if it charged p_{ijt}. This price makes the second-most-productive firm (and all other firms) forgo production of good i because they cannot profitably compete. The product-line owner can nonetheless charge p_{ijt} and make a profit due to its unparalleled productivity.

Formally, in Internet Appendix C, we show that the product-line leader optimally charges $p_{ijt} = w_t(1+\lambda_c)/q_{ijt}$. This "limit price" is the highest price that discourages less productive competitors from paying employees the wage rate w_t to produce good i. We also show that the product-line leader makes profit flow $\pi_{ijt} dt$ by charging this price, where

$$\pi_{ijt} = \frac{\lambda_c}{1+\lambda_c} \omega_j Y_t. \tag{18}$$

From this expression, we see that π_{ijt}: (i) grows at the same rate as aggregate output, Y_t, (ii) is linearly related to the industry j's share ω_j, and (iii) is increasing in the technology class-specific productivity step size λ_c, which is also the net markup. Note that π_{ijt} is independent of the productivity q_{ijt}. Intuitively, if q_{ijt} is high, then the product-line owner has a highly productive competitor because $q^{old} = q_{ijt}/(1+\lambda_c)$ is also high, so the owner must charge a low price that cancels out the potential benefits of high productivity. This property of π_{ijt} implies that the relevant state variable for an incumbent firm's dynamic problem is not the set of productivities of owned product lines but simply the number of them, as in Klette and Kortum (2004).

4.4. Incumbent's dynamic optimization problem

We now summarize the incumbent firm's dynamic optimization problem. Let $V_{cjt}(n)$ be the value of an incumbent firm in technology class c and industry j that owns n product lines at time t. It is the net present value of future cash flows associated with product-line profits, R&D expenses, and litigation activity. Formally, the Hamilton-Jacobi-Bellman (HJB) equation characterizing the incumbent's value function and optimization problem is

$$
\begin{aligned}
r_t V_{cjt}(n) - \dot{V}_{cjt}(n) \quad = \quad & \max_{\{x_{mcjt}\}_{m=1}^{n}} \Bigg\{ \underbrace{\sum_{m=1}^{n} \frac{\lambda_c}{1+\lambda_c} \omega_j Y_t}_{\text{static profit flow}} - \underbrace{\sum_{m=1}^{n} \frac{(1-s_{cj})\chi_c x_{mcjt}^{\psi} Y_t}{1+\sigma M_{ct}}}_{\text{R\&D expenses}} + \underbrace{n \sum_{j'=1}^{J} R_{cj't}}_{\text{litigation rent}} \\
+ \quad & \underbrace{\left(\sum_{m=1}^{n} x_{mcjt}\right)}_{\text{successful innov.}} \times \underbrace{\left(V_{cjt}^{+}(n) - V_{cjt}(n)\right)}_{\mathbb{E}(\Delta V|\text{successful innov.})} + \underbrace{n d_{jt}}_{\text{creative destruc.}} \times \underbrace{\left(V_{cjt}^{-}(n) - V_{cjt}(n)\right)}_{\mathbb{E}(\Delta V|\text{other innov.})} \\
+ \quad & \underbrace{\delta\left(0 - V_{cjt}(n)\right)}_{\text{exogenous exit}\times\Delta V} \Bigg\}. \qquad (19)
\end{aligned}
$$

Equation (19) offers a great deal of intuition. Because each product line m owned by the incumbent comes with a lab for innovation, at each instant t, the incumbent chooses a level of innovation x_{mcjt} for each lab to maximize its net present value. The first line of equation (19) captures the portion of this value that comes from immediate flow profits, which contain three components. The first term on the right side captures the profits from owning n product lines, from equation (18). The second term is R&D expenses, from equation (12). The third term captures the rents that the incumbent firm can extract through type-2 litigation, which we calculate in Section C.5 in Internet Appendix C. When the incumbent firm holds the patent in a type-2 infringement case, it can sue the innovating firm to extract rents. Because the innovating firm can be in any industry, these potential rents are aggregated across all industries. We denote by R_{cjt} the total rent flows from infringements by firms in industry j for a single product line, which we then multiply by the number of product lines n.

The second line in equation (19) captures changes in value when the firm gains or loses product lines. The first term represents the expected gain from new product lines as the Poisson arrival rate of a successful innovation multiplied by the expected value improvement from innovation. To understand this gain, we must account for patent litigation. Absent litigation, a successful innovation would increase the firm's net present value from $V_{cjt}(n)$ to $V_{cjt}(n+1)$. However, patent litigation implies that successful innovation does not necessarily result in a product line. Therefore, we define $V^{+}_{cjt}(n)$ as the expected value of a firm conditional on successful innovation, but before potential patent infringement and litigation outcomes are realized. Therefore, successful innovation increases the firm's expected value by $V^{+}_{cjt}(n) - V_{cjt}(n)$. We describe the litigation subgame that determines $V^{+}_{cjt}(n)$ in the following section.

The second term captures creative destruction, that is, the loss of product lines when competitors innovate. This term represents this loss as the number of product lines, n, times the rate of creative destruction, d_{jt}, times the incumbent's expected loss in value from a potential product line takeover, $V_{cjt}(n)$ to $V^{-}_{cjt}(n)$, where $V^{-}_{cjt}(n)$ is the incumbent's expected value immediately before potential litigation to block the new innovation. We describe the litigation subgame that determines $V^{-}_{cjt}(n)$ in the following section. In Internet Appendix C, we calculate the "creative destruction rate" d_{jt}, which measures the rate at which competing firms innovate on one of the incumbent firm's product lines. Each of the n product lines owned by the firm faces this displacement risk.

Finally, the third line includes the risk of exogenous firm exit at a rate $\delta \geq 0$, which captures firm exit events for reasons other than losing all product lines. When a firm exogenously exits, it is replaced by an identical firm that inherits its product lines.

To characterize the value function, $V_{cjt}(n)$, fully, we must find the equilibrium objects $R_{cjt}, d_{jt}, V^{+}_{cjt}(n)$, and $V^{-}_{cjt}(n)$, which we calculate in Section C.5 and Section C.6 of Internet Appendix C. However, we can make a few observations without these calculations. First, each product line has the same flow profit, so the total flow profit is linear in the number of product lines n. Second, the first-order condition with respect to the innovation rate x_{mcjt} for any lab m implies the following optimal innovation policy

$$x_{mcjt} \quad = \quad \left(\frac{\left(V_{cjt}^{+}(n) - V_{cjt}(n)\right)(1+\sigma M_{ct})}{(1-s_{cj})\chi_c \psi Y_t} \right)^{\frac{1}{\psi-1}} \equiv x_{cjt}(n). \tag{20}$$

This expression implies that the firm chooses the same innovation rate, $x_{cjt}(n)$, for each of its n labs. Accordingly, the total R&D expense and the firm-level arrival rate of successful innovation are also linear in the number of product lines n. As we prove in Internet Appendix C, these properties imply that the firm value function $V_{cjt}(n)$ itself is linear in n. Formally, $V_{cjt}(n) = v_{cj} n Y_t$ for coefficients v_{cj} that we calculate in closed form.

Equation (20) also provides helpful intuition about firm innovation incentives. As technology spillovers σM_{ct} grow, R&D becomes cheaper, so firms innovate more. Similarly, subsidies, s_{cj}, lead to more innovation. If a defendant-friendly litigation system makes it unlikely that an innovating firm's invention will be blocked, then the return to innovating, $V_{cjt}^{+}(n) - V_{cjt}(n)$ is high, and firms innovate more. As such, defendant-friendly reforms can potentially boost innovation. However, a plaintiff-friendly system could also boost innovation in two ways. It raises the rents, R_{cjt}, that a firm can extract by suing other firms for patent infringement, and it helps the firm use patent protection to fend off competitors and retain ownership of its own product lines. Put differently, a defendant-friendly system reduces the probability of forfeiting a successful innovation but also lowers the value of the product line because of reduced intellectual property protection. Our estimation allows us to determine quantitatively which of these countervailing forces dominates.

4.5. Entrepreneur's problem

Next, we characterize the entrant's optimization problem, which is a simplified version of the incumbent's problem. It is static because, by definition, an entrant always has zero product lines. Specifically, in each instant t, the entrepreneur solves

$$\max_{z_t \geq 0} \left\{ -(1-s_e)\nu z_t^{\psi} Y_t + z_t \sum_{c=1}^{C} \sum_{j=1}^{J} \eta_{cj} V_{cjt}^{+}(0) \right\}. \tag{21}$$

The first term in the maximization is the R&D cost incurred by the entrepreneur, where $s_e \in [0,1]$ is the entrant R&D subsidy rate. The second term is the expected return from entrant innovation. The Poisson arrival rate of successful innovation is z_t. Conditional on successful innovation, the new firm has technology class c and industry j with probability η_{cj}.

If there were no litigation, the value of the new firm would be $V_{cjt}(1)$. However, due to litigation risk, the new firm's value equals the expected value of an incumbent firm with zero existing product lines that succeeded in innovation, but before potential patent infringement and consequent litigation outcomes are realized, denoted $V^+_{cjt}(0)$.[12]

The first-order condition with respect to entrant innovation z_t in equation (21) pins down its optimal value as

$$z_t \quad = \quad \left(\frac{\sum_{c=1}^{C}\sum_{j=1}^{J}\eta_{cj}V^+_{cjt}(0)}{(1-s_e)\nu\psi Y_t}\right)^{\frac{1}{\psi-1}}. \tag{22}$$

As with optimal incumbent innovation, both subsidies and defendant-friendly litigation systems raise innovation. However, entrants can still benefit from plaintiff-friendly systems through changes in product line value, which, as shown in equation (26) below, depends on the litigation system.

4.6. Litigation and settlements

We now describe the solution to the litigation subgame that ensues after successful innovation. In a BGP equilibrium, a subgame perfect equilibrium (SPE) of the litigation subgame determines the litigation outcome. In Internet Appendix C, we calculate the SPE strategies in closed form and use them to calculate the pre-litigation value functions $V^+_{cjt}(n)$ and $V^-_{cjt}(n)$. Here, we outline the solution and provide intuition.

Defendant's decision to go to trial: We start with the defendant's decision to go to trial. Its expected payoff from a trial is given by

$$[\tau+(1-\tau)(1-\zeta)]\left(V_{cjt}(n+1)-V_{cjt}(n)\right), \tag{23}$$

[12] Note that $V^+_{cjt}(0) = V^+_{cjt}(0) - V_{cjt}(0)$ since $V_{cjt}(0) = 0$.

where $\zeta \in \{\zeta_1, \zeta_2\}$ depends on the infringement type. $V_{cjt}(n)$ is the value function for a firm with n product lines, technology class c, and industry j. The first component is the defendant's probability of taking over the product line, either by winning the trial or having the court decide not to grant an injunction despite recognizing the infringement. The second component is the defendant's value improvement from gaining an additional product line.

The defendant's alternative to a trial is to accept the plaintiff's settlement offer, s, in which case, the defendant must pay s to the plaintiff. This choice yields a payoff of $V_{cjt}(n+1) - V_{cjt}(n) - s$. The defendant accepts the offer if this payoff is higher than the trial payoff in equation (23). In Internet Appendix C, we calculate a cutoff, $\bar{s}_{cjt}(n,\tau)$, such that the defendant accepts if and only if $s \le \bar{s}_{cjt}(n,\tau)$. This cutoff decreases with the defendant's probability of trial victory, τ, which the plaintiff does not observe.

Plaintiff's choice of settlement offer: We now move backward and examine the plaintiff's choice of a take-it-or-leave-it settlement offer s. The plaintiff's choice of s varies across infringement types.

We first consider settlement offers in the simpler case of type-2 patent infringement. Recall that in type-2 infringement, the plaintiff is not at risk of losing a product line. The plaintiff's payoff is, therefore, equal to the settlement offer of s if the defendant accepts or 0 if the defendant rejects and goes to trial. The plaintiff's problem is thus

$$\max_{s \ge 0} \left\{ s \times \mathbb{P}(s \le \bar{s}_{cjt}(n,\tau)) \right\}, \tag{24}$$

where the second component is the probability that the offer is accepted. The plaintiff faces a tradeoff: a higher settlement offer of s leads to a higher profit if the defendant accepts. However, a higher settlement offer is less likely to be accepted. Importantly, the plaintiff does not know the defendant's probability τ of winning a trial, so the plaintiff does not know whether a settlement offer will be accepted. In Internet Appendix C, we calculate the optimal settlement offer s^* that solves this problem.

To ensure a solution s^* such that $\mathbb{P}(s \le \bar{s}_{cjt}(n,\tau)) \in (0,1)$, for type-2 infringements we assume that the endpoints of the uniform distribution of τ satisfy $1 + \tau_2^l \le 2\tau_2^h$. This assumption implies that settlement offers are sometimes rejected and sometimes accepted in equilibrium.

Next, we consider settlement offers in type-1 patent infringement. This problem differs from type-2 infringement because the plaintiff owns the product line facing possible creative destruction. As such, because settling out of court implies losing its product line, the plaintiff requires a higher value for the settlement than it would in the case of a type-2 infringement. Thus, the plaintiff chooses a settlement offer, s, to solve the following problem

$$\begin{aligned}\max_{s\geq 0} \quad \Bigg\{ \quad & \int_{\tau_1^l}^{1-s/(\zeta_1(V_{cjt}(n^d+1)-V_{cjt}(n^d)))} \left(V_{cjt}(n^p-1)-V_{cjt}(n^p)+s\right)\frac{1}{\tau_1^h-\tau_1^l}d\tau \\ & +\int_{1-s/(\zeta_1(V_{cjt}(n^d+1)-V_{cjt}(n^d)))}^{\tau_1^h} (\tau+(1-\tau)(1-\zeta_1))(V_{cjt}(n^p-1)-V_{cjt}(n^p))\frac{1}{\tau_1^h-\tau_1^l}d\tau\Bigg\},\end{aligned} \tag{25}$$

where n^d and n^p are the numbers of product lines held by the defendant and plaintiff, respectively. The term $V_{cjt}(n^p-1)-V_{cjt}(n^p)$ is negative and reflects the plaintiff's cost of losing the product line. The defendant's value function $V_{cjt}(n^d)$ appears in the integral bounds because of the defendant's subsequent choice of whether to accept the settlement offer. Thus, the first integral represents the value to the plaintiff (the settlement amount minus lost product line value) over the range of defendants who accept. The second integral gives the expected value when settlement is rejected: the probability of losing at trial or lacking injunction protection, multiplied by the loss in value from giving up the product line.

Plaintiff's choice of whether to hire a legal team: Finally, we move backward in time and characterize the plaintiff's decision to hire a legal team. Recall that the plaintiff observes a stochastic cost γY_t of hiring a legal team to pursue a lawsuit. The plaintiff hires a legal team if its expected value from proceeding to the next stage (making a settlement offer) exceeds γY_t. In Internet Appendix C, we calculate equilibrium thresholds for type-1 and type-2 infringement such that the plaintiff hires a legal team if and only if γ is below the respective threshold. Given these thresholds, we can calculate the equilibrium probability that the plaintiff files a lawsuit after an infringement.

In Internet Appendix C, we solve for the SPE and calculate all of the equilibrium objects in closed form. Proposition 1 in Section C.2 of the Internet Appendix C describes the SPE of the type-2-infringement game. Proposition 2 in Section C.3 of the Internet Appendix C describes the SPE of the type-1-infringement game.

The SPE continuation values are influenced by the firm value function $V_{cjt}(n)$ (which establishes the stakes for all parties involved). The continuation values in turn determine the values R_{cjt}, $V^+_{cjt}(n)$, and $V^-_{cjt}(n)$ that appear in the HJB equation. In this way, the litigation equilibrium influences innovation incentives.

4.7. Completing the equilibrium

The previous sections summarized each agent's optimization problem, the SPE of the litigation subgame, and the determination of the real interest rate, r, based on the household's problem. We now summarize how we verify our conjectures and complete the model solution. We focus on the intuition and relegate the formal definition of the BGP equilibrium and proofs to Section C.5 of Internet Appendix C.

Firm value and innovation: We first confirm that the firm's value function $V_{cjt}(n)$ depends linearly on the number of product lines n, the value of a product line v_{cj}, and scales with aggregate output Y_t. Formally, $V_{cjt}(n) = v_{cj} n Y_t$. In equilibrium, the value of product line v_{cj} is given by

$$v_{cj} = \frac{\frac{\lambda_c}{1+\lambda_c}\omega_j + \sum_{j'=1}^{J}\hat{R}_{cj'} - \frac{(1-s_{cj})\chi_c x_{cj}^{\psi}}{1+\sigma M_c}}{\rho + \delta - x_{cj}L_{cj}^{\text{def}} + d_j L_{cj}^{\text{plain}}}. \tag{26}$$

Intuitively, equation (26) resembles the value of a growing perpetuity. The numerator captures the cash flow firm receives from owning a product line. The first term, $\frac{\lambda_c}{1+\lambda_c}\omega_j$, is the firm's profits from selling its product. The second term is the rents that the firm extracts by suing other firms for type-2 patent infringement, where $\hat{R}_{cj} \equiv R_{cjt}/Y_t$ is the descaled flow of litigation rents. The third term is the R&D costs for the lab associated with the product line. Thus, holding all else fixed, a product line becomes more valuable when a change in parameters either increases profits, increases litigation rents, or reduces R&D expenses.

The denominator in equation (26) captures the *effective discount rate*. A higher discount rate, $\rho = r - g$, or exogenous exit rate, δ, reduces the present value of future cash flows. As a departure from the standard growing perpetuity formula, in our model, innovation and litigation not only influence the cash flows in the numerator but also affect the likelihood of the firm securing these cash flows, thereby altering the effective discount rate.

First, higher innovation, x_{cj}, increases the likelihood of securing the product line, thus reducing the effective discount rate. This effect is amplified by lower infringement losses, which are captured by L_{cj}^{def}. Formally, we define L_{cj}^{def} as

$$V_{cjt}^{+}(n) - V_{cjt}(n) \equiv L_{cj}^{\text{def}}\big(V_{cjt}(n+1) - V_{cjt}(n)\big), \tag{27}$$

where we calculate L_{cj}^{def} in closed form in equation (C.45) in Internet Appendix C. The left-hand side captures the change in the expected value of the firm after its own successful innovation. In the absence of patent infringement, a successful innovation allows the firm to gain a product line with certainty ($L_{cj}^{\text{def}} = 1$), and the number of product lines it owns rises from n to $n+1$. Thus, $1 - L_{cj}^{\text{def}}$ represents the proportion of innovation value lost because of the risk of infringing on other firms' intellectual property.

The final component in the denominator of equation (26) captures the impact of creative destruction on the firm's effective discount rate. Recall that d_j is the rate at which an incumbent in industry j faces a threat to its product line due to a competitor's innovation. When this creative destruction rate is high, the incumbent has a high risk of its product line being taken, increasing the effective discount rate. This effect is amplified by weaker IP protection, which is captured by L_{cj}^{plain}. Formally, we define L_{cj}^{plain} through the equation

$$V_{cjt}^{-}(n) - V_{cjt}(n) \equiv L_{cj}^{\text{plain}}\big(V_{cjt}(n-1) - V_{cjt}(n)\big), \tag{28}$$

where we calculate L_{cj}^{plain} in closed form in equation (C.48) in Internet Appendix C.

This expression is analogous to equation (27). The left-hand side captures the change in the expected value of the firm after its competitors' successful innovation. Without intellectual property protection, when competitors successfully innovate, the firm loses the product line with certainty ($L_{cj}^{\text{plain}} = 1$), and the number of product lines it owns falls from n to $n-1$. Therefore, $1 - L_{cj}^{\text{plain}}$ reflects the value gained from using patent protection to fend off competitors and retain ownership of the product line.

In summary, the value v_{cj} of a product line depends on cash flows (profits, R&D expense, and litigation rent extraction) and the effective discount rate, which reflects equilibrium outcomes, such as the growth of the economy and firms' litigation and innovation policies. Given this value, we prove that the equilibrium per-product-line incumbent innovation arrival rate is

$$x_{cj} \quad = \quad \left(\frac{L_{cj}^{\text{def}} v_{cj}(1+\sigma M_c)}{(1-s_{cj})\chi_c \psi}\right)^{\frac{1}{\psi-1}}. \tag{29}$$

Equation (29) indicates that incumbent innovation x_{cj} rises with the value of a product line, as higher expected returns incentivize firms to invest more in innovation. In contrast, higher value losses from infringement (lower L_{cj}^{def}) reduce innovation. Further details are in Internet Appendix C, which includes the derivation of the entrant's optimal innovation decision.

Legal reforms affect firm value and innovation through multiple terms in equations (26) and (29), which capture effects on both cash flows and discount rates. Because these channels can work in opposing directions, a reform's impact on firm and aggregate outcomes is *ex ante* ambiguous.

Product line and firm distribution: Next, we calculate the stationary equilibrium distribution of product lines across industries and technology classes. Formally, we calculate the stationary measure μ_{cj} of all product lines in industry j for which the leader has technology class c. These measures sum to one for each industry, $\sum_{c=1}^{C} \mu_{cj} = 1$, and they depend on the relative equilibrium levels of innovation across technology classes in equation (20).

While not needed to compute the BGP equilibrium, we can also compute the stationary firm size distributions $\varphi_{cj}(n)$ for firms of type (c, j). The details of their derivation are relegated to Section C.7 of Internet Appendix C.

Growth rate: For each technology class c and industry j, we calculate the contribution to aggregate growth made by product lines in that class and industry. We let f_{cj} denote this contribution, which depends on three factors: the probabilities of different types of innovations landing on product lines in class c and industry j, the patent infringement probabilities for these product lines, and the outcomes of the ensuing litigation subgames. We provide details in Internet Appendix C (equation (C.62)). We use these equilibrium objects to calculate the equilibrium growth rate g as

$$g = \sum_{j=1}^{J} \omega_j \sum_{c=1}^{C} \mu_{cj} f_{cj}. \tag{30}$$

Given these closed-form solutions, it is straightforward to solve the model numerically. We simply iterate between calculating the value function, the equilibrium growth rate, and other model objects until all model objects are mutually consistent. The closed-form solutions make this process remarkably fast, aiding our estimation.

4.8. Output and welfare

Finally, we calculate social welfare, which is useful for comparing our estimated equilibrium to counterfactual economies. First, we need to compute the consumption stream of the representative household. From the utility function of the representative household in equation (7), we have

$$W = \int_0^\infty e^{-\rho t} \ln C_t dt = \int_0^\infty e^{-\rho t} \ln(e^{gt} C_0) dt = \frac{\ln C_0}{\rho} + \frac{g}{\rho^2}. \tag{31}$$

This expression shows how welfare depends on the initial level of consumption C_0 and the growth rate of the economy g. Intuitively, welfare tends to be higher when the growth rate of the economy is higher. However, if a much higher fraction of output goes to R&D or litigation expenses in the high-growth economy, then C_0 is lower, so welfare can be lower in the high-growth economy. To compute C_0, we need to calculate the initial output level, Y_0, and the fraction of output spent on R&D and litigation by all firms in the economy. The details are relegated to Section C.8 in Internet Appendix C.

