Are Markups Too High?
Competition, Strategic Innovation, and Industry Dynamics*

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Abstract

To study competition, innovation, and industry dynamics that arise as a result of their interaction, we develop a new oligopolistic general-equilibrium Schumpeterian growth model. This model ties together the endogenous growth, oligopolistic competition, and dynamic industrial organization literatures in a single unified framework, which is used to assess the growth and welfare implications of counterfactuals. Within each industry, there are an endogenously determined number of large firms ("superstars") that compete *à la* Cournot and a continuum of small firms which collectively constitute a competitive fringe. Firms dynamically choose their innovation strategies, cognizant of other firms' choices, and their entry and exit are endogenous. The model is consistent with the macroeconomic trends observed in the United States since the 1970s, such as the domination of industries by a small number of superstar firms, the rise of markups, market concentration, profits, and R&D spending, and the decline in business dynamism, productivity growth, and the labor share. It replicates the empirical relationship between innovation and competition within and across industries. As an application, we estimate the model to disentangle the effects of separate mechanisms on the structural transition observed in the United States, which yields striking results: (1) While the increase in the average markup causes a significant static welfare loss, this loss is overshadowed by the dynamic welfare gains from increased innovation in response to higher profit opportunities. (2) The increasing costs of innovation are found to be the primary determinant of lackluster productivity growth, i.e., ideas are getting harder to find.

Keywords: innovation, markups, growth, strategic investment, industry dynamics, business dynamism.
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1 Introduction

Consumer welfare depends crucially on both competition and long-run productivity growth. Firms' forward-looking entry and innovation decisions determine the market structure in industries in terms of the number of competing firms and their productivities, and consequently, product market competition. In turn, product market competition determines which firms can reap the highest profits as the reward of their innovation efforts. Therefore, competition and growth are inseparably interlinked. Their joint study calls for a theoretical framework that can simultaneously capture the general-equilibrium effects of firms' strategic innovation decisions on economic growth and industrial structure, as well as the rich within-industry dynamics featuring heterogeneity in market shares, markups, and profitability, which in turn influence the (mis)allocation of resources.

Thus motivated, we develop a new unified framework that ties together the endogenous growth, oligopolistic competition, and dynamic industrial organization literatures, combining their most important features without giving up on tractability. It serves as a laboratory to answer a wide range of questions related to competition and growth, which is amenable to extensions in several directions. This framework is particularly well-suited for studying the underlying economic mechanisms behind the recent macroeconomic trends observed in the United States since the 1970s, such as the increase in market concentration and markups that were accompanied by lackluster productivity growth, and its application delivers some surprising new insights.

Our new framework possesses several desirable properties when it comes to questions related to market structure, competition, innovation, and growth, especially in light of the recent literature on rising concentration and markups. At the industry level, it is able to generate rich heterogeneity in market shares, consistent with the empirically-observed industry-level market share distributions. It offers realistic industry dynamics with firm heterogeneity within and across industries, and endogenous firm entry and exit that generate a distribution of industries populated by different numbers of large firms with different relative productivities. The model is also able to generate realistic firm life cycles with gradual changes in productivity and market shares.

At the firm level, the model captures static and dynamic strategic interactions between firms: (1) product market competition which determines relative prices, market shares, markups, profits, labor demand, and labor shares endogenously; (2) dynamic innovation decisions to improve firm productivity over time, optimally chosen in response to the innovation policies of competing firms. Finally, the framework is able to replicate the observed non-linear relationship between innovation and competition:[1] (1) a hump-shaped relationship between industry innovation and market concentration across industries,[2] and (2) a hump-shaped relationship between firm innovation and relative sales within industries.[3] Consistency in these last two dimensions is important, since welfare implications of a change in market concentration do not only depend on static inefficiencies caused by markups, but also on the dynamic inefficiencies in innovation and aggregate productivity growth.