Finally, for two economies A and B, we can define a consumption equivalent welfare change (CEWC) measure ϖ, which is the percentage increase in lifetime consumption that an agent in economy A would need to be indifferent between being in economy A or B

$$W_B = \frac{\ln(C_0^A(1+\varpi))}{\rho} + \frac{g^A}{\rho^2}. \tag{32}$$

Solving for ϖ, we get

$$\varpi = \exp\left(\left(W_B - \frac{g^A}{\rho^2}\right)\rho - \ln(C_0^A)\right) - 1. \tag{33}$$

5. Estimation and Identification

5.1. Data

We use several datasets to estimate model parameters: (i) Compustat North America Fundamentals Annual, which contains annual data from firm financial statements; (ii) the Global Corporate Patent Dataset (GCPD) (Bena, Ferreira, Matos, and Pires, 2017), which contains detailed information on patents as well as a link between patents and the patent-holding firms in Compustat; (iii) the USPTO litigation database, which lists the patents involved in each patent lawsuit filed over the period 2003 to 2016, and (iv) the Federal Judicial Center (FJC) civil-lawsuit database, which includes detailed information on lawsuit outcomes for all patent lawsuits filed in federal courts.

We use these data to construct a firm-year panel from 2003 to 2016. Based on the data, we categorize firms into nine technology classes and four primary industry groups, with details regarding the classification procedure in Section A.1 of Internet Appendix A.

5.2. Parameterizing the model

To map the model to the data, we add some parametric assumptions. First, we assume the litigation cost γ is drawn from an exponential distribution with parameter ξ, with CDF $1-e^{-\xi\gamma}$. Second, we draw λ_c from a uniform distribution with a mean of μ_λ and a standard deviation of σ_λ. Third, the R&D cost parameter χ_c is drawn from a uniform distribution with a mean of μ_χ and a standard deviation of σ_χ. We estimate the values of $\xi, \mu_\lambda, \sigma_\lambda, \mu_\chi$, and σ_χ.

Additionally, we set a few parameters based on literature conventions. The discount rate ρ is set to 0.04, which implies a real interest rate of 6% when the growth rate is 2%. The R&D subsidy s_{cj} is set to 8%, the implied tax subsidy rate on R&D expenditures in the US from the OECD database. The exit rate δ is set to 3% following Acemoglu et al. (2018).

We also use our firm-year panel to estimate some parameters based on observable measures. We directly estimate $\{\omega_j\}$ as the observed share of all sales attributable to each industry j. We estimate $\{\eta_{cj}\}$ as the share of all new-entrant sales attributable to each industry j and technology class c. Additionally, we estimate the model injunction rate, ζ, as the observed injunction rate, which was 95% before eBay ruling in 2006 and 75% after eBay (Seaman, 2015), leading to an average injunction rate of 81% in our sample. We provide details in Internet Appendix A.

5.3. Parameter identification and estimation by GMM

Using generalized method of moments (GMM), we estimate the remaining parameters: (i) the mean μ_λ and standard deviation σ_λ of λ_c; (ii) the mean μ_χ and standard deviation σ_χ of R&D cost scale χ_c; (iii) the R&D cost convexity ψ; (iv) the entrant R&D cost scale ν; (v) the knowledge spillovers σ; (vi) the infringement probabilities κ_1, κ_2 for type-1 and type-2 infringement, respectively; (vii) the bounds τ_2^l, τ_2^h determining the distribution of defendant win rates in type-2 infringements; (viii) the single parameter $(\tau_1^l + \tau_1^h)/2$ sufficient for summarizing defendant win rates in type-1 infringements, and (ix) the litigation cost-scale parameter ξ. We jointly estimate these 13 parameters by minimizing the distance between the 16 model-implied moments in Table 2 and their empirical counterparts, whose detailed construction is in Section A.2 of Internet Appendix A.

We now explain how our empirical moments help us infer the values of these 13 parameters. Technically, each model-implied moment depends jointly on all 13 parameters through the model solution. However, certain moments are more sensitive to particular parameters, aiding our identification. As shown in Figure 2, some of these moments are particularly useful for identifying certain parameters, as the relation between moment and parameter is steep and monotonic.

The mean and standard deviation of the innovation step size, μ_λ and σ_λ: Growth in our model depends on the parameters μ_λ and σ_λ, which govern the distribution of λ_c. These two parameters are primarily identified by two growth-rate moments: the average output growth rate and the standard deviation of sales growth across technology classes. We follow a literature convention by targeting the growth rate of average output instead of average sales because our model captures a balanced growth path for the entire economy, not just the growth of large publicly traded firms.[13] Panel (a) of Figure 2 confirms that our model-implied average output growth increases monotonically with μ_λ. Similarly, our model-implied standard deviation of sales growth across technology classes increases monotonically with σ_λ, as shown in Panel (b) of Figure 2.

Two litigation-related moments also help identify μ_λ and σ_λ. First, we use the correlation between litigation rates and sales growth across technology classes. Intuitively, if firms in technology class c experience litigation more frequently than those in class c', then these lawsuits are likely generated by frequent innovation by class c firms, that is, x_c is likely higher than $x_{c'}$ in equilibrium. By comparing sales growth between technology classes with different litigation rates, we can measure how much additional innovation translates into additional sales growth: a relationship governed by μ_λ in our model. Panel (c) of Figure 2 confirms that this across-technology class correlation increases with μ_λ. Second, we calculate the standard deviation of litigation probabilities across both technology classes and industries. This variation helps identify σ_λ, as differences in λ_c drive litigation rate differences in our

[13]See Akcigit et al. (2016), Celik and Tian (2023), and Terry (2023), among others.

model. Panel (d) confirms that higher σ_λ leads to greater variation in λ_c and, consequently, wider dispersion in litigation rates.

The mean and standard deviation of R&D cost scale, μ_χ and σ_χ: Two moments related to R&D intensity (R&D spending divided by sales) help identify the R&D cost parameters μ_χ and σ_χ. As shown in Panel (e) of Figure 2, average R&D intensity decreases monotonically with μ_χ. Intuitively, higher values of χ_c raise the marginal cost of R&D spending, reducing firms' innovation rates. While a higher value of χ_c mechanically raises R&D spending, Panel (e) shows the reduction in innovation dominates, allowing the mean R&D cost μ_χ to be identified from total R&D spending. Second, the standard deviation of R&D intensity across technology classes identifies σ_χ, with Panel (f) showing a monotonic decline as σ_χ increases. Firms in technology classes with high λ_c typically spend more on R&D due to higher expected benefits. Rising σ_χ increases χ_c in these high-λ_c classes while decreasing χ_c in low-λ_c classes, thereby compressing the distribution of R&D spending across technology classes. The strength of this compression identifies σ_χ.

The R&D cost convexity parameter, ψ: The R&D cost convexity ψ is primarily identified by the skewness of R&D intensity across technology classes. Technology classes with high λ_c innovate more (higher x). As ψ rises, marginal innovation costs increase most sharply in these high-λ_c classes, leading to a disproportionate decline in x where innovation was previously highest. Panel (g) of Figure 2 confirms that R&D intensity skewness declines monotonically with ψ, providing identification.

A litigation-related moment further helps identify the R&D cost convexity ψ: the correlation between R&D intensity and plaintiff probabilities across technology classes. Technology classes with higher litigation rates must have more frequent innovation (x_c) to generate these lawsuits. With μ_χ and σ_χ already identified, differences in R&D spending between high- and low-litigation technology classes reveal information about ψ. Panel (h) of Figure 2 shows this correlation increases monotonically with ψ, aiding identification.

The entrant R&D cost scale parameter, ν: The parameter ν governs R&D costs for potential entrants, determining their innovation intensity. Higher values of ν make R&D more costly, reducing both innovation and firm entry. Panel (i) of Figure 2 shows this monotonic relationship between ν and the model-implied entry rate.

Knowledge spillover parameter, σ: We identify the technology-spillover parameter σ using the slope coefficient from a regression of firm R&D spending on the share of total sales from the firm's technology class. This regressor is correlated with M_{ct}, the fraction of all product lines owned by firms in technology class c at time t. As shown in Equation (12), the R&D cost function, which we repeat here for clarity,

$$\frac{(1-s_{cj})\chi_c x_{ijt}^{\psi} Y_t}{1+\sigma M_{ct}},$$

an increase in M_{ct} reduces the marginal cost of R&D, as M_{ct} reflects knowledge spillovers, so firms optimally innovate more, that is x_{ijt} rises. As the coefficient σ rises, two countervailing forces affect total R&D spending. Because σ is in the denominator, total R&D spending mechanically falls. However, more optimal innovation implies more R&D spending. The convexity of Equation (12) limits the strength of this second effect, so overall, σ has a negative effect on the relationship between R&D spending and M_{ct}, and consequently between R&D spending and share of total sales from the firm's technology class. Panel (j) of Figure 2 confirms this pattern, showing that this regression coefficient declines monotonically with σ.

The infringement probability parameters for type-1 and type-2 infringement, κ_1 and κ_2: We identify these parameters using two litigation-related moments. First, we use the probability of a firm being a plaintiff in a given year. With the parameters governing innovation incentives pinned down, this litigation probability increases monotonically with κ_1, as shown in Panel (k) of Figure 2. Second, we use the probability that the plaintiff and defendant share the same industry, conditional on a lawsuit going to trial. This same-industry lawsuit probability declines monotonically with κ_2, as shown in Panel (l). This relationship emerges because type-1 infringement occurs only within industries, while type-2 infringement can span different industries.

The average defendant-trial-win rate in type-1 infringements, $(\tau_1^l+\tau_1^h)/2$: We identify $(\tau_1^l+\tau_1^h)/2$ using the rate of plaintiff trial victories in same-industry lawsuits (those in which the plaintiff and defendant share the same industry). As expected, Panel (m) of Figure 2 confirms that this plaintiff win rate declines monotonically in the probability $(\tau_1^l+\tau_1^h)/2$ of a defendant victory, which identifies $(\tau_1^l+\tau_1^h)/2$.

The bounds of defendant win rates in type-2 infringements, τ_2^l and τ_2^h: We identify these parameters using two litigation moments. First, we use the probability of a plaintiff victory, conditional on the plaintiff and defendant being in different industries. Panel (n) of Figure 2 shows this probability declines with τ_2^l as the distribution of defendant victory probabilities shifts right. Second, we use the overall settlement rate for lawsuits. As τ_2^h rises, defendants become more likely to win at trial and thus less likely to settle. Panel (o) of Figure 2 confirms this negative relationship between the settlement rate and τ_2^h.

The litigation cost-scale parameter, ξ: We identify ξ using the average ratio of annual firm litigation spending to annual firm revenue. Panel (p) of Figure 2 confirms that as ξ increases (expected litigation costs fall), the ratio of litigation spending to revenue declines monotonically.

5.4. Parameter estimates and model fit

Parameter estimates: Panel A of Table 2 reports the estimated parameter values and the associated standard errors. All of the parameters are precisely estimated because, as shown in Figure 2, our model moments are highly sensitive to the parameters that they identify. Several of our parameters are easily interpretable, as they represent probabilities that are easily matched to the moments in Panel B. For example, type-1 infringement is far more common than type-2 infringement, with $\kappa_1 = 83\%$ and $\kappa_2 = 39.3\%$. This result allows us to match the empirical fact from Panel B of Table 2 that most lawsuits are filed between firms in the same industry. Similarly, the parameters, $(\tau_1^l + \tau_1^h)/2$, τ_2^l, and τ_2^h map intuitively into the moments describing the likelihood of a trial win. For example, defendants are more successful in trial in type-1 infringements, winning roughly 68% of trials, than in type-2 infringements.

The parameters describing technology require further interpretation. First, the estimates of σ_χ and σ_λ imply that both R&D costs, χ_c, and innovation efficacy, λ_c, have meaningful variation across technology classes. This variation validates our focus on how heterogeneous firms select into lawsuits. Second, consistent with Bloom et al. (2013), we estimate positive and statistically significant knowledge spillovers ($\sigma = 0.099$). Third, we estimate $\psi = 1.969$, nearly identical to the value of 2 that is frequently assumed in the literature (Bloom et al., 2002). Fourth, our estimate of entrants' R&D cost scale parameter, ν, is quite close to the mean of the distribution of this parameter for incumbents, μ_χ, (6.5 versus 5.2), suggesting that entrants have similar R&D costs. Finally, our litigation cost scale parameter, ξ, while not technological, is central to the model's ability to match the likelihood a firm is a plaintiff.

Model fit: Panel B of Table 2 reports the targeted moments in the data and the model. The model closely matches the data moments on several important dimensions. First, our model matches observed innovation activity and the resultant economic growth. For example, the average output growth rate is 2.12% in our model, compared to 2.03% in the data. The average R&D intensity is 5.35% in the model, compared to 6.43% in the data. Our model similarly matches the distribution of innovation across technology classes. The standard deviation of R&D intensity across technology classes is 3.27% in our model, compared to 3.60% in the data.

Second, the model matches observed litigation activity. For example, a patent-holding firm in a given year has a 9.49% chance of filing a patent lawsuit in our model, compared to 10.53% in the data. The settlement rate in our model is 55.68%, quite close to the rate in the data (58.32%).

Third, our model closely matches the interaction between innovation and litigation. The across-technology-class correlation between litigation rates and sales growth is 0.694 in our model, quite close to the empirical correlation of 0.713. This correlation is particularly important for our welfare implications. Recall from the illustrative model that a plaintiff-friendly reform, such as increasing the injunction rate, reduces welfare if efficient innovators with high λ_c are frequent litigators. Our model finds a positive correlation between sales growth and litigation, indicating that high-λ_c technology classes are more frequently involved in litigation. Based on the illustrative model, this result implies that increasing the injunction rate would reduce innovation activity and harm welfare. The following section demonstrates that the full model delivers the same prediction.

6. Quantitative Analysis

6.1. Increasing R&D subsidies

We begin by using our model to quantify the effects of one of the most fundamental innovation policies: increasing R&D subsidies. In three separate exercises, we double the R&D subsidy s_{cj} from its baseline value for three groups: (i) the whole sample, (ii) firms in the lowest tercile of research efficiency (measured by λ_c/χ_c), and (iii) firms in the highest tercile of research efficiency. In each exercise, we use the parameters from Table 2 to solve the model and calculate many model moments of interest: social welfare, the output growth rate, the distribution of firm value, and the innovation and litigation decisions of firms.

Table 3 presents the results. As expected, subsidizing R&D reduces the marginal cost of R&D, thereby increasing R&D intensity. Column 2 of Table 3 indicates that doubling the R&D subsidy for all firms in the economy would significantly boost innovation, which would raise the growth rate for the economy, increasing social welfare by 2.76%. This result implies that aggregate investment in innovation is too low relative to a socially efficient benchmark.

However, the degree of over-investment or under-investment varies across firms because of differences in research efficiency. First, we assume that only the least efficient technology classes receive the subsidy. Column 3 of Table 3 shows that subsidizing only inefficient firms reduces welfare by 0.07%. Several effects are at work. The least efficient innovators raise R&D spending, so efficient innovators value any given product line less, knowing that less-innovative competitors are more likely to obtain it. Thus, efficient innovators reduce their innovation, and total incumbent innovation activity falls slightly, as the reduction in innovation by efficient firms outweighs the increase by inefficient firms. Consequently, output growth and welfare decline. When incumbent innovation falls, two effects emerge. First, entrants innovate more because they face less risk of losing product lines to incumbent lawsuits. Second, the shift toward innovation by inefficient firms reduces average product line value, making product-line owners less likely to file lawsuits. The combination of fewer product lines from reduced incumbent innovation and lower product line values reduces litigation overall, as measured by the average probability of becoming a plaintiff.

Next, we consider a scenario where only firms in the most efficient technology classes receive the subsidy. Column 4 of Table 3 shows that this policy improves welfare substantially by 3.022%. The improvement occurs because the reduced cost of R&D leads incumbents to innovate more. There are more product lines in equilibrium, and they are more valuable because R&D activity moves to these highly efficient firms, so growth and welfare rise.

Table 3 also reveals a redistribution effect. The welfare improvement coincides with a transfer from entrants to incumbents. Greater incumbent innovation means entrepreneurs are likelier to lose their product lines, dampening their incentive to innovate, so entrant innovation and entrepreneurial value fall. Product-line owners also file more lawsuits to protect their increasingly valuable products from innovators obtaining them.

Overall, these two exercises underscore the importance of firm heterogeneity in our model. High-efficiency firms underinvest in innovation relative to a socially efficient benchmark, so subsidizing their R&D improves welfare. Conversely, low-efficiency firms overinvest in innovation, so subsidizing their R&D amplifies inefficient activity, thereby reducing welfare. This heterogeneity implies that the effectiveness of any litigation reform depends on the types of firms it affects. Our model, therefore, needs to capture both the impact of the litigation system on innovation and the selection into lawsuits by heterogeneous firms.

6.2. Litigation reforms and the efficacy of R&D subsidies

Table 3 illustrates that R&D subsidies affect both innovation and litigation because the two are endogenously linked in our model. So, a natural question is whether changes to the litigation system alter the effectiveness of R&D subsidies. To explore this connection, we modify four aspects of the legal system: the injunction rate, the cost of filing lawsuits, the likelihood of patent infringement, and the ability to litigate at all, modeled through prohibitively high filing costs. For each modification, we reevaluate the three R&D subsidy scenarios from Section 6.1.

We present the results in Table 4, focusing on welfare for brevity. The first row replicates the result of Section 6.1. Next, starting with the parameters from Table 2, we change one litigation parameter at a time and repeat the same analysis of R&D subsidies. Row 2 shows the results from halving the injunction rate parameter, ζ. Row 3 shows the results from halving the litigation cost scaling parameter ξ, which doubles the average cost of hiring a legal team. Row 4 shows the results from halving the infringement probability parameters κ_1 and κ_2. Row 5 shows the results when ξ is set low enough to make litigation prohibitively expensive.

After these four defendant-friendly litigation reforms, R&D subsidies increase welfare by 2.8% to 4.1% when they affect all firms, decrease welfare by 0.07% to 0.09% when subsidizing only inefficient firms (low λ_c/χ_c), and raise welfare by 3.0% to 4.6% when subsidizing only efficient firms (high λ_c/χ_c). All four reforms amplify R&D subsidy effects because firms invest more in innovation when they face lower litigation risk. This amplification increases both the gains from subsidizing efficient firms and the losses from subsidizing inefficient ones. These results suggest models that omit litigation can overstate the impact of R&D subsidies by ignoring how litigation risk dampens firms' responses to subsidies.

In summary, litigation reforms can significantly enhance the effectiveness of R&D subsidies. However, a litigation reform will have its own baseline impact on the economy that must be considered in addition to the interaction between the reform and R&D subsidies. In the following sections, we examine the impact of litigation reforms in the absence of increased R&D subsidies. For each reform, we study its impact on litigation, innovation, firm dynamics, firm value, economic growth, and social welfare. We quantify these reforms' average effects while highlighting outcome heterogeneity across firms and infringement types.

6.3. The impact of the *eBay v. MercExchange* ruling

While litigation reforms can enhance the effectiveness of R&D subsidies, they can have important effects in their own right, even without subsidies. We now turn to quantifying both the average effects of possible reforms, while also highlighting the substantial heterogeneity in outcomes across firms and types of patent infringement.

First, we use the model to study a historical policy. Before 2006, a plaintiff victory in a patent litigation trial almost always led to an injunction. The pivotal 2006 *eBay v. MercExchange* Supreme Court ruling raised the standard for granting an injunction. According to Chien and Lemley (2012), injunctions were granted in 95% of plaintiff victories before the eBay ruling and 75% after. Therefore, we model the effects of the eBay ruling by adjusting the ξ parameter from 0.95 to 0.75. Using the estimated model, we assess the impact of the eBay ruling on innovation, firm values, litigation, growth, and social welfare.

The consequences of reducing the injunction rate are ex-ante ambiguous. Potential innovators might be deterred by the fear that their future patents will not be protected adequately from infringers. Alternatively, potential innovators might be encouraged by the reduced threat of an injunction blocking their innovation. Columns 1 and 2 of Table 5 show that reducing the injunction rate increases innovation among incumbents and entrants by 4.26% and 2.58%, respectively. Innovation increases because firms worry less that a patent lawsuit will block their innovation. Incumbent firms exhibit a greater increase in innovation compared to entrants, resulting in a slight decline in the contribution of entrants to growth.

The effect of lowering the injunction rate on firm value is nuanced. We find that our simulation of the eBay ruling lowers the average value of a product line by 1.69%. Firms value a product line less after eBay because they are less able to protect it. Even if they win a lawsuit against an infringer, they are less likely to save their monopoly with an injunction.

However, the increased innovation activity described above leads to the creation of new product lines. The average number of product lines per firm increases by 3.34%. The increase in the number of product lines outweighs the decreased value of each product line, increasing overall firm value for incumbents by 1.60%. The reduced injunction threat also offsets the decrease in average product line value, leading to an increase in entrepreneur value of 5.14%.

This rise in innovation activity also widens the dispersion in the number of product lines held by different types of firms. While all firms innovate more after eBay, high-research-efficiency firms are more likely to become product-line leaders and, thus, hold more product lines. Conversely, low-research-efficiency firms hold fewer product lines. This redistribution raises the standard deviation of the number of product lines per firm by 10.09%.

Moreover, the eBay ruling affects the potential gain from litigation, influencing firms' legal strategies. The probability of hiring a legal team after an infringement falls because of the lower potential gain from litigation. However, the increase in overall innovation leads to more infringements and, thus, more creative destruction. The rise in infringements outweighs the reduced likelihood of hiring a legal team, leading to a slight increase in the average rate of lawsuits filed per product line. Together with the increase in the number of product lines, the average probability of a firm being a plaintiff increases by 3.38%.

Regarding aggregate implications, the increase in incumbent and entrant innovation boosts the output growth rate. Higher R&D costs crowd out consumption, which falls slightly. However, for welfare, higher economic growth dominates lower consumption, so social welfare rises by 3.29%. This welfare improvement is substantial. For comparison, recent quantitative evaluations find welfare costs of 0.1% to 1.8% from business cycles (Krusell et al., 2009), about 1% from inflation (Lucas, 2000) and managerial short-termism (Terry, 2023), and about 2.5% from trade (Melitz and Redding, 2015).

Heterogeneous injunction rates for different types of firms: Next, we examine the effects of different standards for granting injunctions for different types of patents. Specifically, we consider a counterfactual in which we reduce the injunction rate ζ from 0.95 to 0.75 for only the most efficient innovators (the highest tercile of technology classes by λ_c/χ_c) while maintaining the pre-eBay rate of 95% for all other patent lawsuits. Column 4 of Table 5 shows that this targeted policy improves welfare by 3.49%, exceeding eBay's 3.29% improvement by concentrating innovation incentives on efficient firms. In contrast, column 3 shows that reducing injunction rates only for the least efficient innovators reduces welfare by 0.035%.