This rich framework allows us to analyze recent trends in the US economy. Since the 1970s, there have been significant changes in firm dynamics within and across industries in the US, with quantitatively

[1] The model can also generate other types of relationship between innovation and competition, depending on parameter values.

[2] See Aghion, Bloom, Blundell, Griffith, and Howitt (2005), and the stylized facts in Section 3.1.

[3] See Section 3.1.

important implications for the macroeconomy at large. Industries are increasingly dominated by a small number of large firms ("superstars", "megafirms") which compete against each other in the product market, as well as through dynamic strategic innovation decisions. At the aggregate level, average markup, market concentration, profits, and R&D spending are increasing, whereas business dynamism and the labor share are in decline.[4] Despite a rise in the late 90s and early 2000s, productivity growth has been slowing down. These sweeping changes have drawn considerable attention from academics, policymakers, and the public alike. Discovering and understanding the economic mechanisms underlying this transformation are key to assess the implications for efficiency, economic growth, and social welfare, which can then be used to formulate optimal policy responses. In this paper, we offer a unified framework to study these trends and uncover the underlying reasons.

Our new quantitative model with all its desirable properties remains highly tractable despite its rich dynamics and heterogeneity, and its quantification yields some striking insights. It can be described as a combination of a detailed oligopolistic competition model with endogenous entry and exit, and a new Schumpeterian growth model that features step-by-step innovation. Within each industry and at any given time, there is an endogenously determined number of large firms – the superstars – and a continuum of small firms which collectively constitute a competitive fringe. Relative to small firms, superstar firms in the model are characterized by higher productivity and larger market shares. Entrepreneurs choose the entry rate of new small businesses. Industry output is a CES aggregate of the production of superstar firms and the competitive fringe, and the superstars compete *à la* Cournot. This specification yields non-degenerate distributions of sales, employment, and markups in each industry, a contribution upon Schumpeterian growth models with Bertrand competition and homogeneous goods. The distributions depend on the number of superstar firms and the distribution of their productivities, as well as the relative productivity of the competitive fringe. These industry characteristics endogenously change over time according to the strategic innovation decisions of the superstars and the small firms. Motivated by higher profits, the superstars undertake costly R&D in order to improve their productivity relative to their competitors. Small firms also spend resources on R&D, and conditional on success, they join the ranks of the superstars. At the same time, a large firm that lags behind can lose its status as a superstar if its productivity relative to the leader falls to a sufficiently low level. Combined, these dynamics generate transitions between industry states, resulting in a stationary distribution along a balanced growth path equilibrium. Aggregate productivity growth in the economy is determined by the innovation decisions in each industry, weighted by this invariant distribution across industry states in a stationary economy. The model can generate the described hump-shaped relationship between innovation and competition within and across industries.

Our model provides a unified framework that simultaneously combines and extends three strands of literature. First, we generalize the Schumpeterian step-by-step innovation models in the endogenous growth literature to have an arbitrary number of large firms instead of a duopoly, and a competitive fringe of small firms to capture the existence of thousands of small firms observed in the real world, which delivers both realistic firm dynamics and market share distributions. Second, we augment static oligopolistic competition models such as Atkeson and Burstein (2008) and Autor, Dorn, Katz, Patterson, and Van Reenen (2020) with a completely endogenous industry structure featuring endogenous productivity growth, entry, and exit, while accounting for dynamic strategic interactions in a Markov Perfect equilibrium.

[4] See Akcigit and Ates (2021) for an extensive summary and discussion of these trends.

Finally, our model constitutes a general equilibrium version of dynamic industrial organization models such as Ericson and Pakes (1995) with endogenous growth, where the ergodic distribution of the industry-states for a single industry simultaneously becomes the cross-sectional stationary distribution across a continuum of industries in the aggregate. This unified framework provides a laboratory for answering questions at the intersection of these three literatures.

We estimate our model via indirect inference using macro and micro level moments related to growth, R&D investment, markup distribution, profitability, productivity, and the relationship between market shares and innovation from three different samples. First, we estimate the model using all available data between 1976 and 2004, and use the estimated equilibrium to clarify the workings of the mechanisms in the model, with a special emphasis on the relationship between competition and innovation, and how it differs from other models in the literature that study similar questions. We also split our data into two sub-samples – an early sub-sample from 1976 to 1994, and a late sub-sample after 1994 – and re-estimate.[5] We use these two estimates to disentangle the mechanisms underlying the structural transition in the US over these three decades. We achieve this by starting from the late period economy, and considering counterfactual economies where parameters governing individual mechanisms are reset back to their early period values. This helps us quantify what channels contribute the most to – or work against – the observed changes in the macroeconomic aggregates.

Perhaps one of the most important questions to ask is whether the observed increase in markups is detrimental to welfare and economic growth. While the static losses from increased markups are well-known and unambiguous, the dynamic effects can go in either direction, as evidenced by the hump-shaped relationship between innovation and competition. Increased competition can boost innovation as firms try to improve their relative productivity ("escape competition"), or lower it if they get discouraged by lower expected profits. Which channel dominates is a quantitative question. First, we show that most of the rise in markups is driven by a decrease in competition from small firms instead of a decrease in competition among large firms. Next, we conduct a counterfactual exercise in which we reset the competition from small firms to its early period level, and compute the consumption-equivalent welfare change. The results are striking: the growth rate goes down by 21.0% of its value, and instead of a net gain, social welfare is reduced by 7.60%. Decomposing the change in welfare into its individual components reveals that although the static efficiency gains would improve welfare by 4.13%, the fall in profitability discourages innovation, and the dynamic losses from the decline in endogenous productivity growth more than offset the static gain. In other words, if markups had stayed the same across the period, economic growth would have been slower. Our results suggest that the dynamic effects of increasing market concentration on innovation and productivity growth should not be ignored when trying to understand the transformation in the US in the last four decades, and the significant increase in markups is not necessarily detrimental to welfare.

Our model also has interesting predictions regarding the cost of innovation over time. While productivity growth increased between the early and late periods, our model suggests that the cost of innovation went up over the same period, both for large and small firms. These results point towards "the ideas are getting harder to find" hypothesis studied in Gordon (2012) and Bloom, Jones, Van Reenen, and Webb (2020).

[5] We restrict our baseline analysis to 1976-2004 due to data limitations and the assumptions that need to be made on how to deal with the Great Recession period. However, our results covering the later period until 2016, presented in Section 7.1, reveal the same qualitative changes.

In other words, the rising cost of innovation has partially offset the positive effect of rising markups on economic growth and welfare. If R&D efficiency had not decreased, productivity growth would have accelerated even faster between the early and late periods.[6]

Our estimated model also delivers several predictions that can be tested, especially related to trends in productivity, market concentration, and the labor share. Although they are not targeted, we show that our estimation correctly predicts the increase in productivity dispersion documented by Barth, Bryson, Davis, and Freeman (2016) and the negative correlation between productivity dispersion and the labor share across industries highlighted in Gouin-Bonenfant (2022). In addition, our model is in line with several facts related to changes in the labor share documented in Autor, Dorn, Katz, Patterson, and Van Reenen (2020). These tests further confirm the external validity of the model.

Our quantitative results are found to be robust to introducing endogenous capital accumulation, as well as using different values for the elasticity of intertemporal substitution. Neither are they sensitive to using different targets to discipline markups. Our baseline estimation targets the sales-weighted markup moments from De Loecker, Eeckhout, and Unger (2020). Targeting cost-weighted markups from Edmond, Midrigan, and Xu (2023) does not lead to sizable differences. Motivated by the recent criticism in Bond, Hashemi, Kaplan, and Zoch (2021) regarding the consistency of the markup estimation methodology developed in De Loecker and Warzynski (2012), we also conduct an estimation which does not rely on any markup targets. Our results are likewise found to be robust to modeling competition à la Bertrand and to assuming decreasing returns to scale in production.