The stark difference in outcomes demonstrates the importance of targeting litigation reforms based on firm innovation efficiency.

Heterogeneous injunction rates for different types of infringement: We examine how reducing injunction rates differentially affects outcomes when targeting type-1 versus type-2 infringement. Table 6 presents four scenarios. We repeat the baseline results and counterfactuals in columns 1 and 2. Column 3 presents the results from reducing the injunction rate from 0.95 to 0.75 for type-1 only, and Column 4 presents the results for type-2 only. Reducing injunction rates for type-1 infringement (column 3) substantially increases incumbent innovation, as incumbents face less risk when developing new products that might infringe on existing patents. However, this policy decreases entrant innovation, as entrepreneurs anticipate a higher likelihood of losing their product lines to incumbents. In contrast, reducing injunction rates for type-2 infringement (column 4) primarily benefits entrants by lowering their litigation risk from non-incumbent patent holders. The effect on incumbent innovation remains modest because, as shown in Table 2, type-2 infringement occurs less frequently than type-1 infringement.

Creative destruction rises in both scenarios, reducing average product-line value and the incentive to hire legal teams. When only type-1 injunction rates fall, litigation opportunities rise as creative destruction rises. Although the incentive to hire a legal team falls, the rise in litigation from creative destruction raises the per-product-line probability of becoming a plaintiff. The opposite occurs when only type-2 rates fall. A smaller rise in creative destruction is dominated by reduced legal hiring, lowering the per-product-line probability of becoming a plaintiff.

The average number of product lines increases when only type-1 injunction rates fall, driven by stronger incumbent innovation. In contrast, the average number of product lines falls when only type-2 rates fall, despite higher entrant innovation, as the incumbent response is weaker. These differences in the number of product lines affect firm values. When type-1 rates fall, more product lines outweigh their lower average value, raising incumbent value. When type-2 rates fall, the modest change in product lines cannot offset lower product-line value, reducing incumbent value.

When only type-1 injunction rates fall, growth improves more because of a larger rise in creative destruction, which is driven by incumbent innovation. The welfare gains are also larger, reflecting both the higher prevalence of type-1 infringement and incumbents' outsized role in driving growth.

6.4. Increasing plaintiff filing costs

Our model allows us to study other potential reforms. For example, one recent proposed reform suggests increasing plaintiff pleading requirements (Gugliuzza, 2015). In our model, this proposal is analogous to increasing the costs of filing for plaintiffs. To examine its impact, we vary the litigation cost parameter, ξ. A lower ξ value corresponds to higher average litigation costs for plaintiffs, making it more costly to file lawsuits. We reduce ξ to half of its baseline value, thereby effectively doubling the average plaintiff filing costs. The values of all other parameters are kept unchanged at their baseline levels in Table 2. Columns 1 and 2 of Table 7 report the impact of increasing filing costs for plaintiffs.

In response to the higher litigation cost, the probability of hiring a legal team declines. Both incumbents and entrants innovate more because they are less concerned about being sued if they infringe on other firms' intellectual property. As in the previous section, we find a larger increase in innovation among incumbents than entrants, diminishing the entrants' growth contribution.

The increase in innovation spurs a rise in creative destruction, which, along with the reduced ability to protect a product line with litigation, lowers average product line value by 1.79%. For entrants, who worry less about protecting product lines, the reduced intellectual property infringement risks outweigh this decline, improving entrepreneur value.

As in the case of reduced injunction rates, the increase in innovation activity from higher plaintiff filing costs boosts both the average and the standard deviation of the number of product lines per firm. However, the average number of product lines per firm only rises by 1.57% and fails to offset the 1.79% reduction in average product line value. The result is a slight decrease in incumbent firm value by 0.25%.

Higher filing costs significantly influence litigation behavior by reducing the average probability of potential plaintiffs filing lawsuits. This reduction is primarily due to a decline in the per-product-line probability of becoming a plaintiff. The lower likelihood of engaging legal teams in the event of infringement is directly attributed to the increased filing costs.

The rise in filing costs also leads to aggregate implications comparable to reduced injunction rates. The rise in innovation by both incumbents and entrants propels the output growth rate. While the increased R&D expenditure tempers consumption levels, the improved growth outweighs the lower consumption level. Consequently, social welfare rises by 2.2%.

Reducing injunction rates and increasing plaintiff filing costs generate similar welfare gains but different distributional effects. Reducing injunction rates raises incumbent value but higher filing costs reduce incumbent value. The contrasting effects on firm values highlight how different legal reforms can achieve comparable welfare improvements through distinct economic channels.

Heterogeneous filing costs for different types of firms: Next, we consider the impact of imposing different filing costs for different firms. Starting with the parameters in Table 2, we halve the ξ parameter for all firms, only lawsuits targeting firms in the lowest tercile of research efficiency (measured by λ_c/χ_c), and only lawsuits targeting firms in the highest tercile of research efficiency. We report the results in columns 3 and 4 of Table 7. We report the results of halving the ξ parameter for lawsuits targeting low-efficiency firms in column 3. In column 4, we report the analogous results from changing only the costs of filing lawsuits against high-efficiency firms.

The results are similar to the analogous heterogeneity exercise for reduced injunction rates. We find that raising the filing costs for only lawsuits targeting low-efficiency firms reduces overall welfare. In contrast, increasing the filing costs for only lawsuits aimed at high-efficiency firms boosts innovation and growth, thereby improving welfare. This finding underscores the importance of tailoring litigation reforms to the specific types of innovative firms that they affect.

Heterogeneous filing costs for different types of infringements: We also examine the effects of applying different filing costs to different types of infringement. We present the results in Table 8. We repeat the baseline results and counterfactuals in columns 1 and 2. To produce column 3, we halve the ξ parameter for only type-1 infringement, and for column 4, we halve the ξ parameter for only type-2 infringement.

Increasing filing costs substantially reduces the incentive to hire a legal team, thereby lowering the average litigation probabilities. Differing from the corresponding heterogeneity analysis for injunction rates, this reduction in litigation rates occurs across columns 2 to 4.

Comparing columns 3 and 4, the type-1-only reduction in filing costs improves incumbent value while the type-2-only reduction lowers incumbent value. Likewise, the type-1-only reduction improves incumbent innovation and welfare more than the type-2-only reduction. The intuition for these results mirrors the intuition for the corresponding results for injunction rates. Overall, this exercise highlights the importance of tailoring litigation reforms to specific types of patent infringement.

6.5. Simulating the effects of reducing patent infringement risk

The tractability and realism of our model make it a useful laboratory for evaluating other patenting trends. In recent years, two key developments have shaped the U.S. patent litigation environment by curbing opportunistic patent enforcement. First, the Patent Trial and Appeal Board (PTAB), introduced under the America Invents Act of 2012, provides an administrative venue for re-examining the validity of granted patents. By offering a more efficient and technically informed review process, especially through Inter Partes Review (IPR), the PTAB improves patent clarity and limits the extent to which a patent holder can pursue litigation. Second, the 2014 Supreme Court decision in *Alice Corp. v. CLS Bank International* tightened the criteria for patent-eligible subject matter, particularly in software and business-method patents. This shift discourages broad, abstract claims and lowers the likelihood that weak patents will survive legal scrutiny, thereby reducing litigation risk.

We use our model to illustrate the effects of policies that reduce patent-infringement risk, such as the introduction of PTAB and the *Alice* decision. We report the results in Table 9. In column 1, we present baseline values for our model objects. We halve the type-1 infringement parameter κ_1 from its baseline value and report the results in column 2. We analogously halve the type-2 infringement parameter κ_2 and report the results in column 3. Finally, we halve both κ_1 and κ_2, reporting the results in column 4.

Halving both types of patent-infringement risk raises incumbent innovation by 7.68% and entrant innovation by 3.30%. When only type-1 risk falls, incumbent innovation rises by 6.31% while entrant innovation falls by 0.83%. In contrast, halving only type-2 risk raises entrant innovation by 3.93% with a smaller 2.56% rise in incumbent innovation. The creative destruction rate rises in all cases by 6.93%, 5.10%, and 2.79%, respectively, leading to welfare gains of 6.68%, 5.65%, and 1.75%. We conclude that reforms to the patent-approval process can achieve welfare improvements similar to those from patent litigation reform.

6.6. Simulating the effects of patent trolls

In recent decades, patent litigation filed by entities commonly known as "patent trolls" has surged. These plaintiffs are often non-practicing entities (NPEs) that accumulate broad or weak patents primarily to engage in rent extraction through infringement lawsuits. Their growing presence has raised significant policy concerns. Critics argue that these patent trolls increase uncertainty, discourage genuine innovation, and divert resources away from productive activities.

In our model, a rise in patent trolls can be modeled through two channels: an increased probability of type-2 infringement where plaintiffs file lawsuits purely for rent extraction, and a higher entry rate for low-research-efficiency firms that may serve as vehicles for opportunistic litigation rather than innovation. We simulate the potential effects of a rise in patent-troll activity on firm and aggregate outcomes. Specifically, we double both the type-2 infringement risk, κ_2, and the entry rate of firms in the lowest tercile of research efficiency.

Table 10 shows that patent trolls reduce entrant innovation by 24.8% and average R&D intensity by 24.6%. As more low-R&D-efficiency firms enter, R&D activities shift toward less capable innovators, with creative destruction falling by 3.9%. While incumbent innovation stays nearly flat (0.3% increase), entrants' contribution to growth falls by 18.4%. The number of product lines rises by 35.8%, and incumbent value rises by 44.2% through greater rent extraction, while entrant value falls by 43.0%. Although lower R&D expenses could theoretically free up resources for consumption, the average probability of being a plaintiff rises by 89.6%, and the per-product-line plaintiff probability rises by 39.6%, so more resources are wasted on litigation. These costs mitigate any potential consumption gains, with consumption rising only 0.03%. Combined with a 5.3% decline in economic growth from reduced innovation and creative destruction, social welfare falls by 2.8%.

7. Conclusion

We develop a novel dynamic general equilibrium model with endogenous growth to quantify the influence of the litigation system on innovation, firm value, growth, and social welfare. The model features heterogeneous firms that innovate while facing potential patent lawsuits. The firms innovate to steal market share from competitors, so they inefficiently internalize the transfer they extract from competitors by innovating better products. This behavior can lead to inefficient overinvestment in innovation. However, innovation also creates positive externalities through technology spillovers that firms do not internalize. Thus, firms can also underinvest in innovation in equilibrium.

We embed a realistic patent litigation model in this dynamic general equilibrium framework. When firms innovate to steal a competitor's product line, they can infringe on an existing patent. The patent holder's decision to file a lawsuit and the joint decision of whether to go to trial are both determined endogenously. In a trial, the court may grant an injunction, stopping the innovating firm from taking over the product line.

By integrating a realistic patent litigation system into a dynamic general equilibrium model with endogenous growth, we can assess how changes in the legal landscape affect firm behavior and social welfare. Using this framework, we estimate the model to evaluate the effectiveness of R&D subsidies, historical patent-litigation reforms, and proposed reforms. Our findings reveal that low-research-efficiency firms tend to overinvest in innovation, while high-research-efficiency firms underinvest. The effectiveness of R&D subsidies is significantly amplified in alternative legal environments with weaker plaintiff rights. Motivated by these insights, we examine the impact of two important legal reforms.

In both cases, we find that defendant-friendly reforms enhance innovation and welfare. The 2006 Supreme Court "eBay ruling," which strengthened defendant rights by lowering injunction rates, improved welfare by 3.29%. Similarly, a proposed reform to increase plaintiff pleading requirements, making it more difficult to file lawsuits, also improves welfare. These results underscore the potentially profound influence of patent litigation reforms.

We also assess the heterogeneous effects of these legal reforms across firms with varying levels of research efficiency and different types of patent infringement. These decompositions emphasize the need to move beyond a one-size-fits-all approach to litigation policies. By tailoring injunction criteria and plaintiff pleading requirements based on firm research efficiency or the type of infringement, policymakers can more effectively address both overinvestment and underinvestment in innovation, fostering a litigation environment that enhances dynamic efficiency, growth, and welfare.

Our research contributes to the discourse on patent litigation reform, offering policymakers and stakeholders guidance on crafting reforms that align with the patent system's original purpose of promoting technological progress and economic well-being. Our rich yet highly tractable dynamic framework can be applied to study a wide range of patent-related issues, providing a versatile tool for analyzing the interplay between legal reforms and innovation dynamics.

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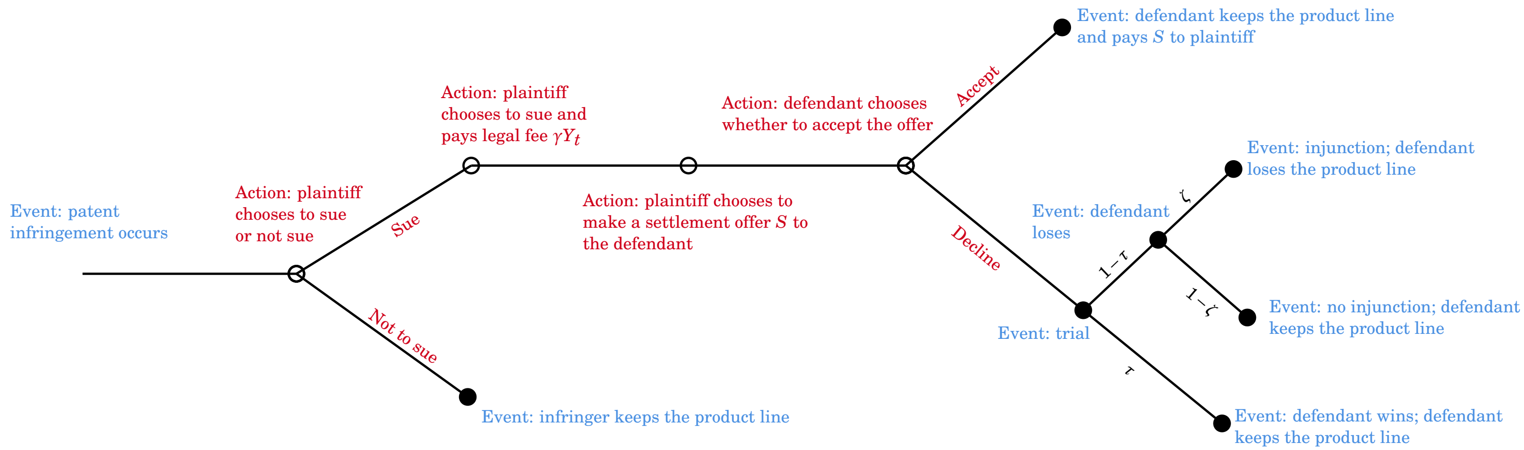

Figure 1: Litigation Subgame Timeline

Notes: This figure illustrates the timeline of the litigation subgame. Conditional on patent infringement, the plaintiff has to make a decision of whether to hire a legal team. A legal team is necessary for making a settlement offer or going to court. The cost of hiring a legal team is γY_t, where $\gamma > 0$ is a random variable drawn from the distribution $\Gamma(\gamma)$, and the Y_t term ensures that litigation costs grow at the same rate as output in a BGP equilibrium. If the plaintiff chooses to pay γY_t and hire the legal team, it then makes a take-it-or-leave-it settlement offer to the defendant. The defendant has private information about its chances of winning the trial. Let $\tau \in [0,1]$ denote the probability that the defendant wins the trial. This probability is drawn from the exogenous distributions $T_1(\tau)$ and $T_2(\tau)$ for type-1 and type-2 infringements, respectively. Given its private information τ, the defendant can accept the settlement or refuse. Refusal leads to a trial. With probability τ, the defendant wins the trial and the product line takeover is realized. With the complementary probability $1-\tau$, the defendant loses. If the defendant loses, then the court decides on whether to grant an injunction or not. With probability $\zeta_1 \in [0,1]$, an injunction is granted for a type-1 infringement, thus blocking the product line takeover. With probability $1-\zeta_1$, there is no injunction, so the defendant can still take over the product line. The same probability is denoted $\zeta_2 \in [0,1]$ in the case of type-2 infringements. The parameters ζ_1 and ζ_2 are policy parameters that capture the inclination of a court to grant an injunction in the case of a proven patent infringement.

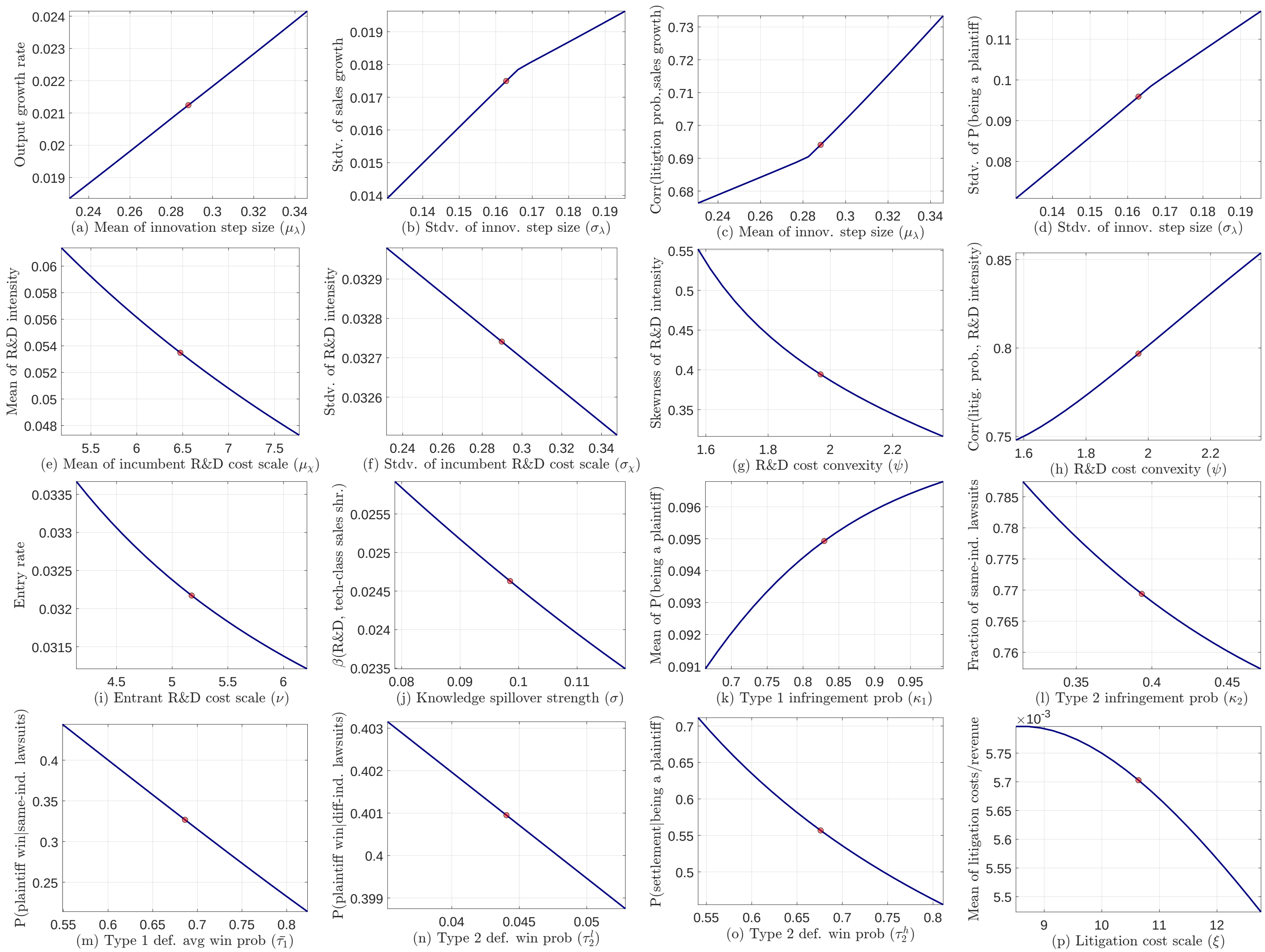

Figure 2: Model Identification

Notes: This figure illustrates the comparative statics of the most informative moments that help identify each parameter. For each model parameter, we vary the value of the parameter while holding other parameters unchanged, solve the model, and calculate the value of one moment. The red dots indicate the baseline parameter values and model moments.

Table 1: Illustrative Model

Notes: This table shows the results of a simple example from our illustrative model. Panel A shows our assumed parameter values. In Panel B, we consider a case in which only efficient innovators ($c = 1, \lambda_1 = 3$) use the litigation system: $\alpha_{1j} > 0, \alpha_{2j} = 0$. We report the gap between the private and socially optimal levels of innovation and the changes in innovation and welfare if ζ increases by 0.2. In Panel C, we repeat the exercise assuming only inefficient innovators use litigation: $\alpha_{2j} > 0, \alpha_{1j} = 0$.

Parameters	
χ	4.5
σ	2
ζ	0.75
π	1
λ_1	3
λ_2	0.1

Case 1: efficient innovators litigate	
α_{1j}	0.05
α_{2j}	0
$x^*_{1j} - x^s_{1j}$	-0.02
Δx^*_{1j} from $\zeta \uparrow 0.2$	-0.0036
Δ Welfare from $\zeta \uparrow 0.2$	-0.02

Case 2: inefficient innovators litigate	
α_{1j}	0
α_{2j}	0.05
$x^*_{2j} - x^s_{2j}$	0.13
Δx^*_{2j} from $\zeta \uparrow 0.2$	-0.001
Δ Welfare from $\zeta \uparrow 0.2$	0.0007

Table 2: Model Estimation

Notes: The table reports estimation results. Panel A reports the values of the parameters, whereas Panel B provides an overview of the values of the targeted moments in the data and the estimated model. See Section 5 and Appendix A for the definition and construction of data moments.