Solving for the non-stationary equilibria of our model is significantly more challenging than in standard macroeconomic models, given that it requires calculating the complete time paths of 86 continuous state variables given our choices.[7] Despite the complexity, the tractability of our framework and our choice to use a continuous-time setting render the computation of non-stationary equilibria feasible. We find that our welfare results remain robust to taking the transitional dynamics into account.

For further elucidation, we also solve the (unconstrained as well as constrained) social planner's problem dynamically, which is once again complex, yet feasible.[8] Consistent with our results regarding the welfare effect of increasing markups, the solution to the planner's problem reveals that the dynamic inefficiencies due to under-investment in innovation are much more severe than the static inefficiencies due to market power, even though the latter are quite significant on their own. This result suggests that the optimal design of corporate taxation and R&D subsidies, studied in Akcigit, Hanley, and Stantcheva (2022), is first-order for social welfare and economic growth.

This paper is related to the literature on the welfare cost of markups and a recent body of literature that investigates the increase in market concentration and the associated increase in markups and profit shares over the last few decades in the US as highlighted in Barkai (2020), Gutiérrez and Philippon (2017a), Eggertsson, Robbins, and Wold (2021), Hall (2018), De Loecker, Eeckhout, and Unger (2020),

[6]In Section 7.1, we extend our analysis to the most recent period from 2006 to 2016, which has been characterized by a productivity slowdown. We show that our main results remain unchanged and that this period also features an increase in the cost of innovation, which more than offsets the positive effect of increased concentration on innovation and growth.

[7]For comparison, computing non-stationary equilibria in the canonical neoclassical growth model requires finding the complete time path of a single continuous state variable, the capital stock K_t.

[8]Even though we simplify the full dynamic problem significantly through our derivations, this still leaves 253 positive scalars to be simultaneously solved for through global optimization methods.

and Grullon, Larkin, and Michaely (2019).[9] Gutiérrez and Philippon (2017b) and Gutiérrez, Jones, and Philippon (2021) show that the increase in market concentration and the decline in competition can explain underinvestment by US firms. Covarrubias, Gutiérrez, and Philippon (2020) distinguish between so-called *good* and *bad* concentration. For instance, market concentration can be *good* if it is associated with technological change while increased concentration due to barriers to entry would be *bad*. They further suggest that market concentration in the US turned from being efficient in the 1990s to being inefficient after 2000. Rossi-Hansberg, Sarte, and Trachter (2021) find that the observed positive trend in market concentration at the national level in the US has been accompanied by a corresponding negative trend in average local market concentration. Baqaee and Farhi (2020) estimate that misallocation due to large and dispersed markups results in a TFP loss as large as 15%. Recent work by Edmond, Midrigan, and Xu (2023) is closely related to our paper. Using a dynamic model with size-dependent markups, they find sizeable welfare losses from markups. Weiss (2020) studies the role played by intangible capital on markup trends and welfare in a model with oligopolistic competition and a one-time investment in intangible capital by firms. Compared to these papers, our model has continuous and strategic investment in productivity growth, and oligopolistic competition between an endogenous number of heterogeneous innovative firms. Our Schumpeterian structure delivers significant differences regarding the dynamics, growth, and welfare. De Loecker, Eeckhout, and Mongey (2022) also study the causes and consequences of market power in general equilibrium. Our model differs from their approach, as we endogenize productivity growth and model strategic interactions between firms in innovation. This difference is significant, since allowing for endogenous productivity growth can completely overturn the welfare implications of higher markups as we discussed earlier.