Panel A: Parameter Estimates

Calibrated Parameters	Description	Values	
ρ	Discount rate	0.04	
δ	Exogenous exit rate	0.03	
ζ	Avg. injunction rate	81%	
Estimated Parameters	**Description**	**Values**	**Std. Err.**
μ_λ	Mean of innov. step size	0.288	0.003
σ_λ	Stdv. of innov. step size	0.163	0.002
μ_χ	Mean of incumbent R&D cost scale	6.474	0.549
σ_χ	Stdv. of incumbent R&D cost scale	0.290	0.049
ψ	R&D cost convexity	1.969	0.027
ν	Entrant R&D cost scale	5.175	0.445
σ	Knowledge spillover strength	0.099	0.013
κ_1	Type-1 infringement prob.	0.830	0.011
κ_2	Type-2 infringement prob.	0.393	0.007
$(\tau_1^l + \tau_1^h)/2$	Type-1 def. avg. win prob.	0.686	0.005
τ_2^l	Type-2 def. win prob. (lb)	0.044	0.022
τ_2^h	Type-2 def. win prob. (ub)	0.676	0.008
ξ	Litigation cost scale	10.637	0.389

Panel B: Moments

Moment	Data	Model	Std. Err.
Output growth rate	2.03%	2.12%	0.002
Mean of R&D intensity	6.43%	5.35%	0.002
Stdv. of sales growth	1.79%	1.75%	0.004
Stdv. of R&D intensity	3.60%	3.27%	0.002
Skewness of R&D intensity	0.546	0.394	0.098
Mean of entry rate	4.20%	3.22%	0.001
β(R&D spending, tech-class share)	0.028	0.025	0.004
Mean prob. of being a plaintiff	10.53%	9.49%	0.004
Stdv. prob. of being a plaintiff	8.63%	9.59%	0.011
Fraction of same-industry lawsuits	78.22%	76.94%	0.042
Prob(plaintiff win | same-industry lawsuits)	35.44%	32.68%	0.056
Prob(plaintiff win | diff.-industry lawsuits)	40.91%	40.09%	0.099
Prob(settlement | being a plaintiff)	58.32%	55.68%	0.003
Mean litigation costs/revenue	0.59%	0.57%	0.0001
Corr(litigation prob., sales growth)	0.713	0.694	0.256
Corr(litigation prob., R&D intensity)	0.620	0.797	0.105

Table 3: The Impact of Increasing R&D Subsidies

Notes: This table presents the implications of increasing R&D subsidies. To simulate the influence of R&D subsidies, we conduct three exercises. We double the subsidy parameter s_{cj} from its baseline value for one of three groups: (i) the whole sample, (ii) firms in the lowest tercile of research efficiency (measured by λ_c/χ_c), and (iii) firms in the highest tercile of research efficiency, respectively. The values of other parameters are kept unchanged at their baseline values. We evaluate the impact of these R&D subsidies on innovation, firm value, firm dynamics, litigation, growth, and social welfare.

	(1) Baseline	(2) Whole Sample		(3) Low λ_c/χ_c Subsample		(4) High λ_c/χ_c Subsample	
		Counterfactual	**% Change**	**Counterfactual**	**% Change**	**Counterfactual**	**% Change**
incumbent innovation	0.0498	0.0544	9.164%	0.0498	-0.008%	0.0544	9.239%
avg. R&D intensity	5.348%	6.220%	16.291%	5.357%	0.164%	6.052%	13.157%
entrant innovation	0.0408	0.0399	-2.364%	0.0409	0.064%	0.0396	-2.909%
avg. entry rate	3.217%	3.399%	5.667%	3.210%	-0.215%	3.448%	7.188%
contribution of entrants to growth	17.617%	16.169%	-8.217%	17.618%	0.005%	16.159%	-8.277%
creative destruction rate	6.005%	6.437%	7.204%	6.005%	0.005%	6.435%	7.174%
avg. incumbent value	0.6353	0.6850	7.814%	0.6323	-0.481%	0.7055	11.042%
avg. entrepreneur value	0.0534	0.0510	-4.567%	0.0535	0.119%	0.0504	-5.595%
avg. product line value	0.1951	0.1940	-0.569%	0.1947	-0.195%	0.1958	0.342%
avg. number of product lines	3.2564	3.5309	8.430%	3.2470	-0.287%	3.6036	10.663%
stdv. number of product lines	6.6644	7.6593	14.929%	6.6321	-0.485%	7.9606	19.449%
avg. plaintiff prob.	9.493%	11.361%	19.675%	9.441%	-0.543%	11.817%	24.484%
per product line plaintiff prob.	2.915%	3.217%	10.370%	2.908%	-0.257%	3.279%	12.489%
prob of hiring legal team	85.625%	85.642%	0.020%	85.546%	-0.092%	85.856%	0.270%
output growth rate	2.124%	2.296%	8.093%	2.121%	-0.144%	2.308%	8.650%
consumption	0.2271	0.2235	-1.560%	0.2271	0.006%	0.2234	-1.604%
output	0.2513	0.2514	0.0355%	0.2513	-0.0076%	0.2514	0.052%
CEWC	—	2.763%	—	-0.070%	—	3.022%	—

Table 4: Welfare Effects of R&D Subsidies across Different Legal Environments

Notes: This table reports the effects of R&D subsidies in different legal environments on social welfare. To simulate the influence of R&D subsidies, we conduct three exercises by doubling the subsidy parameter s_{cj} from its baseline value for one of three groups: (i) the whole sample, (ii) firms in the lowest tercile of research efficiency (measured by λ_c/χ_c), and (iii) firms in the highest tercile of research efficiency. All other parameters are held constant at their baseline values. Row (1) reports the change in consumption-equivalent welfare in the baseline model. Row (2) provides the welfare change in an economy where the injunction rate parameter ζ is set to half of its baseline value. Row (3) reports the welfare change in an economy where ξ (litigation cost scaling parameter) is halved from its baseline value, which effectively doubles the average expenses associated with hiring a legal team. Row (4) shows the welfare change in an economy where $\kappa_{1,2}$ (the infringement probability parameters) are reduced to half of their baseline values. Row (5) presents the welfare change in an alternative economy without litigation risk, where ξ is set to a very small value, making it prohibitively expensive for firms to hire legal teams.

	(1)	(2)	(3)
Welfare Changes	**Whole Sample**	**Low λ_c/χ_c Subsample**	**High λ_c/χ_c Subsample**
(1) **CEWC Baseline**	**2.763%**	**-0.070%**	**3.022%**
(2) CEWC lower injunction risk ($\zeta = \zeta^*/2$)	3.562%	-0.085%	4.004%
(3) CEWC higher filing cost ($\xi = \xi^*/2$)	3.073%	-0.079%	3.419%
(4) CEWC lower infringement risk ($\kappa_{1,2} = \kappa^*_{1,2}/2$)	3.378%	-0.081%	3.773%
(5) CEWC no litigation risk ($\xi \to 0$)	4.079%	-0.092%	4.604%

Table 5: Impact of Reducing the Injunction Rate

Notes: This table reports the impact of the 2006 eBay ruling. To model the effects of the eBay ruling, we adjust the ζ parameter from 0.95 to 0.75 while keeping other parameters unchanged at their baseline values. We evaluate its effects on innovation, firm values, firm dynamics, litigation, growth, and social welfare. Column (1) reports the values of the variables of interest for the baseline model. Column (2) reports the results of reducing the injunction rate for all firms from 0.95 to 0.75. In column (3), we reduce the injunction rate parameter for firms in the lowest tercile of research efficiency (measured by λ_c/χ_c) from 0.95 to 0.75 while keeping the rate for other firms unchanged at 0.95. In column (4), we reduce the injunction rate parameter for firms in the highest tercile of research efficiency (measured by λ_c/χ_c) from 0.95 to 0.75 while keeping the rate for other firms unchanged at 0.95. "Counterfactual" refers to the level of the variable in question after we change ζ to 0.75, and "% Change" refers to the percent change from the baseline level.

	(1)	(2)		(3)		(4)	
	Baseline	**Whole Sample**		**Low λ_c/χ_c Subsample**		**High λ_c/χ_c Subsample**	
		Counterfactual	**% Change**	**Counterfactual**	**% Change**	**Counterfactual**	**% Change**
incumbent innovation	0.0484	0.0504	4.256%	0.0483	-0.094%	0.0507	4.766%
avg. R&D intensity	5.130%	5.436%	5.953%	5.128%	-0.050%	5.385%	4.960%
entrant innovation	0.0401	0.0411	2.575%	0.0402	0.331%	0.0403	0.601%
avg. entry rate	3.091%	3.270%	5.808%	3.089%	-0.051%	3.288%	6.389%
contribution of entrants to growth	17.961%	17.466%	-2.756%	18.037%	0.420%	17.124%	-4.663%
creative destruction rate	5.840%	6.072%	3.968%	5.839%	-0.021%	6.077%	4.052%
avg. incumbent value	0.6286	0.6386	1.603%	0.6251	-0.555%	0.6613	5.213%
avg. entrepreneur value	0.0515	0.0542	5.138%	0.0518	0.648%	0.0521	1.209%
avg. product line value	0.1975	0.1942	-1.685%	0.1972	-0.173%	0.1961	-0.713%
avg. number of product lines	3.1824	3.2888	3.344%	3.1702	-0.383%	3.3723	5.969%
stdv. number of product lines	6.2293	6.8579	10.091%	6.1880	-0.663%	7.1971	15.536%
avg. plaintiff prob.	9.243%	9.555%	3.376%	9.153%	-0.976%	10.095%	9.223%
per product line plaintiff prob.	2.904%	2.905%	0.031%	2.887%	-0.596%	2.994%	3.071%
prob. of hiring legal team	89.152%	83.793%	-6.011%	88.638%	-0.577%	85.399%	-4.209%
output growth rate	2.016%	2.170%	7.670%	2.014%	-0.085%	2.181%	8.174%
consumption	0.2281	0.2267	-0.624%	0.2281	0.008%	0.2265	-0.686%
output	0.2513	0.2513	0.0065%	0.2513	-0.0052%	0.2513	0.022%
CEWC	—	3.292%	—	-0.035%	—	3.490%	—

Table 6: Decomposing the Impact of Reducing the Injunction Rate by Infringement Types

Notes: This table decomposes the impact of reducing the injunction rate while keeping other parameters unchanged at their baseline values. We evaluate its effects on innovation, firm values, firm dynamics, litigation, growth, and social welfare. Column (1) reports the values of the variables of interest for the baseline model. In column (2), we reduce the injunction rate for both type-1 and type-2 infringements to 0.75. In column (3), we reduce the injunction rate parameter for type-1 patent infringement to 0.75 while keeping the rate for type-2 patent infringement unchanged at 0.95. Finally, in column (4), we reduce the injunction rate parameter for type-2 patent infringement to 0.75 while keeping the rate for type-1 patent infringement unchanged at 0.95.

	(1) Baseline	(2) Both Types		(3) Type 1 Only		(4) Type 2 Only	
		Counterfactual	**% Change**	**Counterfactual**	**% Change**	**Counterfactual**	**% Change**
incumbent innovation	0.0484	0.0504	4.256%	0.0500	3.278%	0.0491	1.450%
avg. R&D intensity	5.130%	5.436%	5.953%	5.188%	1.121%	5.384%	4.940%
entrant innovation	0.0401	0.0411	2.575%	0.0400	-0.259%	0.0412	2.730%
avg. entry rate	3.091%	3.270%	5.808%	3.238%	4.754%	3.139%	1.579%
contribution of entrants to growth	17.961%	17.466%	-2.756%	17.263%	-3.888%	18.039%	0.432%
creative destruction rate	5.840%	6.072%	3.968%	5.996%	2.671%	5.938%	1.669%
avg. incumbent value	0.6286	0.6386	1.603%	0.6607	5.109%	0.6109	-2.814%
avg. entrepreneur value	0.0515	0.0542	5.138%	0.0512	-0.502%	0.0543	5.449%
avg. product line value	0.1975	0.1942	-1.685%	0.1974	-0.073%	0.1940	-1.763%
avg. number of product lines	3.1824	3.2888	3.344%	3.3474	5.185%	3.1483	-1.071%
stdv. number of product lines	6.2293	6.8579	10.091%	7.2821	16.902%	5.9129	-5.079%
avg. plaintiff prob.	9.243%	9.555%	3.376%	10.145%	9.759%	8.794%	-4.860%
per product line plaintiff prob.	2.904%	2.905%	0.031%	3.031%	4.348%	2.793%	-3.830%
avg. prob. of hiring a legal team	89.152%	83.793%	-6.011%	86.652%	-2.804%	86.011%	-3.523%
output growth rate	2.016%	2.170%	7.670%	2.136%	5.955%	2.063%	2.332%
consumption	0.2281	0.2267	-0.624%	0.2270	-0.479%	0.2276	-0.205%
output	0.2513	0.2513	0.006%	0.2513	0.007%	0.2513	0.003%
CEWC	—	3.292%	—	2.553%	—	0.974%	—

Table 7: The Impact of Increasing Plaintiff Filing Costs

Notes: This table decomposes the impact of increasing plaintiff filing costs while keeping other parameters unchanged at their baseline values. We evaluate its effects on innovation, firm values, firm dynamics, litigation, growth, and social welfare. Column (1) reports the values of the variables of interest for the baseline model. In column (2), we halve the ξ parameter from its baseline value for all lawsuits. In column (3), we halve the ξ parameter from its baseline value for lawsuits targeting firms in the in the lowest tercile of research efficiency (measured by λ_c/χ_c), effectively doubling the average expense associated with hiring a legal team for these cases. In column (4), we halve the ξ parameter from its baseline value for lawsuits targeting firms in the highest tercile of research efficiency (measured by λ_c/χ_c), effectively doubling the average expense associated with hiring a legal team for these cases.

	(1)	(2)		(3)		(4)	
	Baseline	**Whole Sample**		**Low λ_c/χ_c Subsample**		**High λ_c/χ_c Subsample**	
		Counterfactual	**% Change**	**Counterfactual**	**% Change**	**Counterfactual**	**% Change**
incumbent innovation	0.0498	0.0511	2.519%	0.0498	-0.091%	0.0513	2.981%
avg. R&D intensity	5.348%	5.569%	4.125%	5.346%	-0.047%	5.524%	3.284%
entrant innovation	0.0408	0.0417	2.154%	0.0410	0.302%	0.0410	0.461%
avg. entry rate	3.217%	3.334%	3.634%	3.215%	-0.055%	3.351%	4.165%
contribution of entrants to growth	17.617%	17.354%	-1.491%	17.687%	0.398%	17.049%	-3.225%
creative destruction rate	6.005%	6.152%	2.457%	6.003%	-0.024%	6.158%	2.553%
avg. incumbent value	0.6353	0.6337	-0.249%	0.6320	-0.513%	0.6535	2.865%
avg. entrepreneur value	0.0534	0.0557	4.286%	0.0537	0.591%	0.0539	0.924%
avg. product line value	0.1951	0.1916	-1.792%	0.1948	-0.156%	0.1933	-0.930%
avg. number of product lines	3.2564	3.3075	1.571%	3.2447	-0.358%	3.3811	3.831%
stdv. number of product lines	6.6644	6.9960	4.976%	6.6233	-0.618%	7.3043	9.601%
avg. plaintiff prob.	9.493%	7.565%	-20.311%	9.354%	-1.468%	8.222%	-13.389%
per product line plaintiff prob.	2.915%	2.287%	-21.544%	2.883%	-1.114%	2.432%	-16.584%
prob. of hiring legal team	85.625%	64.573%	-24.586%	84.483%	-1.334%	68.924%	-19.504%
output growth rate	2.124%	2.230%	4.993%	2.122%	-0.087%	2.241%	5.500%
consumption	0.2271	0.2260	-0.472%	0.2271	0.009%	0.2259	-0.532%
output	0.2513	0.2513	0.0014%	0.2513	-0.0048%	0.2513	0.015%
CEWC	—	2.203%	—	-0.037%	—	2.416%	—

Table 8: Decomposing the Impact of Increasing Plaintiff Filing Costs by Infringement Types

Notes: This table decomposes the impact of increasing plaintiff filing costs while keeping other parameters unchanged at their baseline values. We evaluate its effects on innovation, firm values, firm dynamics, litigation, growth, and social welfare. Column (1) reports the values of the variables of interest for the baseline model. In column (2), we halve the ξ parameter from its baseline value for both type-1 and type-2 infringements. In column (3), we halve the ξ parameter from its baseline value for type-1 infringement, effectively doubling the average expense associated with hiring a legal team for the plaintiff in type-1 infringement cases. In column (4), we halve the ξ parameter from its baseline value for type-2 infringement, effectively doubling the average expense associated with hiring a legal team for the plaintiff in type-2 infringement cases.

	(1)	(2)		(3)		(4)	
	Baseline	**Both Types**		**Type 1 Only**		**Type 2 Only**	
		Counterfactual	**% Change**	**Counterfactual**	**% Change**	**Counterfactual**	**% Change**
incumbent innovation	0.0498	0.0511	2.519%	0.0509	2.127%	0.0501	0.597%
avg. R& D intensity	5.348%	5.569%	4.125%	5.387%	0.718%	5.532%	3.436%
entrant innovation	0.0408	0.0417	2.154%	0.0408	-0.140%	0.0417	2.239%
avg. entry rate	3.217%	3.334%	3.634%	3.322%	3.263%	3.237%	0.616%
contribution of entrants to growth	17.617%	17.354%	-1.491%	17.123%	-2.800%	17.785%	0.956%
creative destruction rate	6.005%	6.152%	2.457%	6.109%	1.741%	6.057%	0.876%
avg. incumbent value	0.6353	0.6337	-0.249%	0.6554	3.170%	0.6154	-3.129%
avg. entrepreneur value	0.0534	0.0557	4.286%	0.0533	-0.269%	0.0558	4.455%
avg. product line value	0.1951	0.1916	-1.792%	0.1944	-0.338%	0.1920	-1.587%
avg. number of product lines	3.2564	3.3075	1.571%	3.3710	3.519%	3.2053	-1.567%
stdv. number of product lines	6.6644	6.9960	4.976%	7.4080	11.158%	6.3038	-5.412%
avg. plaintiff prob.	9.493%	7.565%	-20.311%	8.859%	-6.677%	8.158%	-14.065%
per product line plaintiff prob.	2.915%	2.287%	-21.544%	2.628%	-9.850%	2.545%	-12.696%
avg. prob. of hiring a legal team	85.625%	64.573%	-24.586%	74.697%	-12.762%	74.420%	-13.086%
output growth rate	2.124%	2.230%	4.993%	2.213%	4.196%	2.148%	1.097%
consumption	0.2271	0.2260	-0.472%	0.2262	-0.377%	0.2268	-0.129%
output	0.2513	0.2513	0.001%	0.2513	0.005%	0.2513	-0.001%
CEWC	—	2.203%	—	1.868%	—	0.454%	—

Table 9: Simulating the Effects of Reducing Patent Infringement Risk

Notes: This table evaluates the impact of reducing patent infringement risk while keeping other parameters fixed at their baseline values. We evaluate its effects on innovation, firm values, firm dynamics, litigation, growth, and social welfare. This analysis is relevant to policies such as the introduction of Patent Trial and Appeal Board (PTAB) and the *Alice* decision in improving patent clarity and reducing litigation risks. Column (1) presents the baseline values for the variables of interest. Column (2) examines the effects of halving both κ_1 and κ_2 simultaneously. Column (3) reports the results of halving the type-1 infringement risk parameter, κ_1, from its baseline value. Column (4) reflects the results of halving the type-2 infringement risk parameter, κ_2, from its baseline value.

	(1) **Baseline**	(2) **Both Types**		(3) **Type 1 Only**		(4) **Type 2 Only**	
		Counterfactual	**% Change**	**Counterfactual**	**% Change**	**Counterfactual**	**% Change**
incumbent innovation	0.0498	0.0537	7.677%	0.0530	6.309%	0.0511	2.558%
avg. R&D intensity	5.348%	5.845%	9.288%	5.446%	1.823%	5.778%	8.034%
entrant innovation	0.0408	0.0422	3.302%	0.0405	-0.829%	0.0424	3.928%
avg. entry rate	3.217%	3.554%	10.481%	3.503%	8.897%	3.306%	2.757%
contribution of entrants to growth	17.617%	16.548%	-6.064%	16.170%	-8.210%	17.678%	0.349%
creative destruction rate	6.005%	6.421%	6.933%	6.310%	5.095%	6.172%	2.791%
avg. incumbent value	0.6353	0.6687	5.260%	0.6997	10.141%	0.6142	-3.321%
avg. entrepreneur value	0.0534	0.0570	6.625%	0.0526	-1.604%	0.0576	7.889%
avg. product line value	0.1951	0.1913	-1.922%	0.1951	0.008%	0.1906	-2.309%
avg. number of product lines	3.2564	3.4948	7.323%	3.5863	10.132%	3.2226	-1.036%
stdv. number of product lines	6.6644	8.0899	21.390%	8.7339	31.052%	6.2225	-6.632%
avg. plaintiff probability	9.493%	5.658%	-40.402%	7.878%	-17.014%	7.075%	-25.473%
per product line plaintiff probability	2.915%	1.619%	-44.469%	2.197%	-24.649%	2.195%	-24.693%
avg. prob. of hiring a legal team	85.625%	85.172%	-0.529%	85.892%	0.312%	84.793%	-0.972%
output growth rate	2.124%	2.422%	14.031%	2.376%	11.870%	2.206%	3.843%
consumption	0.2271	0.2249	-0.976%	0.2253	-0.804%	0.2264	-0.301%
output	0.2513	0.2513	0.020%	0.2514	0.021%	0.2513	0.010%
CEWC	—	6.684%	—	5.651%	—	1.754%	—

Table 10: Simulating the Impact of Rising Patent Trolls

Notes: This table simulates the potential effects of a rise in patent trolls on innovation, firm values, firm dynamics, litigation, growth, and social welfare. Patent trolls are modeled as a combination of an increase in type-2 infringement probability—cases where the plaintiff is not the owner of the product line and files lawsuits for rent-extraction purposes—along with a rise in the entry rate of low R&D efficiency firms. To simulate these effects, we double the type-2 infringement risk and the entry rate of firms in the lowest tercile of research efficiency (measured by λ_c/χ_c). Column (1) reports the baseline values for the variables of interest. Column (2) presents the effects of a rise in patent trolls, combining both the increased type-2 infringement risk and the higher entry rate of low R&D efficiency firms. Column (3) isolates the effect of a rise in type-2 infringement probability by doubling the type-2 infringement risk. Column (4) shows the impact of a rise in low research efficiency firms by doubling the entry rate of firms in the lowest tercile of research efficiency.