Our paper also adds to the endogenous growth literature studying the relationship between competition and innovation. In early Schumpeterian models of growth through innovation (e.g., Aghion and Howitt (1992)), more product market competition reduces rents and hence the incentives to invest in R&D and innovation. Aghion, Harris, Howitt, and Vickers (2001) propose a model of step-by-step innovation by superstar firms. By introducing an additional incentive to escape competition, this model can generate an inverted-U shape relationship between the degree of market competition (measured by the elasticity of substitution between products) and innovation. Using a similar model, Aghion, Bloom, Blundell, Griffith, and Howitt (2005) also generate an inverted-U shape relationship between competition and innovation.[10] They further provide empirical evidence for such an inverted-U shape relationship between innovation activity and industry competition using data from the UK. We also undertake an empirical analysis in which we look at the relationship between competition and innovation using data from the US. We verify that the findings in Aghion, Bloom, Blundell, Griffith, and Howitt (2005) also hold in the US using several measures of innovation. Furthermore, we also document a robust inverted-U relationship between the relative sales of a firm and its innovation. Our model is able to replicate the two hump-shaped relationships within and across industries without relying on exogenous heterogeneity,[11] and in the presence of endogenous firm entry.[12]

[9] De Loecker and Eeckhout (2018) also document similar trends for markups in other regions of the world.

[10] See Gilbert (2006) for an extensive review of the literature on competition and innovation.

[11] For instance, in Aghion, Bloom, Blundell, Griffith, and Howitt (2005), exogenous heterogeneity in collusion across industries is assumed to generate the inverted-U result.

[12] Etro (2007) argues that the industry-level inverted-U relationship breaks down when endogenous entry is introduced to

Within the class of endogenous growth models, the closest papers to ours are Peters and Walsh (2019), Akcigit and Ates (2021, 2023), Liu, Mian, and Sufi (2022), Olmstead-Rumsey (2022), Aghion, Bergeaud, Boppart, Klenow, and Li (2023), and De Ridder (2023).[13] Peters and Walsh (2019) propose an extension of Peters (2020) to study the role of the decline in the growth rate of the labor force on the observed decline in business dynamism, increased market power, and slower productivity growth.[14] Building on Acemoglu and Akcigit (2012), Akcigit and Ates (2021, 2023) propose a model of endogenous growth and firm dynamics with heterogeneous markups and show that declining knowledge diffusion played a significant role in declining business dynamism. Liu, Mian, and Sufi (2022) show that a decline in the interest rate can lead to a rise in market concentration and a slowdown in productivity growth. Olmstead-Rumsey (2022) shows, using an endogenous growth model in which the quality of new ideas is heterogeneous, that the decline in small firms' innovativeness can be linked to recent trends in market concentration and productivity growth. Aghion, Bergeaud, Boppart, Klenow, and Li (2023) explain the rise in concentration and profits through falling firm-level costs of spanning multiple markets due to accelerating IT advances. De Ridder (2023) shows that the rise in intangible inputs can explain the recent trends in productivity growth and business dynamism as it causes a shift from variable to fixed costs. In both Aghion, Bergeaud, Boppart, Klenow, and Li (2023) and De Ridder (2023), the rise in concentration can be temporarily associated with an increase in productivity growth. In the long run, productivity growth decreases.

Most of the listed papers assume Bertrand competition with homogeneous goods in each industry.[15] Markups depend directly on the productivity gap between the leader and the follower. Unless firms are neck-and-neck, the leader takes over the whole industry. Our paper differs from the literature in its rigorous treatment of within-industry dynamics as discussed earlier. Our model features non-degenerate distributions of sales, employment, markups, and profits within each industry. It allows for an arbitrarily high and endogenous number of oligopolistically-competing firms in each industry.[16] These advances upon the previous literature allow us to hit the hump-shaped relationship between competition and innovation within and across industries, which helps discipline the estimated growth and welfare implications of higher market concentration and markups. The new competitive fringe feature of our model allows for realistic market share distributions and firm life cycles where entrants do not immediately become industry leaders, and we can distinguish new business entry from the emergence of new superstars.[17] Apart from the technical differences, our study also differs in its quantitative approach. Instead of searching for a single mechanism which can jointly explain all the changes in the macroeconomic aggregates to some

Aghion, Harris, Howitt, and Vickers (2001). Our model provides a counterexample. Other counterexamples can be found in Bento (2014, 2020)

[13]In other related work, Corhay, Kung, and Schmid (2021) use a Romer-type endogenous growth model to study the asset pricing implications of increasing markups.