	(1)	(2)		(3)		(4)	
	Baseline	**Rise in Patent Trolls**		**Higher Type-2 Risk**		**More Low R&D Firms**	
		Counterfactual	**% Change**	**Counterfactual**	**% Change**	**Counterfactual**	**% Change**
incumbent innovation	0.0498	0.0500	0.325%	0.0482	-3.345%	0.0511	2.504%
avg. R&D intensity	5.348%	4.032%	-24.611%	4.536%	-15.195%	4.754%	-11.116%
entrant innovation	0.0408	0.0307	-24.801%	0.0371	-9.105%	0.0338	-17.201%
avg. entry rate	3.217%	3.282%	2.017%	3.096%	-3.773%	3.349%	4.084%
contribution of entrants to growth	17.617%	14.379%	-18.381%	16.946%	-3.810%	15.149%	-14.010%
creative destruction rate	6.005%	5.768%	-3.946%	5.745%	-4.324%	5.954%	-0.846%
avg. incumbent value	0.6353	0.9160	44.179%	0.7019	10.481%	0.8196	29.009%
avg. entrepreneur value	0.0534	0.0305	-42.975%	0.0443	-17.133%	0.0368	-31.087%
avg. product line value	0.1951	0.2072	6.207%	0.2039	4.490%	0.2000	2.487%
avg. number of product lines	3.2564	4.4206	35.752%	3.4431	5.733%	4.0991	25.878%
stdv. number of product lines	6.6644	12.8794	93.255%	7.9529	19.333%	10.9749	64.678%
avg. plaintiff prob.	9.493%	17.995%	89.566%	13.930%	46.740%	12.911%	36.002%
per product line plaintiff prob.	2.915%	4.071%	39.641%	4.046%	38.783%	3.150%	8.043%
avg. prob of hiring legal team	85.625%	86.351%	0.848%	86.772%	1.340%	85.374%	-0.293%
output growth rate	2.124%	2.011%	-5.329%	2.005%	-5.604%	2.099%	-1.206%
consumption	0.2271	0.2272	0.031%	0.2280	0.394%	0.2265	-0.242%
output	0.2513	0.2511	-0.0595%	0.2513	-0.0005%	0.2511	-0.076%
CEWC	—	-2.760%	—	-2.550%	—	-0.879%	—

Internet Appendices:
The Efficiency of Patent Litigation

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A. Data, Moment Construction, and Estimation Procedure

A.1. Data

We use several data sets in our estimation. First, we download the Compustat North America Fundamentals Annual dataset. We obtain a firm-year dataset with the following variables: (i) total sales (revenue); (ii) R&D spending; (iii) industry (SIC code); (iv) firm name.

Second, we download the Global Corporate Patent Dataset (GCPD) (Bena, Ferreira, Matos, and Pires, 2017).[5] For the period 1980 to 2017, the GCPD provides a comprehensive link between patents awarded by the U.S. Patent and Trademark Office (USPTO) and the publicly listed Compustat firms that received those patents.

Third, we download the USPTO patent-litigation database,[6] which covers all patent lawsuits filed in federal courts over the period 2003 to 2016. For each lawsuit, the dataset includes identifiers for all of the infringed patents.

We merge these three data sets together. We construct a firm-year panel covering the period 2003 to 2016. It includes all Compustat observations for all firms that hold at least one patent in the GCPD.[7] We assign each firm a technology class c and an industry j by the following procedure. We use the first digit of the GCPD technology-class classification to construct nine different potential technology classes. We assign each firm a time-invariant technology class using the GCPD classification.[8] We use SIC codes from Compustat to construct the Fama-French twelve industries.[9] We exclude firms in Finance, Utilities, or Other. We then aggregate the remaining nine industries into four industry groups.[10] Thus,

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[5] See https://patents.darden.virginia.edu/.

[6] See https://www.uspto.gov/ip-policy/economic-research/research-datasets/patent-litigation-docket-reports-data.

[7] We exclude firm years with missing SIC codes, assets, sales or Compustat identifiers. We exclude firm years with under $50 million in sales.

[8] If a firm has multiple patents, we take the modal technology class across its patents. If there is a tie, we consider the firm's technology class to be missing. If the firm's modal technology class is the "missing" technology class, we consider the firm's technology class to be missing.

[9] See https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library/det_12_ind_port.html.

[10] Based on the Fama-French industry classification, our analysis focuses on four primary industry groups: (i) Manufacturing: This encompasses Fama-French industry classifications 1 (Consumer NonDurables – Food, Tobacco, Textiles, Apparel, Leather, Toys), 2 (Consumer Durables – Cars, TVs, Furniture, Household Appliances), and 3 (Manufacturing – Machinery, Trucks, Planes, Office Furniture, Paper, Commercial Printing); (ii) Extraction and Chemicals: This group includes Fama-French industry classifications 4 (Oil, Gas, and Coal Extraction and Products) and 5 (Chemicals and Allied Products); (iii) Information and Communication

we have nine technology classes and four industry groups in our data. Since we analogously solve our model assuming there are nine technology classes and four industry groups, the reduction in the number of industry groups eases computation.

Our merging links each firm in our sample to all of its patents over the period 1980 to 2017. It also links each patent to all of the lawsuits filed over that patent. We can thus construct an indicator equal to one for firm i in year t if firm i files a patent lawsuit in year t.

For a separate empirical moment, we download quarterly data on year-over-year US GDP growth from the Federal Reserve Bank of St. Louis.[11]

Finally, we construct some moments using a separate lawsuit-level dataset. To construct this dataset, we begin with the Federal Judicial Center (FJC) database.[12] This contains every civil lawsuit filed in federal courts. Using the FJC classification, we isolate patent lawsuits. For each patent lawsuit, the FJC contains an indicator equal to one if a lawsuit is settled out of court. It also contains an indicator equal to one if a lawsuit goes to trial. Further, it contains a variable specifying whether the plaintiff or defendant won in trial. We merge the FJC data with the USPTO litigation database to obtain identifiers for litigated patents. Using the patent identifiers, Compustat, and the GCPD, we identify the name and industry j of the plaintiff. Finally, we use defendant names in the FJC data to identify defendants in Compustat.[13] Our final lawsuit-level dataset contains the industry of the plaintiff, the industry of the defendant, the outcome of the lawsuit (settlement versus trial), and the trial outcome (plaintiff or defendant victory) for lawsuits ending in trials.

A.2. Empirical Moments

We use the data described above to calculate 16 empirical moments. We estimate our model by minimizing the distance between these empirical moments and their model-implied

Technology (ICT): This encompasses Fama-French industry classifications 6 (Business Equipment – Computers, Software, and Electronic Equipment) and 7 (Telephone and Television Transmission); (iv) Services: This group includes Fama-French industry classifications 9 (Wholesale, Retail, and Some Services such as Laundries, Repair Shops) and 10 (Healthcare, Medical Equipment, and Drugs). We exclude firms categorized under the 'other' industry classification. Additionally, we exclude firms operating in the financial and utility sectors due to their heavy regulation and distinct business models compared to other industries.

[11] See https://fred.stlouisfed.org/series/USAGDPRQPSMEI.

[12] See https://www.fjc.gov/research/idb/civil-cases-filed-terminated-and-pending-sy-1988-present.

[13] We verify name matches both algorithmically and manually. We only keep matches for which we are highly confident of the accuracy.

counterparts. We now describe the empirical moments.

First, we calculate the average annual GDP growth rate in our sample period: it is 2.03%.

Second, we use our firm-year panel to calculate several moments. We calculate firm-year-level sales growth,[14] take an average for each technology class, then take a standard deviation across technology classes. The corresponding standard deviation, 1.79%, corresponds to the variation in sales growth across technology classes. Next, we calculate R&D intensity as the ratio of R&D spending to sales.[15] We calculate a sample average of 6.43%. We then take the average R&D intensity for each technology class and take a standard deviation across technology classes: we find this standard deviation is 3.6%. We similarly calculate that the skewness of R&D intensity across technology classes is 0.546. Next, we measure whether firms in technologies with a higher share of sales in the economy tend to have higher R&D spending. For each technology class in each year, we calculate the share of all sales attributable to firms in that technology class. We regress the logarithm of R&D spending on this sales share, controlling for year fixed effects, and find a regression coefficient on the sales share that is equal to 0.028.[16]

Next, we study our indicator for a firm being a plaintiff in a patent lawsuit in a given year. The average for this indicator is 10.53% in our sample. We take an average for each industry j and technology class c. Taking a standard deviation across industries and technology classes, we find the standard deviation is 8.63%. We then explore how high-litigation technology classes differ from low-litigation technology classes. We take an average of our litigation indicator for each technology class. We similarly take the average sales growth and R&D intensity at the technology class level. Taking a correlation across technology classes, we find a correlation between litigation and sales growth equal to 0.713. We find a correlation between litigation and R&D intensity equal to 0.62.

We also use the firm-year panel to calculate the rate at which new firms enter the economy. For each firm-year observation, we define an indicator equal to one if that year is the first time that the firm appears in our panel. Excluding 2003, the first year in our panel, we

[14] We calculate this as the sales for firm i in year t minus the sales for the same firm in the previous year, divided by sales in the previous year. The previous year is the most recent year prior to t that appears in Compusat.

[15] We impute zero for missing R&D values. We winsorize R&D intensity at 5% and 95%.

[16] We normalize the dependent variable, the logarithm of R&D spending, by subtracting the sample mean of the log R&D spending and dividing by the standard deviation of the log R&D spending.

calculate that the mean of this indicator is equal to 4.2%.

We then turn to our lawsuit-level dataset. Conditional on a plaintiff having the same industry as the defendant and a lawsuit going to trial, we find that 35.44% of plaintiffs win in trial.[17] Conditional on a plaintiff having a different industry than the defendant and a lawsuit going to trial, we find that 40.91% of plaintiffs win in trial. Across all of these cases that end in trial, the plaintiff and defendant share the same industry in 78.22% of cases. Overall, we find that 58.32% of lawsuits end in settlement.[18]

Next, we use a US courts survey to measure an annual time series of corporate litigation spending. Specifically, in each year over the period 2003 to 2008, the survey reports the average ratio of litigation spending to revenue for public US corporations. Calculating an average across years, we find that an average public US firm spends 0.59% of its revenue on litigation costs annually.[19]

Finally, we directly calibrate ω_j to match the share of all sales attributable to industry j.[20] Likewise, we directly calibrate η_{cj} to match the share of all new-entrant sales attributable to industry j.[21] Additionally, the injunction rate is pinned down by the observed injunction rate - this was 95% before eBay ruling in 2006 and 75% after eBay (Seaman, 2015), leading to an average injunction rate of 81% in our sample.[22]

[17] When we calculate these statistics, we keep only cases in which either the plaintiff wins a trial or the defendant wins a trial. We exclude cases in which the FJC indicates that both parties had a partial victory. We exclude six lawsuits in which there were multiple plaintiffs that had different industries.

[18] When we calculate the settlement rate, we use our sample of patent lawsuits from the FJC. We exclude cases that are transferred or dismissed for reasons other than settlement. We therefore only keep cases that are settled or conclude with a judgment.

[19] See Figure 7 in https://www.uscourts.gov/sites/default/files/litigation_cost_survey_of_major_companies_0.pdf. We use the time series for consistent reporters over the period 2003 to 2008.

[20] In each industry j and year t, we calculate the fraction of all sales attributable to firms in industry j. For each j, we average across years to calculate ω_j.

[21] We call a firm i a new entrant in year t if it is the first year over the period 2003 to 2016 in which the firm appears in Compustat. In each industry j and each year t, we calculate the fraction of new-entrant sales attributable to industry j. Excluding the first year 2003, in which all firms are new entrants, we average across years to construct a share η_j of new-entrant sales. We then divide by the number of technology classes to construct η_{cj}.

[22] In our sample, 31.5% of observations correspond to the period 2003 to 2006. We calculate $95\% \times 31.5\% + 75\% \times (1 - 31.5\%) = 81\%$.

A.3. Calculating the GMM Weighting Matrix and Objective

We estimate our parameters by GMM. We calculate the covariance matrix Ω for our 16 empirical moments. We calculate the efficient weighting matrix W as the inverse of Ω. We define a vector $\boldsymbol{\theta}$ containing the 13 estimated model parameters. We define a vector M_{emp} containing our 16 empirical moments. We define a vector $M(\boldsymbol{\theta})$ containing the model counterparts, which depend on the vector $\boldsymbol{\theta}$. We estimate $\boldsymbol{\theta}$ by minimizing the GMM objective:

$$\boldsymbol{\theta}_{\text{GMM}} = \underset{\boldsymbol{\theta}}{\arg\min}\,(M_{\text{emp}} - M(\boldsymbol{\theta}))' W (M_{\text{emp}} - M(\boldsymbol{\theta})). \tag{A.1}$$

We calculate GMM asymptotic standard errors using the standard sandwich formula.

We calculate the covariance matrix Ω by bootstrapping. We face a challenge: some moments are calculated in a firm-year panel while others are calculated at a different unit of observation (e.g., the time series of GDP growth or the settlement rate across lawsuits). To overcome this challenge, we must make a simplifying assumption: the covariance between two moments calculated in different samples is zero. This assumption allows us to stack covariance matrices. Specifically, we first bootstrap a firm-clustered covariance matrix for 10 moments calculated in our firm-year panel. We then stack this on top of a covariance matrix for three moments calculated in a smaller sample of lawsuits that go to trial. We impute zeros for the covariances between the first 10 moments and the latter three moments. We do the same for the following moments: the average GDP growth rate, the average lawsuit settlement rate, and the average ratio of litigation spending to revenue, which is based on time-series data from an external source. Our approach is similar to the approaches used in Acemoglu et al. (2018) and Bordalo et al. (2020). For each bootstrapping exercise, we bootstrap 500 samples from our original sample, clustering by firm for the firm-year panel.

B. Illustrative Model Proof

Define the operator G that maps the vector $[x^s, x^c]$ to a vector Gx defined by

$$Gx^*_{cj} = (2-\alpha_{cj}\zeta)\frac{\pi+\lambda_c+\sigma x^*_{cj'}}{4\chi}. \tag{B.1}$$

$$Gx^s_{cj} = \frac{2-\alpha_{cj}\zeta}{4\chi}(\lambda_c+\sigma x^s_{cj'})+\sigma x^s_{cj'}\frac{2-\alpha_{cj'}\zeta}{4\chi}. \tag{B.2}$$

The private equilibrium and social optimum are given by a fixed point of G. Moreover, if $\sigma < \chi$, this is clearly a contraction mapping, since each element of $Gx - Gy$ is the product of (i) another element of the vector $x - y$, and (ii) $\sigma/\chi < 1$, and (iii) either $(2-\alpha_{cj}\zeta)/4 < 1$ or $1 - \frac{\alpha_{cj}\zeta+\alpha_{cj'}\zeta}{4} \leq 1$.

First, note that as $\sigma \to 0$,

$$x^*_{cj} = \frac{2-\alpha_{cj}\zeta}{4\chi}(\pi+\lambda_c) > \frac{2-\alpha_{cj}\zeta}{4\chi}\lambda_c = x^s_{cj}. \tag{B.3}$$

Next, note that as $\pi \to 0$,

$$x^*_{cj} = (2-\alpha_{cj}\zeta)\frac{\lambda_c+\sigma x^*_{cj'}}{4\chi} > 0. \tag{B.4}$$

while

$$x^s_{cj} = \frac{2-\alpha_{cj}\zeta}{4\chi}(\lambda_c+\sigma x^s_{cj'})+\sigma x^s_{cj'}\frac{2-\alpha_{cj'}\zeta}{4\chi} > 0. \tag{B.5}$$

Thus, as $\pi \to 0$, G has the property that whenever the vector x has $x^*_{cj} < x^s_{cj}$, the same holds for the corresponding elements of Gx. Since G is a contraction mapping, we can start at any point and iteratively apply G to find the fixed point. It follows that $x^*_{cj} \leq x^s_{cj}$ at the fixed point. From inspection of the above equations, we see that it cannot be that $x^s_{cj} = x^*_{cj}$, so $x^*_{cj} < x^s_{cj}$.

As $\pi \to \infty$, we have $x^*_{cj} \to \infty$ while x^s_{cj} is fixed, so $x^*_{cj} > x^s_{cj}$.

Next, note that as $\lambda_c \to 0$,

$$x^*_{cj} = (2-\alpha_{cj}\zeta)\frac{\pi+\lambda_c+\sigma x^*_{cj'}}{4\chi} \geq \frac{2-\alpha_{cj}\zeta}{4\chi}\pi > 0, \tag{B.6}$$

while

$$x_{cj}^s = x_{cj'}^s \left(\frac{2-\alpha_{cj}\zeta}{4\chi}\sigma + \sigma\frac{2-\alpha_{cj'}\zeta}{4\chi} \right), \tag{B.7}$$

implying that

$$x_{cj}^s = x_{cj}^s \left(\frac{2-\alpha_{cj}\zeta}{4\chi}\sigma + \sigma\frac{2-\alpha_{cj'}\zeta}{4\chi} \right)\left(\frac{2-\alpha_{cj'}\zeta}{4\chi}\sigma + \sigma\frac{2-\alpha_{cj}\zeta}{4\chi} \right), \tag{B.8}$$

and thus $x_{cj}^s = 0$ absent a knife edge case. This implies $x_{cj}^s = 0 < x_{cj}^*$ as $\lambda_c \to 0$.

C. Theory Appendix

C.1. Additional Calculations for the Model

C.1.1. Product-line owner's static pricing problem

Bertrand competition implies that only the productivity leader for any differentiated good produces a positive quantity. Because the monopoly price tends to infinity, the final good production function assures that the leader always chooses to follow a limit pricing strategy. For a leader with productivity q_{ijt} and technology class c, the productivity of the most productive competitor is $q_{ijt}/(1+\lambda_c)$, and the limit price is therefore $w_t(1+\lambda_c)/q_{ijt}$. At this price, if the competitor produced $y > 0$, it would need labor $l = y(1+\lambda_c)/q^{new}$ and would therefore make profit

$$yp - lw = y(w_t(1+\lambda_c)/q^{new}) - w(y(1+\lambda_c)/q^{new}) = 0.$$

The static profit flow of the leader for good i in industry j at time t is

$$\pi_{ijt} = \left(p_{ijt} - \frac{w_t}{q_{ijt}}\right) y_{ijt} = \left(\frac{w_t(1+\lambda_c)}{q_{ijt}} - \frac{w_t}{q_{ijt}}\right) \frac{q_{ijt}\omega_j Y_t}{w_t(1+\lambda_c)} = \frac{\lambda_c}{1+\lambda_c}\omega_j Y_t, \tag{C.1}$$

where the first equality uses $y = ql$ and the second equality uses the final good producer's demand at the limit price.

C.1.2. The defendant's settlement acceptance cutoff

The defendant accepts a settlement offer if and only if

$$\begin{aligned} V_{cjt}(n+1) - V_{cjt}(n) - s \quad &> \quad [\tau + (1-\tau)(1-\zeta)]\left(V_{cjt}(n+1) - V_{cjt}(n)\right) \\ (1-\tau)\zeta\left(V_{cjt}(n+1) - V_{cjt}(n)\right) \quad &> \quad s \end{aligned} \tag{C.2}$$

where $(1-\tau)\zeta$ is the probability of an injunction conditional on a trial. $\zeta \in \{\zeta_1, \zeta_2\}$ depends on the infringement type. Equation (C.2) shows that the defendant accepts the settlement offer only if the value of removing the injunction likelihood is above a threshold value of s, denoted $\bar{s}_{cjt}(n,\tau)$, that leaves the defendant indifferent.

C.2. Proof of Proposition 1

Proposition 1. *When successful innovation by a firm with technology class c in industry j with n product lines leads to a type 2 patent infringement, the following are true in the subgame perfect equilibrium of the litigation game:*

1. *The ex-ante probability that the plaintiff hires a legal team is*

$$p^{LT}_{2,cjt} \equiv \mathbb{P}\left(\gamma \leq \frac{(1-\tau^l_2)^2\zeta_2(V_{cjt}(n+1)-V_{cjt}(n))}{4(\tau^h_2-\tau^l_2)Y_t}\right) \tag{C.3}$$

2. *The optimal settlement offer made by the plaintiff is*

$$s^* \equiv \frac{(1-\tau^l_2)\zeta_2(V_{cjt}(n+1)-V_{cjt}(n))}{2} \tag{C.4}$$

3. *The defendant accepts the settlement if $\tau \leq \tau^* \equiv \frac{1+\tau^l_2}{2}$, and rejects otherwise. The ex-ante acceptance probability is $\mathbb{P}(s^* < \bar{s}_{cjt}(n,\tau)) = \frac{1}{2}\frac{1-\tau^l_2}{\tau^h_2-\tau^l_2}$.*

4. *The expected payoff of the plaintiff is*

$$W^{plain}_{2,cjt} \equiv p^{LT}_{2,cjt}\frac{(1-\tau^l_2)^2\zeta_2(V_{cjt}(n+1)-V_{cjt}(n))}{4(\tau^h_2-\tau^l_2)} - Y_t\int_0^{\frac{(1-\tau^l_2)^2\zeta_2(V_{cjt}(n+1)-V_{cjt}(n))}{4(\tau^h_2-\tau^l_2)Y_t}} \gamma d\Gamma(\gamma) \tag{C.5}$$

5. *The expected payoff of the defendant is*

$$\begin{aligned} W^{def}_{2,cjt} \equiv & \Bigg((1-p^{LT}_{2,cjt}) + p^{LT}_{2,cjt}\Bigg[\left(1-\frac{(1-\tau^l_2)\zeta_2}{2}\right)\left(\frac{1}{2}\frac{1-\tau^l_2}{\tau^h_2-\tau^l_2}\right) + (1-\zeta_2)\left(1-\frac{1}{2}\frac{1-\tau^l_2}{\tau^h_2-\tau^l_2}\right) \\ & + \frac{\zeta_2}{2(\tau^h_2-\tau^l_2)}\left((\tau^h_2)^2 - \frac{(1+\tau^l_2)^2}{4}\right)\Bigg]\Bigg)(V_{cjt}(n+1)-V_{cjt}(n)) \end{aligned} \tag{C.6}$$

Proof. We consider the decision problem of a plaintiff facing a type 2 patent infringement in which they don't face any risk of losing product lines. They must choose a take-it-or-leave-it settlement offer s without knowing the realization of τ – the defendant's probability of winning at court. Their problem is written as

$$\max_{s\geq 0}\left\{s\mathbb{P}(s\leq \bar{s}_{cjt}(n,\tau))\right\} \tag{C.7}$$

where the second term is the probability that the offer is accepted. Let T_2 denote the distribution of τ_2, which is uniform with bounds τ_2^l, τ_2^h. We can rewrite this probability as

$$\begin{aligned}
\mathbb{P}(s\leq \bar{s}_{cjt}(n,\tau)) &= \mathbb{P}\left(s\leq (1-\tau)\zeta_2\left(V_{cjt}(n+1)-V_{cjt}(n)\right)\right) \\
&= \mathbb{P}\left(\tau\leq 1-\frac{s}{\zeta_2\left(V_{cjt}(n+1)-V_{cjt}(n)\right)}\right) \\
&= \int_{-\infty}^{1-s/(\zeta_2\left(V_{cjt}(n+1)-V_{cjt}(n)\right))} dT_2(\tau)
\end{aligned} \tag{C.8}$$

which, under the distributional assumption, becomes

$$\mathbb{P}(s\leq \bar{s}_{cjt}(n,\tau)) = \begin{cases} 0 & \text{if } 1-s/(\zeta_2(V_{cjt}(n+1)-V_{cjt}(n)))<\tau_2^l \\ 1 & \text{if } 1-s/(\zeta_2(V_{cjt}(n+1)-V_{cjt}(n)))>\tau_2^h \\ \frac{1-s/(\zeta_2(V_{cjt}(n+1)-V_{cjt}(n))-\tau_2^l}{\tau_2^h-\tau_2^l} & \text{otherwise} \end{cases} \tag{C.9}$$

Note that the optimal s must be such that $\tau_2^l\leq 1-s/(\zeta_2(V_{cjt}(n+1)-V_{cjt}(n)))\leq \tau_2^h$.[23] Then we can rewrite the objective function over this range as

$$\frac{s(1-\tau_2^l)-s^2/(\zeta_2(V_{cjt}(n+1)-V_{cjt}(n))}{\tau_2^h-\tau_2^l} \tag{C.10}$$

with the first order condition delivering

$$\begin{aligned}
1-\tau_2^l &= \frac{2s}{\zeta_2(V_{cjt}(n+1)-V_{cjt}(n))} \\
s &= \frac{(1-\tau_2^l)\zeta_2(V_{cjt}(n+1)-V_{cjt}(n))}{2}\equiv s^*
\end{aligned} \tag{C.11}$$

[23]Below τ_2^l, the probability of acceptance is zero, and so are the extracted rents. Above τ_2^h, the plaintiff is asking for a smaller payment even though it does not increase the probability of acceptance, thus losing out on rents. Both are suboptimal.

which pins down the optimal s if the solution is interior. Given this expression, the cut-off τ for which the defendant is indifferent is given as

$$\tau^* = \frac{1+\tau_2^l}{2} \tag{C.12}$$

If $\tau^* \leq \tau_2^h$, then the solution is interior, and the optimal s is given by equation (C.4). If not, then we have a corner solution:

$$s = (1-\tau_2^h)\zeta_2(V_{cjt}(n+1) - V_{cjt}(n)) \tag{C.13}$$

In the case of an interior solution, the identity for the acceptance probability becomes

$$\mathbb{P}(s < \bar{s}_{cjt}(n,\tau)) = \frac{1}{2}\frac{1-\tau_2^l}{\tau_2^h - \tau_2^l} \tag{C.14}$$

which is independent of c, j, t, and n. The optimal expected rent is then

$$s\mathbb{P}(s < \bar{s}_{cjt}(n,\tau)) = \frac{(1-\tau_2^l)^2\zeta_2(V_{cjt}(n+1) - V_{cjt}(n))}{4(\tau_2^h - \tau_2^l)} \tag{C.15}$$

In the case of a corner solution, the probability of acceptance is unity, and the optimal expected rent is simply equal to equation (C.13). Given our assumption that $1+\tau_2^l \leq 2\tau_2^h$, the solution is always interior.