[14]In a model without endogenous growth, Hopenhayn, Neira, and Singhania (2022) also show that declining population growth can explain a large share of the observed decline in firm entry and productivity slowdown.

[15]Two exceptions are Liu, Mian, and Sufi (2022) and Olmstead-Rumsey (2022) which propose models of endogenous growth with Bertrand competition between two firms with differentiated products. However, they maintain the perpetual duopoly assumption that pervades the step-by-step innovation literature, as opposed to allowing an endogenous number of superstars in each industry as we do.

[16]Peretto (1996) features a model with an endogenous number of firms in the aggregate economy, but focuses on a symmetric equilibrium where all firms have the same productivity, innovation, and sales. Impullitti and Licandro (2018) also consider a model of endogenous growth with oligopolistic competition. In contrast to our framework, they focus on industries with a common (endogenous) number of homogeneous firms.

[17]Ghazi (2021) proposes a model with a competitive fringe and a single dominant firm but does not consider innovation by small firms.

degree, our estimation tightly hits all the changes, and we use the model to disentangle the contribution of each channel. This provides some guidance on which mechanisms should be further investigated to better understand the underlying sources of the structural transition, such as the widening productivity gap between the superstars and the rest, and the increasing costs of R&D.

Our paper is related to the broader literature on the decline in business dynamism. A large body of that literature shows that the entry rate of new firms has significantly decreased in the US since the early 1980s (see for instance, Hathaway and Litan (2014), Decker, Haltiwanger, Jarmin, and Miranda (2016), Pugsley and Sahin (2018)). The decline in firm entry can further affect productivity growth. For instance, Lentz and Mortensen (2008), Garcia-Macia, Hsieh, and Klenow (2019) and Acemoglu, Akcigit, Alp, Bloom, and Kerr (2018) find that a significant share of aggregate TFP growth and job creation is due to firm entry and young firms. Our quantitative exercise reveals that not only the share of output by small firms decreases but also that these firms are less likely to become superstar firms due to higher R&D costs.

Our results also add to the recent literature on the slowdown of TFP growth in the US. Gordon (2012, 2014) argues that the significant slowdown in the rate of productivity growth in the US since the 1970s is due to diminishing returns from R&D. Bloom, Jones, Van Reenen, and Webb (2020) show empirical evidence for a steady decline in R&D productivity over the course of the 20th century. As ideas are getting harder to find, sustaining a constant economic growth rate has required a simultaneous increase in research inputs. Our quantitative results suggest that the cost of R&D has been increasing since the late 1970s for both small and superstar firms.

Another important trend over the last decades in the US relates to the evolution of factor shares. In particular, the share of income paid to workers has steadily decreased over the last decades as highlighted in, among others, Karabarbounis and Neiman (2013) and Elsby, Hobijn, and Şahin (2013). At the same time, the profit share has followed an opposite trajectory (Barkai (2020)). A large literature has highlighted several potential sources for the decrease in the labor share such as technological change and automation (see, for instance, Zeira (1998), Acemoglu and Restrepo (2018), Martinez (2019)), globalization (Elsby, Hobijn, and Şahin (2013)), the decline in the relative price of capital (Karabarbounis and Neiman (2013)), increased cost of housing (Rognlie (2015)) or a rise in productivity dispersion (Gouin-Bonenfant (2022)). Barkai (2020) and De Loecker, Eeckhout, and Unger (2020) relate the decrease in the labor share to the rise in profitability and markups. Autor, Dorn, Katz, Patterson, and Van Reenen (2020) and Kehrig and Vincent (2021) attribute this trend to the rise of superstar firms which tend to display high markups and low labor shares. Our model is also able to generate a decline in the labor share over the same time period.