Given the optimal expected rent expression, we can now turn to the plaintiff's decision to hire a legal team or not. The plaintiff will choose to hire a legal team if the expected rent is higher than the cost γY_t where γ is drawn from the distribution $\Gamma(\gamma)$. The probability of hiring a legal team is given by

$$p_{2,cjt}^{LT} \equiv \mathbb{P}\left(\gamma \leq \frac{(1-\tau_2^l)^2\zeta_2(V_{cjt}(n+1) - V_{cjt}(n))}{4(\tau_2^h - \tau_2^l)Y_t}\right) \tag{C.16}$$

and the expected rents conditional on a type 2 patent infringement minus legal team cost are given as

$$W_{2,cjt}^{plain} \equiv p_{2,cjt}^{LT}\frac{(1-\tau_2^l)^2\zeta_2(V_{cjt}(n+1)-V_{cjt}(n))}{4(\tau_2^h-\tau_2^l)} - Y_t\int_0^{\frac{(1-\tau_2^l)^2\zeta_2(V_{cjt}(n+1)-V_{cjt}(n))}{4(\tau_2^h-\tau_2^l)Y_t}} \gamma d\Gamma(\gamma) \quad \text{(C.17)}$$

Turning to the defendant's side, conditional on a type 2 patent infringement, they will receive a settlement offer only if the plaintiff chooses to hire a legal team, the probability of which is $p_{2,cjt}^{LT}$. Conditional on receiving a settlement offer, their expected payoff is

$$\begin{aligned}
& \int_{\tau_2^l}^{\tau^*}\left(V_{cjt}(n+1)-V_{cjt}(n)-s^*\right)dT_2(\tau) + \int_{\tau^*}^{\tau_2^h}[\tau+(1-\tau)(1-\zeta_2)]\left(V_{cjt}(n+1)-V_{cjt}(n)\right)dT_2(\tau) \\
= & \left(1-\frac{(1-\tau_2^l)\zeta_2}{2}\right)(V_{cjt}(n+1)-V_{cjt}(n))\left(\frac{1}{2}\frac{1-\tau_2^l}{\tau_2^h-\tau_2^l}\right) \\
& +(1-\zeta_2)(V_{cjt}(n+1)-V_{cjt}(n))\left(1-\frac{1}{2}\frac{1-\tau_2^l}{\tau_2^h-\tau_2^l}\right) \\
& +\zeta_2(V_{cjt}(n+1)-V_{cjt}(n))\int_{\tau^*}^{\tau_2^h}\frac{\tau}{\tau_2^h-\tau_2^l}d\tau \\
= & \left[\left(1-\frac{(1-\tau_2^l)\zeta_2}{2}\right)\left(\frac{1}{2}\frac{1-\tau_2^l}{\tau_2^h-\tau_2^l}\right)+(1-\zeta_2)\left(1-\frac{1}{2}\frac{1-\tau_2^l}{\tau_2^h-\tau_2^l}\right)\right. \\
& \left.+\frac{\zeta_2}{2(\tau_2^h-\tau_2^l)}\left((\tau_2^h)^2-\frac{(1+\tau_2^l)^2}{4}\right)\right](V_{cjt}(n+1)-V_{cjt}(n)) \quad \text{(C.18)}
\end{aligned}$$

Therefore, the defendant's expected payoff conditional on a type 2 patent infringement and before learning whether they will receive a settlement offer can be written as

$$\begin{aligned}
W_{2,cjt}^{def} \equiv & \left((1-p_{2,cjt}^{LT})+p_{2,cjt}^{LT}\left[\left(1-\frac{(1-\tau_2^l)\zeta_2}{2}\right)\left(\frac{1}{2}\frac{1-\tau_2^l}{\tau_2^h-\tau_2^l}\right)+(1-\zeta_2)\left(1-\frac{1}{2}\frac{1-\tau_2^l}{\tau_2^h-\tau_2^l}\right)\right.\right. \\
& \left.\left.+\frac{\zeta_2}{2(\tau_2^h-\tau_2^l)}\left((\tau_2^h)^2-\frac{(1+\tau_2^l)^2}{4}\right)\right]\right)(V_{cjt}(n+1)-V_{cjt}(n)) \quad \text{(C.19)}
\end{aligned}$$

Note that the whole expression is linear in the expected change in firm value if they take over the product line, $V_{cjt}(n+1)-V_{cjt}(n)$, which will be of use in deriving a closed-form expression for the firm value function. □

C.3. Proof of Proposition 2

Proposition 2. *Suppose $V_{cjt}(n)$ is linear in n. When successful innovation by a firm with technology class c in industry j with n^d product lines leads to a type 1 patent infringement on the IP of an incumbent with n product lines, the following are true in the subgame perfect equilibrium of the litigation game:*

1. *The ex-ante probability that the plaintiff hires a legal team is*

$$p_{1,cjt}^{LT} \equiv \mathbb{P}\left(\gamma \leq \left(-\zeta_1 + \zeta_1 \frac{\tau_1^h + \tau_1^l}{2}\right) \frac{(V_{cjt}(n-1) - V_{cjt}(n))}{Y_t}\right) \tag{C.20}$$

2. *Due to adverse selection, the plaintiff never chooses to settle out of court. That is, the plaintiff offers $s^* = 0$, and the defendant always rejects, independent of the realization of τ.*

3. *The expected payoff of the plaintiff is*

$$\begin{aligned} W_{1,cjt}^{plain} \quad \equiv \quad & p_{1,cjt}^{LT}(V_{cjt}(n-1) - V_{cjt}(n)) \left(1 - \zeta_1 + \zeta_1 \frac{\tau_1^h + \tau_1^l}{2}\right) - Y_t \int_0^{\left(-\zeta_1 + \zeta_1 \frac{\tau_1^h + \tau_1^l}{2}\right) \frac{(V_{cjt}(n-1) - V_{cjt}(n))}{Y_t}} \gamma d\Gamma(\gamma) \\ & + (1 - p_{1,cjt}^{LT})(V_{cjt}(n-1) - V_{cjt}(n)) \end{aligned} \tag{C.21}$$

4. *The expected payoff of the defendant is*

$$W_{1,cjt}^{def} \equiv p_{1,cjt}^{LT}(V_{cjt}(n^d+1) - V_{cjt}(n^d)) \left(1 - \zeta_1 + \zeta_1 \frac{\tau_1^h + \tau_1^l}{2}\right) + (1 - p_{1,cjt}^{LT})(V_{cjt}(n^d+1) - V_{cjt}(n^d)) \tag{C.22}$$

Proof. We consider the decision problem of a plaintiff facing a type 1 patent infringement which means they are the owner of the product line that is facing the risk of creative destruction. We further know that the plaintiff and the defendant share the same technology class c. Unlike a type 2 infringement, this time the plaintiff cares about more than the potential settlement they can extract from the defendant, since settling out of court also means they lose their product line for sure. Assume the distribution $T_1(\tau)$ is the continuous uniform distribution $U(\tau_1^l, \tau_1^h)$ with $0 \leq \tau_1^l < \tau_1^h \leq 1$. Then the plaintiff's problem is written as

$$\begin{aligned} \max_{s \geq 0} \quad \Bigg\{ \quad & \int_{\tau_1^l}^{1 - s/(\zeta_1(V_{cjt}(n^d+1) - V_{cjt}(n^d)))} \left(V_{cjt}(n-1) - V_{cjt}(n) + s\right) dT_1(\tau) \\ & + \int_{1 - s/(\zeta_1(V_{cjt}(n^d+1) - V_{cjt}(n^d)))}^{\tau_1^h} (\tau + (1-\tau)(1-\zeta_1))(V_{cjt}(n-1) - V_{cjt}(n)) dT_1(\tau) \Bigg\} \end{aligned} \tag{C.23}$$

where the first integral is the expected payoff from defendants who accept the settlement and the second integral is the expected payoff from those who reject. The term $V_{cjt}(n-1)-V_{cjt}(n)$ is negative, and reflects the cost of losing the product line. In the cases when the defendant accepts, the plaintiff is gives up their $(1-\tau)\zeta_1$ chance of retaining their product line in exchange for a settlement amount s.

Note that there is an inherent adverse selection problem here: Conditional on a settlement offer s, only firms with the lowest chance of winning the trial τ will accept. From the defendant's problem, we know that a defendant strictly prefers the settlement offer if and only if

$$(1-\tau)\zeta_1(V_{cjt}(n^d+1)-V_{cjt}(n^d)) \quad > \quad s \tag{C.24}$$

where n^d stands for the defendant's number of product lines. On the other hand, the difference in the plaintiff's payoff in the case of acceptance is

$$(1-\tau)\zeta_1(V_{cjt}(n-1)-V_{cjt}(n))+s \tag{C.25}$$

Consider the defendant with the threshold τ^* who is indifferent. Then the abovementioned difference becomes

$$(1-\tau^*)\zeta_1(V_{cjt}(n-1)-V_{cjt}(n))+(1-\tau^*)\zeta_1(V_{cjt}(n^d+1)-V_{cjt}(n^d)) \tag{C.26}$$

which is exactly zero if $V_{cjt}(n)-V_{cjt}(n-1)=V_{cjt}(n^d+1)-V_{cjt}(n^d)$, that is, if the value change from having one more product line in industry j for firms with technology class c is the same regardless of how many product lines the company owns, n. We will later on show that this is exactly the case in a stationary equilibrium, since the value function of the firm will turn out to be linear in n. But this highlights the adverse selection problem: Even in the best case scenario, the plaintiff gains exactly zero from the firm with the highest probability of winning the trial among those who accept. For all other firms who accept that have a probability of winning the trial below the threshold firm with $\tau<\tau^*$, the abovementioned difference becomes

$$(1-\tau)\zeta_1(V_{cjt}(n-1)-V_{cjt}(n))+(1-\tau^*)\zeta_1(V_{cjt}(n^d+1)-V_{cjt}(n^d)) \tag{C.27}$$

which is strictly negative if $V_{cjt}(n) - V_{cjt}(n-1) = V_{cjt}(n^d+1) - V_{cjt}(n^d)$. This means the plaintiff would be making no extra return from the threshold firm that accepts, and would make an extra loss from every other firm that accepts. As a consequence, it is optimal for a plaintiff to always make settlement offers that will be rejected by every defendant – the adverse selection problem completely undermines any chance of out-of-court settlements for type 1 patent infringements.[24]

Having figured out that plaintiffs will always pick a high enough settlement amount s such that every defendant will reject, we can calculate the expected payoffs for the agents. The payoff of the plaintiff from going to court is:

$$\begin{aligned}
& \int_{\tau_1^l}^{\tau_1^h} (\tau + (1-\tau)(1-\zeta_1))(V_{cjt}(n-1) - V_{cjt}(n)) dT_1(\tau) \\
= \; & (V_{cjt}(n-1) - V_{cjt}(n)) \left(1 - \zeta_1 + \zeta_1 \int_{\tau_1^l}^{\tau_1^h} \tau dT_1(\tau)\right) \\
= \; & (V_{cjt}(n-1) - V_{cjt}(n)) \left(1 - \zeta_1 + \zeta_1 \frac{\tau_1^h + \tau_1^l}{2}\right) \qquad \text{(C.28)}
\end{aligned}$$

Given this expression, we can turn to the plaintiff's decision to hire a legal team or not. If the plaintiff does not hire a legal team, then their payoff is simply $V_{cjt}(n-1) - V_{cjt}(n)$ since they will lose their product line for certain. Therefore, they will strictly prefer to hire a legal team if and only if

$$\begin{aligned}
(V_{cjt}(n-1) - V_{cjt}(n)) \left(1 - \zeta_1 + \zeta_1 \frac{\tau_1^h + \tau_1^l}{2}\right) - \gamma Y_t \; &> \; (V_{cjt}(n-1) - V_{cjt}(n)) \\
\left(-\zeta_1 + \zeta_1 \frac{\tau_1^h + \tau_1^l}{2}\right) (V_{cjt}(n-1) - V_{cjt}(n)) \; &> \; \gamma Y_t \qquad \text{(C.29)}
\end{aligned}$$

Then the probability of hiring a legal team is given by

$$p_{1,cjt}^{LT} \equiv \mathbb{P}\left(\gamma \leq \left(-\zeta_1 + \zeta_1 \frac{\tau_1^h + \tau_1^l}{2}\right) \frac{(V_{cjt}(n-1) - V_{cjt}(n))}{Y_t}\right) \qquad \text{(C.30)}$$

[24]Note that this result owes to two facts: (1) The defendant and the plaintiff have the same technology class c in type 1 patent infringements, and (2) the firm value function is linear in n.

and the expected payoff of the plaintiff conditional on a type 1 patent infringement minus legal team cost is given as

$$
\begin{aligned}
W_{1,cjt}^{plain} \equiv\ & p_{1,cjt}^{LT}(V_{cjt}(n-1)-V_{cjt}(n))\left(1-\zeta_1+\zeta_1\frac{\tau_1^h+\tau_1^l}{2}\right)-Y_t\int_0^{\left(-\zeta_1+\zeta_1\frac{\tau_1^h+\tau_1^l}{2}\right)\frac{(V_{cjt}(n-1)-V_{cjt}(n))}{Y_t}}\gamma d\Gamma(\gamma) \\
& +(1-p_{1,cjt}^{LT})(V_{cjt}(n-1)-V_{cjt}(n)) \qquad \text{(C.31)}
\end{aligned}
$$

where the first term is the probability to hire a legal team times the expected returns to the plaintiff not including legal team costs, the second term is the expected legal team costs, and the third term is the probability not to hire a legal team times the expected returns, which is simply the value change from losing a product line for certain.

Now, let's turn to the payoff of the defendant. We know the settlement will always be sufficiently high such that every defendant rejects. Then, given τ, the defendant's payoff from going to court is:

$$
(\tau+(1-\tau)(1-\zeta_1))(V_{cjt}(n^d+1)-V_{cjt}(n^d)) \qquad \text{(C.32)}
$$

and taking expectation over τ before its realization, we have

$$
\begin{aligned}
\mathbb{E}\left[(\tau+(1-\tau)(1-\zeta_1))(V_{cjt}(n^d+1)-V_{cjt}(n^d))\right] &= (V_{cjt}(n^d+1)-V_{cjt}(n^d))\left(1-\zeta_1+\zeta_1\int_{\tau_1^l}^{\tau_1^h}\tau dT_1(\tau)\right) \\
&= (V_{cjt}(n^d+1)-V_{cjt}(n^d))\left(1-\zeta_1+\zeta_1\frac{\tau_1^h+\tau_1^l}{2}\right) \qquad \text{(C.33)}
\end{aligned}
$$

Then, given the probability of the plaintiff hiring a legal team, the expected payoff of the defendant conditional on a type 1 patent infringement is

$$
W_{1,cjt}^{def} \equiv p_{1,cjt}^{LT}(V_{cjt}(n^d+1)-V_{cjt}(n^d))\left(1-\zeta_1+\zeta_1\frac{\tau_1^h+\tau_1^l}{2}\right)+(1-p_{1,cjt}^{LT})(V_{cjt}(n^d+1)-V_{cjt}(n^d)) \qquad \text{(C.34)}
$$

where the first term is the probability to hire a legal team times the expected returns to the defendant, and the second term is the complementary probability times the value change from adding a product line for certain.

□

C.4. Characterizing the incumbent value function

Having solved the subgame perfect equilibrium of the litigation game for both types of patent infringements, we are now ready to characterize the value function of an incumbent.

The rent flow for a single product line from type 2 patent infringements by others in industry j, R_{cjt}, is given by

$$R_{cjt} = p_{cjt}^{rent} W_{2,cjt}^{plain} \tag{C.35}$$

where p_{cjt}^{rent} is the Poisson arrival rate of a type 2 patent infringement from firms in industry j, calculated in general equilibrium. This arrival rate depends on the innovation choices of all other firms with the same technology class c in industry j, as well as the share of product lines that belong to firms with technology classs c across all industries. p_{cjt}^{rent} is increasing in the prior (since more firms innovating means there are more potential infringements) as well as the probability of a type 2 infringement κ_2, and it is decreasing in the latter (since the same amount of infringements is spread over a larger mass of potentially infringed product lines).

The value difference conditional on successful innovation, but before the litigation subgame, denoted as $V_{cjt}^{+}(n) - V_{cjt}(n)$, is given by

$$\begin{aligned} V_{cjt}^{+}(n) - V_{cjt}(n) \quad = \quad & p_{cjt}^{def} \kappa_1 W_{1,cjt}^{def} + (1 - p_{cjt}^{def}) \kappa_2 W_{2,cjt}^{def} \\ & + [(p_{cjt}^{def}(1-\kappa_1) + (1 - p_{cjt}^{def})(1-\kappa_2)](V_{cjt}(n+1) - V_{cjt}(n)) \end{aligned} \tag{C.36}$$

where p_{cjt}^{def} is the probability that the firm innovates on the product line of another firm with the same technology class c in its industry, which is again determined in general equilibrium. The first term is the probability of a type 1 patent infringement times the associated defendant payoff, $W_{1,cjt}^{def}$, calculated earlier. Likewise, the second term is the probability of a type 2 patent infringement times the associated defendant payoff, $W_{2,cjt}^{def}$. The last term is the probability that no infringement happens times the value change from adding a new product line for certain.

The value difference conditional on being innovated on (i.e., value loss from creative destruction), but before the litigation subgame, denoted as $V^{-}_{cjt}(n)-V_{cjt}(n)$, is given by

$$\begin{aligned} V^{-}_{cjt}(n)-V_{cjt}(n) \quad = \quad & p^{plain}_{cjt}\kappa_1 W^{plain}_{1,cjt}+(1-p^{plain}_{cjt})\kappa_2 p^{inj}_{cjt}(V_{cjt}(n)-V_{cjt}(n)) \\ & +(1-p^{plain}_{cjt}\kappa_1-(1-p^{plain}_{cjt})\kappa_2 p^{inj}_{cjt})(V_{cjt}(n-1)-V_{cjt}(n)) \end{aligned} \quad \text{(C.37)}$$

where p^{plain}_{cjt} is the probability that the incoming innovation belongs to a firm with the same technology class c, in which case a type 1 infringement is possible with probability κ_1. The first term is this joint probability times the associated plaintiff payoff, $W^{plain}_{1,cjt}$. The second term is the probability that the incoming innovation belongs to a firm with a different technology class, in which case a type 2 infringement is possible with probability κ_2. In such an event, the innovating firm interacts with a third firm whose patent is infringed, and the probability of an injunction being granted in the litigation subgame is denoted as p^{inj}_{cjt}. In this case, the incumbent retains its product line, and therefore there is no value loss (i.e., the second term equals zero, and it is kept only for clarity). The last term is the remaining probability times the value change from losing a product line for certain.

C.5. Proof of Theorem 1

Definition 1. *A balanced growth path (BGP) equilibrium of this economy is an equilibrium in which:*

1. *The aggregate variables Y_t, C_t, A_t and the real wage rate w_t grow at the constant rate $g>0$.*

2. *The real interest rate r, the industry-specific creative destruction rates $\{d_j\}_{j=1}^{J}$, the fraction of product lines owned by technology class c firms $\{M_c\}_{c=1}^{C}$ and the probabilities $\{\{p^{rent}_{cj}, p^{LT}_{1,cj}, p^{LT}_{2,cj}, p^{def}_{cj}, p^{plain}_{cj}, p^{inj}_{cj}\}_{j=1}^{J}\}_{c=1}^{C}$ are time-invariant.*

Theorem 1. *In a BGP equilibrium, the value function of an incumbent firm with technology class c in industry j who is the leader in n product lines at time t is given by*

$$V_{cj}(n) = v_{cj} n Y_t, \quad \text{(C.38)}$$

where $v_{cj} > 0$ is an industry- and technology-class-specific time-invariant scalar given by

$$v_{cj} \quad = \quad \frac{\frac{\lambda_c}{1+\lambda_c}\omega_j + \sum_{j'=1}^{J} \hat{R}_{cj'} - \frac{(1-s_{cj})\chi_c x_{cj}^{\psi}}{1+\sigma M_c}}{\rho + \delta - x_{cj} L_{cj}^{def} + d_j L_{cj}^{plain}}. \tag{C.39}$$

In particular, x_{cj} is the time-invariant per-product-line incumbent innovation arrival rate given by

$$x_{cj} \quad = \quad \left(\frac{L_{cj}^{def} v_{cj}(1+\sigma M_c)}{(1-s_{cj})\chi_c \psi} \right)^{\frac{1}{\psi-1}}, \tag{C.40}$$

and $\hat{R}_{cj}$, L_{cj}^{def}, and L_{cj}^{plain} are time-invariant terms that summarize the implications of the litigation subgame on firm value, defined in Equations (C.42), (C.45), and (C.48), respectively. Likewise, z is the time-invariant entrant innovation arrival rate given by

$$z \quad = \quad \left(\frac{\sum_{c=1}^{C} \sum_{j=1}^{J} \eta_{cj} L_{cj}^{def} v_{cj}}{(1-s_e)\nu\psi} \right)^{\frac{1}{\psi-1}}. \tag{C.41}$$

The guess-and-verify method will be used. Suppose the value function takes the specified form. Then, we can plug it into the various terms that show up in Equation (19) and recover the equations that pin down the values of the scalars v_{cj} for all technology classes c and all industries j.

First, consider the expected rent flow from type 2 patent infringements by firms in industry j on our firm's IP, nR_{cjt}. Let $\Gamma(\gamma)$ denote the CDF of the litigation cost γ. Using

Equations (C.5) and (C.35), we get:

$$
\begin{aligned}
R_{cjt} &= p_{cj}^{rent}\left(p_{2,cj}^{LT}\frac{(1-\tau_2^l)^2\zeta_2(V_{cjt}(n^d+1)-V_{cjt}(n^d))}{4(\tau_2^h-\tau_2^l)} - Y_t\int_0^{\frac{(1-\tau_2^l)^2\zeta_2(V_{cjt}(n^d+1)-V_{cjt}(n^d))}{4(\tau_2^h-\tau_2^l)Y_t}}\gamma d\Gamma(\gamma)\right)\\
R_{cjt} &= p_{cj}^{rent}\left(p_{2,cj}^{LT}\frac{(1-\tau_2^l)^2\zeta_2(v_{cj}(n^d+1)Y_t-v_{cj}n^dY_t)}{4(\tau_2^h-\tau_2^l)} - Y_t\int_0^{\frac{(1-\tau_2^l)^2\zeta_2(v_{cj}(n^d+1)Y_t-v_{cj}n^dY_t)}{4(\tau_2^h-\tau_2^l)Y_t}}\gamma d\Gamma(\gamma)\right)\\
R_{cjt} &= p_{cj}^{rent}\left(p_{2,cj}^{LT}\frac{(1-\tau_2^l)^2\zeta_2 v_{cj}}{4(\tau_2^h-\tau_2^l)} - \int_0^{\frac{(1-\tau_2^l)^2\zeta_2 v_{cj}}{4(\tau_2^h-\tau_2^l)}}\gamma d\Gamma(\gamma)\right)Y_t\\
nR_{cjt} &= p_{cj}^{rent}\left(p_{2,cj}^{LT}\frac{(1-\tau_2^l)^2\zeta_2 v_{cj}}{4(\tau_2^h-\tau_2^l)} - \int_0^{\frac{(1-\tau_2^l)^2\zeta_2 v_{cj}}{4(\tau_2^h-\tau_2^l)}}\gamma d\Gamma(\gamma)\right)nY_t\\
nR_{cjt} &\equiv \hat{R}_{cj}nY_t \qquad \text{(C.42)}
\end{aligned}
$$

where the last line implicitly defines the normalized term $\hat{R}_{cj}$ for convenience.

Second, consider the value difference conditional on successful innovation, but before the litigation subgame, $V_{cjt}^+(n)-V_{cjt}(n)$. As gleaned from Equation (C.36), we must first obtain the expected payoffs of the defendant conditional on type 1 and type 2 patent infringements, denoted as $W_{1,cjt}^{def}$ and $W_{2,cjt}^{def}$, respectively. Plugging the guess in Equation (C.22) yields:

$$
\begin{aligned}
W_{1,cjt}^{def} &= p_{1,cj}^{LT}(V_{cjt}(n^d+1)-V_{cjt}(n^d))\left(1-\zeta_1+\zeta_1\frac{\tau_1^h+\tau_1^l}{2}\right)+(1-p_{1,cj}^{LT})(V_{cjt}(n^d+1)-V_{cjt}(n^d))\\
&= p_{1,cj}^{LT}(v_{cj}(n^d+1)Y_t-v_{cj}n^dY_t)\left(1-\zeta_1+\zeta_1\frac{\tau_1^h+\tau_1^l}{2}\right)+(1-p_{1,cj}^{LT})(v_{cj}(n^d+1)Y_t-v_{cj}n^dY_t)\\
&= \left(p_{1,cj}^{LT}\left(1-\zeta_1+\zeta_1\frac{\tau_1^h+\tau_1^l}{2}\right)+(1-p_{1,cj}^{LT})\right)v_{cj}Y_t\\
&\equiv \hat{W}_{1,cj}^{def}v_{cj}Y_t \qquad \text{(C.43)}
\end{aligned}
$$

where the last line implicitly defines the normalized term $\hat{W}_{1,cj}^{def}$ which depends on the probability $p^{LT}_{1,cj}$. Likewise, plugging the guess in Equation (C.6) yields:

$$
\begin{aligned}
W^{def}_{2,cjt} &= \Bigg((1-p^{LT}_{2,cj}) + p^{LT}_{2,cj}\Bigg[\Bigg(1-\frac{(1-\tau^l_2)\zeta_2}{2}\Bigg)\Bigg(\frac{1}{2}\frac{1-\tau^l_2}{\tau^h_2-\tau^l_2}\Bigg) + (1-\zeta_2)\Bigg(1-\frac{1}{2}\frac{1-\tau^l_2}{\tau^h_2-\tau^l_2}\Bigg) \\
&\quad + \frac{\zeta_2}{2(\tau^h_2-\tau^l_2)}\Bigg((\tau^h_2)^2 - \frac{(1+\tau^l_2)^2}{4}\Bigg)\Bigg]\Bigg)v_{cj}Y_t \\
&\equiv \hat{W}^{def}_{2,cj}v_{cj}Y_t
\end{aligned}
\tag{C.44}
$$

where the last line implicitly defines the normalized term $\hat{W}^{def}_{2,cj}$ which depends on the probability $p^{LT}_{2,cj}$. Using Equations (C.36), (C.43), and (C.44), we get:

$$
\begin{aligned}
V^+_{cjt}(n) - V_{cjt}(n) &= p^{def}_{cjt}\kappa_1 W^{def}_{1,cjt} + (1-p^{def}_{cjt})\kappa_2 W^{def}_{2,cjt} \\
&\quad + [(p^{def}_{cj}(1-\kappa_1) + (1-p^{def}_{cj})(1-\kappa_2)](V_{cjt}(n+1) - V_{cjt}(n)) \\
&= \Big(p^{def}_{cj}\kappa_1\hat{W}^{def}_{1,cj} + (1-p^{def}_{cj})\kappa_2\hat{W}^{def}_{2,cj} \\
&\quad + [(p^{def}_{cj}(1-\kappa_1) + (1-p^{def}_{cj})(1-\kappa_2)]\Big)v_{cj}Y_t \\
&\equiv L^{\text{def}}_{cj}v_{cj}Y_t
\end{aligned}
\tag{C.45}
$$

where the last line implicitly defines L^{def}_{cj}. Notice that, in the absence of any patent infringement – that is, $\kappa_1 = \kappa_2 = 0$ – we have $L^{\text{def}}_{cj} = 1$, and the whole expression simplifies to $v_{cj}Y_t$ alone. Therefore, $1 - L^{\text{def}}_{cj}$ captures the fraction of the value of a successful innovation that is lost due to the risk of infringing on other firms' IP.

Given Equation (C.45), we can calculate the optimal innovation rate $x_{cjt}(n)$ using Equation (20) as:

$$
\begin{aligned}
x_{cjt}(n) &= \left(\frac{\Big(V^+_{cjt}(n) - V_{cjt}(n)\Big)(1+\sigma M_{ct})}{(1-s_{cj})\chi_c\psi Y_t}\right)^{\frac{1}{\psi-1}} \\
x_{cjt}(n) &= \left(\frac{L^{\text{def}}_{cj}v_{cj}(1+\sigma M_c)}{(1-s_{cj})\chi_c\psi}\right)^{\frac{1}{\psi-1}} \equiv x_{cj}
\end{aligned}
\tag{C.46}
$$

Note that this optimal innovation rate is independent of both the number of product lines owned by the firm, n, and time, t. The latter owes to the fact that the term M_{ct} must be time-invariant in a BGP equilibrium since the firm distribution across industries, technology classes, and number of product lines is stationary.

Third, consider the value difference conditional on being innovated on (i.e., value loss from creative destruction), but before the litigation subgame, denoted as $V^{-}_{cjt}(n) - V_{cjt}(n)$. As gleaned from Equation (C.37), we must first obtain the expected payoff of the plaintiff conditional on type 1 patent infringement, denoted as $W^{plain}_{1,cjt}$. Plugging the guess in Equation (C.21) yields:

$$
\begin{aligned}
W^{plain}_{1,cjt} &= p^{LT}_{1,cj}(V_{cjt}(n-1)-V_{cjt}(n))\left(1-\zeta_1+\zeta_1\frac{\tau^h_1+\tau^l_1}{2}\right) - Y_t\int_0^{\left(-\zeta_1+\zeta_1\frac{\tau^h_1+\tau^l_1}{2}\right)\frac{(V_{cjt}(n-1)-V_{cjt}(n))}{Y_t}} \gamma d\Gamma(\gamma) \\
&\quad +(1-p^{LT}_{1,cj})(V_{cjt}(n-1)-V_{cjt}(n)) \\
&= p^{LT}_{1,cj}(-v_{cj}Y_t)\left(1-\zeta_1+\zeta_1\frac{\tau^h_1+\tau^l_1}{2}\right) - Y_t\int_0^{\left(-\zeta_1+\zeta_1\frac{\tau^h_1+\tau^l_1}{2}\right)(-v_{cj})} \gamma d\Gamma(\gamma) \\
&\quad +(1-p^{LT}_{1,cj})(-v_{cj}Y_t) \\
&= \left(p^{LT}_{1,cj}\left(1-\zeta_1+\zeta_1\frac{\tau^h_1+\tau^l_1}{2}\right) + \frac{1}{v_{cj}}\int_0^{\left(-\zeta_1+\zeta_1\frac{\tau^h_1+\tau^l_1}{2}\right)(-v_{cj})} \gamma d\Gamma(\gamma)\right. \\
&\quad \left. +(1-p^{LT}_{1,cj})\right)(-v_{cj}Y_t) \\
&\equiv \hat{W}^{plain}_{1,cj}(-v_{cj}Y_t) \qquad \text{(C.47)}
\end{aligned}
$$

where the last line implicitly defines the normalized term $\hat{W}^{plain}_{1,cj}$ which depends on the probability $p^{LT}_{1,cj}$ and v_{cj}. Using Equations (C.37) and (C.47), we get:

$$
\begin{aligned}
V^{-}_{cjt}(n) - V_{cjt}(n) &= p^{plain}_{cj}\kappa_1 W^{plain}_{1,cjt} + (1-p^{plain}_{cj}\kappa_1 - (1-p^{plain}_{cj})\kappa_2 p^{inj}_{cj})(V_{cjt}(n-1)-V_{cjt}(n)) \\
&= p^{plain}_{cj}\kappa_1 \hat{W}^{plain}_{1,cj}(-v_{cj}Y_t) + (1-p^{plain}_{cj}\kappa_1 - (1-p^{plain}_{cj})\kappa_2 p^{inj}_{cj})(-v_{cj}Y_t) \\
&= \left(p^{plain}_{cj}\kappa_1 \hat{W}^{plain}_{1,cj} + (1-p^{plain}_{cj}\kappa_1 - (1-p^{plain}_{cj})\kappa_2 p^{inj}_{cj})\right)(-v_{cj}Y_t) \\
&\equiv L^{\text{plain}}_{cj}(-v_{cj}Y_t) \qquad \text{(C.48)}
\end{aligned}
$$

where the last line implicitly defines L^{plain}_{cj}. Notice that, in the absence of any patent infringement – that is, $\kappa_1 = \kappa_2 = 0$ – we have $L^{\text{plain}}_{cj} = 1$, and the whole expression simplifies to $-v_{cj}Y_t$ alone. Therefore, $1 - L^{\text{plain}}_{cj}$ captures the value gain to the owner of a product line from the possibility of using a patent infringement case to fight off an entrant, and by doing so, retain the ownership of their product line.

Before we move on to the HJB equation, there are a few additional expressions that need to be computed. First, notice that the summation of the static profit flows from owned product lines is simply:

$$\sum_{m=1}^{n} \frac{\lambda_c}{1+\lambda_c} \omega_j Y_t = \frac{\lambda_c}{1+\lambda_c} \omega_j n Y_t \tag{C.49}$$

Second, the time derivative of the value function is:

$$\dot{V}_{cjt}(n) = \frac{d}{dt}(v_{cj} n Y_t) = v_{cj} n \frac{dY_t}{dt} = g v_{cj} n Y_t \tag{C.50}$$

Third, the total R&D bill is given as:

$$\sum_{m=1}^{n} \frac{(1-s_{cj})\chi_c x_{mcjt}^{\psi} Y_t}{1+\sigma M_{ct}} = \frac{(1-s_{cj})\chi_c x_{cj}^{\psi}}{1+\sigma M_c} n Y_t \tag{C.51}$$

Given all the previous derivations, we are now ready to plug in all expressions to the HJB equation given in Equation (19). This yields:

$$\begin{aligned}
r_t V_{cjt}(n) - \dot{V}_{cjt}(n) = \max_{\{x_{mcjt}\}_{m=1}^{n}} & \Bigg\{ \sum_{m=1}^{n} \frac{\lambda_c}{1+\lambda_c} \omega_j Y_t + n \sum_{j'=1}^{J} R_{cj't} \\
& - \sum_{m=1}^{n} \frac{(1-s_{cj})\chi_c x_{mcjt}^{\psi} Y_t}{1+\sigma M_{ct}} + \left(\sum_{m=1}^{n} x_{mcjt} \right) \left(V_{cjt}^{+}(n) - V_{cjt}(n) \right) \\
& + n d_{jt} \left(V_{cjt}^{-}(n) - V_{cjt}(n) \right) + \delta \left(0 - V_{cjt}(n) \right) \Bigg\} \\
(r-g) v_{cj} n Y_t \quad = \quad & \frac{\lambda_c}{1+\lambda_c} \omega_j n Y_t + \sum_{j'=1}^{J} \hat{R}_{cj'} n Y_t \\
& - \frac{(1-s_{cj})\chi_c x_{cj}^{\psi}}{1+\sigma M_c} n Y_t + x_{cj} L_{cj}^{\text{def}} v_{cj} n Y_t \\
& - d_j L_{cj}^{\text{plain}} v_{cj} n Y_t - \delta v_{cj} n Y_t
\end{aligned}$$

As can be seen, all the terms are linear in nY_t. Dividing both sides by nY_t and reorganizing, we get:

$$
\begin{aligned}
(r-g)v_{cj} &= \frac{\lambda_c}{1+\lambda_c}\omega_j + \sum_{j'=1}^{J}\hat{R}_{cj'} - \frac{(1-s_{cj})\chi_c x_{cj}^{\psi}}{1+\sigma M_c} + x_{cj}L_{cj}^{\text{def}}v_{cj} \\
&\quad -d_j L_{cj}^{\text{plain}}v_{cj} - \delta v_{cj} \\
\left(r-g+\delta - x_{cj}L_{cj}^{\text{def}} + d_j L_{cj}^{\text{plain}}\right)v_{cj} &= \frac{\lambda_c}{1+\lambda_c}\omega_j + \sum_{j'=1}^{J}\hat{R}_{cj'} - \frac{(1-s_{cj})\chi_c x_{cj}^{\psi}}{1+\sigma M_c} \\
v_{cj} &= \frac{\frac{\lambda_c}{1+\lambda_c}\omega_j + \sum_{j'=1}^{J}\hat{R}_{cj'} - \frac{(1-s_{cj})\chi_c x_{cj}^{\psi}}{1+\sigma M_c}}{\rho+\delta - x_{cj}L_{cj}^{\text{def}} + d_j L_{cj}^{\text{plain}}} \qquad \text{(C.52)}
\end{aligned}
$$

where the last line uses $r-g=\rho$ that must hold in a BGP equilibrium due to the Euler equation of the representative household. Given the probabilities $p_{cj}^{rent}, p_{1,cj}^{LT}, p_{2,cj}^{LT}, p_{cj}^{def}, p_{cj}^{plain}, p_{cj}^{inj}$, the growth rate g, the fraction of product lines owned by technology class c firms M_c, and the creative destruction rate d_j, Equation (C.52) pins down the exact values of the scalars v_{cj} for all c and j, and thus concludes the proof for the incumbents.

Given the value function of incumbents, the optimal entrant innovation arrival rate z chosen by the entrepreneurs can also be calculated in closed-form. Using Equation (22), we get

$$
\begin{aligned}
z_t &= \left(\frac{\sum_{c=1}^{C}\sum_{j=1}^{J}\eta_{cj}(V_{cjt}^{+}(0)-V_{cjt}(0))}{(1-s_e)\nu\psi Y_t}\right)^{\frac{1}{\psi-1}} \\
&= \left(\frac{\sum_{c=1}^{C}\sum_{j=1}^{J}\eta_{cj}L_{cj}^{\text{def}}v_{cj}Y_t}{(1-s_e)\nu\psi Y_t}\right)^{\frac{1}{\psi-1}} \\
&= \left(\frac{\sum_{c=1}^{C}\sum_{j=1}^{J}\eta_{cj}L_{cj}^{\text{def}}v_{cj}}{(1-s_e)\nu\psi}\right)^{\frac{1}{\psi-1}} \equiv z \qquad \text{(C.53)}
\end{aligned}
$$

which is time-invariant and the same for all entrepreneurs.

To compute the full BGP equilibrium, the values of these endogenous probabilities must also be calculated. Two of these, the litigation probabilities $p_{1,cj}^{LT}$ and $p_{2,cj}^{LT}$ can be computed without any reference to the stationary distribution of firms. Using Equation (C.20), we have:

$$
\begin{aligned}
p^{LT}_{1,cj} &= \mathbb{P}\left(\gamma \le \left(-\zeta_1 + \zeta_1 \frac{\tau^h_1 + \tau^l_1}{2}\right) \frac{(V_{cjt}(n-1) - V_{cjt}(n))}{Y_t}\right) \\
&= \mathbb{P}\left(\gamma \le \left(-\zeta_1 + \zeta_1 \frac{\tau^h_1 + \tau^l_1}{2}\right)(-v_{cj})\right) \qquad \text{(C.54)}
\end{aligned}
$$

Likewise, using Equation (C.3), we have:

$$
\begin{aligned}
p^{LT}_{2,cj} &= \mathbb{P}\left(\gamma \le \frac{(1-\tau^l_2)^2 \zeta_2 (V_{cjt}(n+1) - V_{cjt}(n))}{4(\tau^h_2 - \tau^l_2) Y_t}\right) \\
&= \mathbb{P}\left(\gamma \le \frac{(1-\tau^l_2)^2 \zeta_2 v_{cj}}{4(\tau^h_2 - \tau^l_2)}\right) \qquad \text{(C.55)}
\end{aligned}
$$

The remaining endogenous variables must be computed numerically, consistent with the stationary firm distribution in the economy.

C.6. Proof of Proposition 3

Proposition 3. *In a BGP equilibrium, the following are true:*

1. *The industry-specific creative destruction rate d_j in industry j is*

$$d_j = \sum_{c=1}^{C} (\mu_{cj} x_{cj} + \eta_{cj} z) \qquad \text{(C.56)}$$

2. *The probability for plaintiffs of type (c, j) that the incoming innovation belongs to a firm with the same technology class, p^{plain}_{cj}, is*

$$p^{plain}_{cj} = \frac{\mu_{cj} x_{cj} + \eta_{cj} z}{\sum_{c'=1}^{C} (\mu_{c'j} x_{c'j} + \eta_{c'j} z)} \qquad \text{(C.57)}$$

3. *The probability for defendants of type (c, j) to innovate on the product line of another firm with the same technology class c in its industry, p^{def}_{cj}, is*

$$p^{def}_{cj} = \mu_{cj} \qquad \text{(C.58)}$$

4. *The Poisson arrival rate of a type 2 patent infringement for plaintiffs with technology class c from firms in industry j, p_{cj}^{rent}, is*

$$p_{cj}^{rent} = \frac{(\mu_{cj}x_{cj} + \eta_{cj}z)(1-\mu_{cj})\kappa_2}{\sum_{j'=1}^{J}\mu_{cj'}} \tag{C.59}$$

5. *The probability that an injunction is granted conditional on a type 2 infringement from the perspective of the owner of the product line, p_{cj}^{inj}, is*

$$p_{cj}^{inj} = \frac{\sum_{c'\neq c}(\mu_{c'j}x_{c'j}+\eta_{c'j}z)p_{2,c'j}^{LT}}{\sum_{c'\neq c}(\mu_{c'j}x_{c'j}+\eta_{c'j}z)}\left[\left(\tau_2^h - \frac{1+\tau_2^l}{2} - \frac{(\tau_2^h)^2}{2} + \frac{(1+\tau_2^l)^2}{8}\right)\frac{1}{\tau_2^h-\tau_2^l}\right]\zeta_2 \tag{C.60}$$

6. *The time-invariant output growth rate g is given by*

$$g = \sum_{j=1}^{J}\omega_j\sum_{c=1}^{C}\mu_{cj}f_{cj} \tag{C.61}$$

where

$$\begin{aligned} f_{cj} &= (\mu_{cj}x_{cj}+\eta_{cj}z)\left[1-\kappa_1 p_{1,cj}^{LT}\left(1-\frac{\tau_1^h+\tau_1^l}{2}\right)\zeta_1\right]\ln(1+\lambda_c) \\ &+ \sum_{c'\neq c}(\mu_{c'j}x_{c'j}+\eta_{c'j}z)\left[1-\kappa_2 p_{c'j}^{inj2}\right]\ln(1+\lambda_c) \end{aligned} \tag{C.62}$$

7. *Define $P(\Theta,\Theta')$ as the transition rate from product lines of type $\Theta = (c,j)$ (origin) to $\Theta' = (c',j')$ (destination). The stationary values of μ_{cj} are pinned down by the following linear system of equations*

$$P^T\mu = \mu \tag{C.63}$$

$$\sum_{c=1}^{C}\mu_{cj} = 1, \forall j \tag{C.64}$$

which consists of $CJ+J$ equations.

Proof. To close the model, we need to derive the equations that pin down the values of endogenous variables in a BGP equilibrium, such as the growth rate g, the stationary product line distribution across industries and technology classes $\{\{\mu_{cj}\}_{c=1}^{C}\}_{j=1}^{J}$, and the associated probabilities of various events discussed earlier.

Recall that $\mu_{cjt} \in [0,1]$ denotes the measure of all product lines in industry j for which the leader has technology class c at time t, with $\sum_{c=1}^{C} \mu_{cjt} = 1$. In a stationary equilibrium, μ_{cjt} are time-invariant, so time subscripts will be suppressed from here on. Under this definition, total incumbent innovation by firms of technology class c in industry j is $\mu_{cj}x_{cj}$, and the total entrant innovation for the same is $\eta_{cj}z$.

The industry-specific creative destruction rate d_j in industry j depends on total innovation in that industry by both incumbents and entrants with any technology class. This is given by

$$d_j = \sum_{c=1}^{C} (\mu_{cj}x_{cj} + \eta_{cj}z) \tag{C.65}$$

The probability for plaintiffs of type (c, j) that the incoming innovation belongs to a firm with the same technology class, denoted p_{cj}^{plain}, can be calculated as

$$p_{cj}^{plain} = \frac{\mu_{cj}x_{cj} + \eta_{cj}z}{\sum_{c'=1}^{C}(\mu_{c'j}x_{c'j} + \eta_{c'j}z)} \tag{C.66}$$

which is the fraction of total innovation in industry j originating from firms of type (c, j) to that of total innovation in industry j irrespective of technology class.

The probability for defendants of type (c, j) to innovate on the product line of another firm with the same technology class c in its industry, p_{cj}^{def} is simply

$$p_{cj}^{def} = \mu_{cj} \tag{C.67}$$

since $\sum_{c=1}^{C} \mu_{cj} = 1$.

To calculate the the Poisson arrival rate of a type 2 patent infringement for plaintiffs with technology class c from firms in industry j, denoted p_{cj}^{rent}, we need to do an accounting of the measure of type 2 patent infringements that happen in technology class c in industry j, and the measure of eligible plaintiffs across all industries. The prior is calculated as

$$(\mu_{cj}x_{cj} + \eta_{cj}z)(1 - \mu_{cj})\kappa_2 \tag{C.68}$$

where the first factor is the total innovation in industry j originating from firms of type (c, j), the second factor is the probability that such innovation lands on a product line with technology class $c' \neq c$, and the third factor is the probability of a type 2 patent infringement occuring under this scenario. The latter is simply the sum of all product lines belonging to firms with technology class c across all industries, i.e., $\sum_{j=1}^{J} \mu_{cj}$. Then we can calculate p_{cj}^{rent} as

$$p_{cj}^{rent} = \frac{(\mu_{cj} x_{cj} + \eta_{cj} z)(1 - \mu_{cj})\kappa_2}{\sum_{j'=1}^{J} \mu_{cj'}} \tag{C.69}$$

Recall that the probability that an injunction is granted conditional on a type 2 infringement from the perspective of the owner of the product line was denoted p_{cj}^{inj}. In type 2 infringements, the technology class c' of the innovating firm matters for the injunction probability, since it also influences the rents the third-party plaintiff can extract. Define $p_{c'j}^{inj2}$ as the probability of an injunction conditional on the innovating firm having technology class $c' \neq c$. Then, this probability is calculated as

$$\begin{aligned}
p_{c'j}^{inj2} &= p_{2,c'j}^{LT} \left(\int_{\tau_2^l}^{\tau^*} 0 dT_2(\tau) + \int_{\tau^*}^{\tau_2^h} (1-\tau) dT_2(\tau) \right) \zeta_2 \\
&= p_{2,c'j}^{LT} \left[\left(\tau_2^h - \tau^* - \frac{(\tau_2^h)^2}{2} + \frac{(\tau^*)^2}{2} \right) \frac{1}{\tau_2^h - \tau_2^l} \right] \zeta_2 \\
&= p_{2,c'j}^{LT} \left[\left(\tau_2^h - \frac{1+\tau_2^l}{2} - \frac{(\tau_2^h)^2}{2} + \frac{(1+\tau_2^l)^2}{8} \right) \frac{1}{\tau_2^h - \tau_2^l} \right] \zeta_2
\end{aligned} \tag{C.70}$$

where the first factor is the probability that the plaintiff hires a legal team, the second factor is the probability that the defendant rejects the settlement offer and loses at court, and the third factor is the probability that an injunction is granted. Given this, we can calculate p_{cj}^{inj} as

$$\begin{aligned}
p_{cj}^{inj} &= \frac{\sum_{c' \neq c} (\mu_{c'j} x_{c'j} + \eta_{c'j} z) p_{c'j}^{inj2}}{\sum_{c' \neq c} (\mu_{c'j} x_{c'j} + \eta_{c'j} z)} \\
&= \frac{\sum_{c' \neq c} (\mu_{c'j} x_{c'j} + \eta_{c'j} z) p_{2,c'j}^{LT}}{\sum_{c' \neq c} (\mu_{c'j} x_{c'j} + \eta_{c'j} z)} \left[\left(\tau_2^h - \frac{1+\tau_2^l}{2} - \frac{(\tau_2^h)^2}{2} + \frac{(1+\tau_2^l)^2}{8} \right) \frac{1}{\tau_2^h - \tau_2^l} \right] \zeta_2
\end{aligned} \tag{C.71}$$

To calculate the growth rate of the economy, we must tally not only successful innovations, but also the rate at which successful innovations convert to product line takeovers (i.e., the fraction of successful innovations that are not blocked by an injunction), and the technology classes of both the incumbent and the innovator, since the productivity gains λ_c are heterogeneous, and so are the markups charged over marginal cost.

From the definition of the production technology, we have:

$$\begin{aligned}
\ln Y_t &= \sum_{j=1}^{J} \omega_j \left(\int_0^1 \ln y_{ijt} di \right) \\
\frac{\ln Y_{t+\Delta t} - \ln Y_t}{\Delta t} &= \sum_{j=1}^{J} \omega_j \left(\int_0^1 \frac{\ln y_{ijt+\Delta t} - \ln y_{ijt}}{\Delta t} di \right) \\
g_t &= \lim_{\Delta t \to 0} \sum_{j=1}^{J} \omega_j \left(\int_0^1 \frac{\ln y_{ijt+\Delta t} - \ln y_{ijt}}{\Delta t} di \right)
\end{aligned} \tag{C.72}$$

Hence, to figure out the output growth rate g_t, we must focus on how log output in each product line $\ln y_{ijt}$ changes over time. From the incumbent firm's static problem, we know

$$\begin{aligned}
\ln y_{ijt} &= \ln \left(\frac{\omega_j Y_t q_{ijt}}{w_t(1+\lambda_c)} \right) \\
&= \ln \omega_j + \ln \left(\frac{Y_t}{w_t} \right) + \ln q_{ijt} - \ln(1+\lambda_c)
\end{aligned} \tag{C.73}$$

The first term is the function of a parameter, and thus constant. The second term is a function of the relative wage w_t/Y_t, which is time-invariant in a BGP equilibrium. The third term is log productivity, which increases upon successful innovation that is not blocked. The fourth term is the markup distortion, which can change upon successful innovation that is not blocked if the innovator has a different technology class $c' \neq c$.

Now, consider the case of some product line i in industry j owned by a firm with technology class c. The probability that the product line is lost to a firm with the same technology class c over a small time interval Δt is

$$(\mu_{cj} x_{cj} + \eta_{cj} z) \Delta t \left[1 - \kappa_1 p_{1,cj}^{LT} \left(1 - \frac{\tau_1^h + \tau_1^l}{2} \right) \zeta_1 \right] \tag{C.74}$$

where the term outside the brackets is the probability of a successful innovation, whereas the term inside the brackets is the probability that an injunction is not granted. An injunction is only granted if there is an infringement (prob. κ_1), the plaintiff pays the legal team cost (prob. $p^{LT}_{1,cj}$), the defendant loses (prob. $1-(\tau^h_1+\tau^l_1)/2$), and the court grants an injunction (prob. ζ_1). In this scenario, since both firms have the same technology class, the markup distortion is unchanged. However, log productivity increases by $\ln(1+\lambda_c)$.

For any technology class $c' \neq c$, the probability that the product line is lost to a firm with the technology class c' over a small time interval Δt is

$$(\mu_{c'j}x_{c'j}+\eta_{c'j}z)\Delta t\left[1-\kappa_2 p^{inj2}_{c'j}\right] \tag{C.75}$$

where the term outside the brackets is the probability of a successful innovation, whereas the term inside the brackets is the probability that an injunction is not granted, which uses the $p^{inj2}_{c'j}$ defined in Equation (C.70). In this scenario, the markup distortion changes from $\ln(1+\lambda_c)$ to $\ln(1+\lambda_{c'})$. Log productivity also increases by $\ln(1+\lambda_{c'})$. The net effect on log output for the product line is therefore $\ln(1+\lambda_{c'})+\ln(1+\lambda_c)-\ln(1+\lambda_{c'})=\ln(1+\lambda_c)$, same as the previous scenario.

Given these observations, for some product line i in industry j owned by a firm with technology class c, we can write:

$$\begin{aligned}\frac{\ln y_{ijt+\Delta t}-\ln y_{ijt}}{\Delta t} &= (\mu_{cj}x_{cj}+\eta_{cj}z)\left[1-\kappa_1 p^{LT}_{1,cj}\left(1-\frac{\tau^h_1+\tau^l_1}{2}\right)\zeta_1\right]\ln(1+\lambda_c) \\ &\quad+\sum_{c'\neq c}(\mu_{c'j}x_{c'j}+\eta_{c'j}z)\left[1-\kappa_2 p^{inj2}_{c'j}\right]\ln(1+\lambda_c)\equiv f_{cj}\end{aligned} \tag{C.76}$$

which is called f_{cj} for convenience. Then, we can plug in these expressions in Equation (C.72) to obtain

$$g = \sum_{j=1}^{J}\omega_j\sum_{c=1}^{C}\mu_{cj}f_{cj} \tag{C.77}$$

which pins down the output growth rate in a BGP equilibrium.

Finally, we need to pin down the equations that determine $\mu_{cj}, \forall c,j$. To this purpose, define a joint product line type as $\Theta=(c,j)$, and define $P(\Theta,\Theta')$ as the transition rate from product lines of type $\Theta=(c,j)$ (origin) to $\Theta'=(c',j')$ (destination). First, note that no event can change the industry of a product line. Therefore, we have

$$P((c,j),(c',j')) = 0, \forall c, \forall j, \forall c', \forall j' \neq j \tag{C.78}$$

Second, if the innovating firm has the same technology class as the incumbent, the type of the product line does not change even if ownership does, so it requires no explicit accounting. So that leaves the third case to consider, with $j = j'$ and $c' \neq c$. In this case, we have:

$$P((c,j),(c',j)) = (\mu_{c'j} x_{c'j} + \eta_{c'j} z)\left[1 - \kappa_2 p_{c'j}^{inj2}\right], \forall c, \forall j, \forall c' \neq c \tag{C.79}$$

in agreement with Equation (C.75). Finally, we have the case $j = j'$ and $c = c'$ which is implicitly defined as

$$P((c,j),(c,j)) = 1 - \sum_{c' \neq c} P((c,j),(c',j)) \tag{C.80}$$

Using the transition matrix defined by $P(\Theta, \Theta')$, we can pin down the stationary values of μ_{cj} by solving the linear system of equations

$$P^T \mu = \mu \tag{C.81}$$

$$\sum_{c=1}^{C} \mu_{cj} = 1, \forall j \tag{C.82}$$

which consists of $CJ + J$ equations. □

C.7. Firm size distributions

To compute the firm size distributions, we need to calculate the product line takeover probabilities conditional on successful innovation for every firm type. Define this takeover probability for a firm with technology class c in industry j at time t as $p_{cjt}^{take} \in [0,1]$. This probability is calculated as:

$$\begin{aligned}
p_{cjt}^{take} &= p_{cjt}^{def}\kappa_1\left[p_{1,cjt}^{LT}\left(1-\zeta_1+\zeta_1\frac{\tau_1^h+\tau_1^l}{2}\right)+\left(1-p_{1,cjt}^{LT}\right)\right] \\
&+(1-p_{cjt}^{def})\kappa_2\left\{p_{2,cjt}^{LT}\left[\left(\frac{1}{2}\frac{1-\tau_2^l}{\tau_2^h-\tau_2^l}\right)+(1-\zeta_2)\left(1-\frac{1}{2}\frac{1-\tau_2^l}{\tau_2^h-\tau_2^l}\right)\right.\right. \\
&\left.\left.+\frac{\zeta_2}{2(\tau_2^h-\tau_2^l)}\left((\tau_2^h)^2-\frac{(1+\tau_2^l)^2}{4}\right)\right]+\left(1-p_{2,cjt}^{LT}\right)\right\} \\
&+\left(p_{cjt}^{def}(1-\kappa_1)+(1-p_{cjt}^{def})(1-\kappa_2)\right) \qquad (C.83)
\end{aligned}$$

where the first term is the probability of a type 1 infringement times the conditional takeover probability, the second term is the same for type 2 infringements, and the third term is the complementary event that no infringement occurs, in which case the takeover is assured.

We also need to calculate the flow rate of losing a product line for incumbent firms. Define the per product line product line loss flow rate for a firm with technology class c in industry j at time t as $p_{cjt}^{loss} > 0$. This flow rate is calculated as:

$$\begin{aligned}
p_{cjt}^{loss} &= (\mu_{cjt}x_{cjt}+\eta_{cj}z_t)\left[1-\kappa_1 p_{1,cjt}^{LT}\left(1-\frac{\tau_1^h+\tau_1^l}{2}\right)\zeta_1\right] \\
&+\sum_{c'\neq c}(\mu_{c'jt}x_{c'jt}+\eta_{c'j}z_t)\left[1-\kappa_2 p_{c'jt}^{inj2}\right] \qquad (C.84)
\end{aligned}$$

Define the mass of firms with technology class c in industry j at time t that own n product lines as $\varphi_{cjt}(n) \geq 0$. Using previously-calculated expressions, we can write the ordinary differential equations that govern the evolution of these expressions. Due to new firm entry and endogenous firm exit, $n = 1$ is a special case, which is given by:

$$\dot{\varphi}_{cjt}(1) = z_t\eta_{cj}+2p_{cjt}^{loss}\varphi_{cjt}(2)-(x_{cjt}p_{cjt}^{take}+p_{cjt}^{loss})\varphi_{cjt}(1) \qquad (C.85)$$

where the first term corresponds to new entrants with a single product line, the second term corresponds to firms with two product lines losing one of them, and the third term corresponds to outflows of firms with a single product line due to both successful takeovers, as well as losses.

For all the other cases with $n \geq 2$, we have the general expression:

$$\begin{aligned} \dot{\varphi}_{cjt}(n) &= (n-1)x_{cjt}p_{cjt}^{take}\varphi_{cjt}(n-1) + (n+1)p_{cjt}^{loss}\varphi_{cjt}(n+1) \\ &\quad -n(x_{cjt}p_{cjt}^{take} + p_{cjt}^{loss})\varphi_{cjt}(n) \end{aligned} \tag{C.86}$$

where the first term corresponds to firms with $n-1$ product lines succeeding in taking over a new product line, the second term corresponds to firms with $n+1$ product lines losing one of them, and the third term corresponds to outflows of firms with n product lines due to both successful takeovers, as well as losses.

In a stationary equilibrium, we have $\dot{\varphi}_{cjt}(n) = 0, \forall c, j, t, n$. Therefore, the firm size distributions are time-invariant; that is, $\varphi_{cjt}(n) \equiv \varphi_{cj}(n), \forall c, j, t, n$. Using the previous equations, we can pin down these time-invariant firm size distributions. For any technology class c and industry j, we have the following equations:

$$0 = z\eta_{cj} + 2p_{cj}^{loss}\varphi_{cj}(2) - (x_{cj}p_{cj}^{take} + p_{cj}^{loss})\varphi_{cj}(1) \tag{C.87}$$

$$\begin{aligned} 0 &= (n-1)x_{cj}p_{cj}^{take}\varphi_{cj}(n-1) + (n+1)p_{cj}^{loss}\varphi_{cj}(n+1) \\ &\quad -n(x_{cj}p_{cj}^{take} + p_{cj}^{loss})\varphi_{cj}(n), \forall n \geq 2 \end{aligned} \tag{C.88}$$

In addition, we also know

$$z\eta_{cj} = p_{cj}^{loss}\varphi_{cj}(1) \tag{C.89}$$

$$\sum_{n=1}^{\infty} n\varphi_{cj}(n) = \mu_{cj} \tag{C.90}$$

where the first equation is due to firm entry being equal to firm exit in a stationary equilibrium, and the second equation is an accounting identity that ensures that the total number of product lines owned by firms with technology class c in industry j equals μ_{cj}. Together, equations (C.87), (C.88), and (C.89) pin down $\varphi_{cj}(n), \forall n \geq 1$.

C.8. Computing output and welfare

We would like to compute social welfare in counterfactual economies and compare them against the estimated equilibrium. To calculate welfare, we need to compute the consumption stream of the representative household. In a BGP equilibrium, two components must be known: the growth rate of consumption g, and the initial consumption level C_0. This requires us to compute initial output Y_0 and aggregate spending on R&D. In turn, computing initial output requires computing the (time-invariant) relative wage rate w_t/Y_t. We will compute these in reverse order.

To calculate the relative wage rate, we will use the labor market clearing condition. First, recall that the output y_{ijt} of firm i in industry j at time t is given by:

$$y_{ijt} = \frac{\omega_j Y_t}{p_{ijt}} = \frac{\omega_j Y_t q_{ijt}}{w_t(1+\lambda_c)} \tag{C.91}$$

Then, the labor demand of this firm becomes

$$l_{ijt} = \frac{y_{ijt}}{q_{ijt}} = \frac{\omega_j Y_t}{w_t(1+\lambda_c)} \tag{C.92}$$

which is independent of the firm's productivity q_{ijt}. Since the representative household supplies labor $L = 1$ inelastically, labor market clearing requires:

$$\begin{aligned}
1 &= \sum_{j=1}^{J} \int_0^1 l_{ijt} di \\
1 &= \sum_{j=1}^{J} \int_0^1 \frac{\omega_j Y_t}{w_t(1+\lambda_c)} di \\
\frac{w_t}{Y_t} &= \sum_{j=1}^{J} \omega_j \int_0^1 \frac{1}{(1+\lambda_c)} di \\
\frac{w_t}{Y_t} &= \sum_{j=1}^{J} \omega_j \sum_{c=1}^{C} \frac{\mu_{cj}}{(1+\lambda_c)} \qquad \text{(C.93)}
\end{aligned}$$

which delivers the time-invariant relative wage rate.

The level of output Y_t at time t is given by:

$$\begin{aligned}
\ln Y_t &= \sum_{j=1}^{J} \omega_j \left(\int_0^1 \ln y_{ijt} di \right) \\
\ln Y_t &= \sum_{j=1}^{J} \omega_j \left(\int_0^1 \ln \left(\frac{\omega_j Y_t q_{ijt}}{w_t (1+\lambda_c)} \right) di \right) \\
\ln Y_t &= -\ln \frac{w_t}{Y_t} + \sum_{j=1}^{J} \omega_j \ln(\omega_j) + \sum_{j=1}^{J} \omega_j \left(\int_0^1 \ln \left(\frac{q_{ijt}}{(1+\lambda_c)} \right) di \right) \\
\ln Y_t &= -\ln \frac{w_t}{Y_t} + \sum_{j=1}^{J} \omega_j \ln(\omega_j) - \sum_{j=1}^{J} \omega_j \sum_{c=1}^{C} \mu_{cj} \ln(1+\lambda_c) + \sum_{j=1}^{J} \omega_j \left(\int_0^1 \ln q_{ijt} di \right) \qquad \text{(C.94)}
\end{aligned}$$

where the last term is the log productivity level of the economy at time t, i.e., the weighted sum of the log productivity level in each industry j, where the weights are the Cobb-Douglas shares ω_j. In our counterfactual experiments, we shall hold the initial log productivity level at time $t = 0$ fixed across economies. Without loss of generality, it is normalized to zero.[25]

Let L_{cj} denote the normalized per product line expected litigation cost for product lines owned by firms in industry j with technology class c:

$$L_{cj} = d_j p_{cj}^{plain} \kappa_1 \left(\int_0^{\left(-\zeta_1 + \zeta_1 \frac{\tau_1^h + \tau_1^l}{2}\right)(-v_{cj})} \gamma d\Gamma(\gamma) \right) + \sum_{j'=1}^{J} \left(p_{cj'}^{rent} \int_0^{\frac{(1-\tau_2^l)^2 \zeta_2 v_{cj'}}{4(\tau_2^h - \tau_2^l)}} \gamma d\Gamma(\gamma) \right) \qquad \text{(C.95)}$$

Then the aggregate litigation spending in the whole economy is calculated as

$$\sum_{j=1}^{J} \sum_{c=1}^{C} \mu_{cj} L_{cj} Y_t \qquad \text{(C.96)}$$

From the goods market clearing, we can compute the time-invariant consumption to

[25] This is equivalent to setting all $q_{ij0} = 1$.

output ratio C_t/Y_t as follows:

$$\begin{aligned} Y_t &= C_t + \sum_{j=1}^{J}\sum_{c=1}^{C}\mu_{cj}\frac{\chi_c x_{cj}^{\psi} Y_t}{1+\sigma M_c} + \nu z^{\psi} Y_t + \sum_{j=1}^{J}\sum_{c=1}^{C}\mu_{cj}L_{cj}Y_t \\ \frac{C_t}{Y_t} &= 1 - \sum_{j=1}^{J}\sum_{c=1}^{C}\mu_{cj}\frac{\chi_c x_{cj}^{\psi}}{1+\sigma M_c} - \nu z^{\psi} - \sum_{j=1}^{J}\sum_{c=1}^{C}\mu_{cj}L_{cj} \end{aligned} \tag{C.97}$$

where the second and third terms are the total incumbent and entrant R&D spending to output ratios, respectively, and the last term is the aggregate litigation spending to output ratio. Then, the initial output level is simply $C_0 = Y_0(C_0/Y_0)$.

We are now ready to compute social welfare in a BGP equilibrium. From the utility function of the representative household in equation (7), we have:

$$W = \int_0^{\infty} e^{-\rho t}\ln C_t dt = \int_0^{\infty} e^{-\rho t}\ln(e^{gt}C_0)dt = \frac{\ln C_0}{\rho} + \frac{g}{\rho^2} \tag{C.98}$$

which shows how the welfare depends on the initial level of consumption C_0 and the growth rate of the economy g.

For two economies A and B, we can define a consumption equivalent welfare change measure (ϖ) which corresponds to the percentage increase in lifetime consumption that an agent in economy A would need to be indifferent between being in economy A or B:

$$W_B = \frac{\ln(C_0^A(1+\varpi))}{\rho} + \frac{g^A}{\rho^2} \tag{C.99}$$

Solving for ϖ, we get:

$$\varpi = \exp\left(\left(W_B - \frac{g^A}{\rho^2}\right)\rho - \ln(C_0^A)\right) - 1 \tag{C.100}$$