The paper is organized as follows: In Section 2, we introduce our new framework and derive equilibrium conditions. We also discuss non-stationary equilibria and the social planner's problem. In Section 3, we discuss the empirical relationship between competition and innovation that is used to discipline our estimation. We also estimate the model. Section 4 describes some properties and new features of our model. Section 5 applies our new framework to the study of the sources and consequences of recent macroeconomic trends in the US, especially related to the rise in market concentration and markups. We further validate the model in Section 6 and check its robustness in Section 7. Section 8 concludes.

2 Model

2.1 Environment

Preferences Time is continuous and indexed by $t \in \mathbb{R}_+$. The economy is populated by an infinitely-lived representative consumer who discounts the future at rate $\rho > 0$. The representative consumer maximizes lifetime utility:

$$U = \int_0^\infty e^{-\rho t} \ln(C_t)\, dt \tag{1}$$

where C_t is consumption of the final good at time t, the price of which is normalized to one.

The household inelastically supplies one unit of labor in exchange for an endogenously determined wage rate w_t. Households own all the assets in the economy and face the following budget constraint:

$$\dot{A}_t = r_t A_t + w_t - C_t \tag{2}$$

where A_t is household wealth and r_t is the rate of return on assets.

Final Good Production The final good Y_t is produced competitively using inputs from a measure one of industries:

$$\ln(Y_t) = \int_0^1 \ln(y_{jt})\ dj \tag{3}$$

where y_{jt} is production of industry j at time t.

Industry Production Each industry is populated by an endogenous number of superstar firms ($N_{jt} \in \{1, ..., \bar{N}\}$), each producing a differentiated variety, as well as by a competitive fringe composed of a mass m_{jt} of small firms producing a homogeneous good (small firm k in the fringe of industry j at time t produces y_{ckjt}). Given that there is a continuum of small firms and their products are homogeneous, each small firm in the competitive fringe is a price taker.[18] Superstars compete in quantities. In particular, we allow for strategic interaction among superstars as variety production is the result of a static Cournot game. Total production of industry j at time t is given by:

$$y_{jt} = \left(\sum_{i=1}^{N_{jt}} y_{ijt}^{\frac{\eta-1}{\eta}} + \tilde{y}_{cjt}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \tag{4}$$

where y_{ijt} is the production of superstar firm i in industry j at time t, $\tilde{y}_{cjt} = \int_{F_{jt}} y_{ckjt}\, dk$ is the production of the competitive fringe in industry j at time t, F_{jt} is the set of small firms in the fringe in industry j at time t and $\eta > 1$ is the elasticity of substitution between varieties.

[18] For a discussion of the importance and interpretation of the competitive fringe, see Section J in the Revision Appendix.

Variety Production Superstar firms in each industry produce their own variety using a linear production technology in labor:

$$y_{ijt} = q_{ijt} l_{ijt} \tag{5}$$

where q_{ijt} is the productivity of superstar firm i in industry j at time t and l_{ijt} is labor.

Similarly for every small firm k in the competitive fringe:

$$y_{ckjt} = q_{cjt} l_{ckjt} \tag{6}$$

We assume that each small firm in the fringe in a given industry has the same productivity q_{cjt}. Superstar firms within each industry differ according to their level of productivity which can be built over time through R&D and innovation.

R&D and Innovation Each superstar can perform R&D to improve the productivity of its variety. To generate a Poisson rate z_{ijt} of success in R&D, firm i must pay a cost in units of the final good equal to:

$$R_{ijt} = \chi z_{ijt}^{\phi} Y_t. \tag{7}$$

where ϕ is the superstar R&D cost convexity parameter with $\phi > 1$, and $\chi > 0$ is the scale parameter.

If a superstar successfully innovates at time t, its productivity is multiplied by $(1 + \lambda)$. We assume that the maximum number of productivity steps between any two superstar firms within an industry is $\bar{n} \geq 1$. For the competitive fringe, we assume that the relative productivity of small firms with respect to the leader is a constant, denoted by $\zeta = \frac{q_{cjt}}{q_{jt}^{leader}}$. Note that ζ can be any positive real number, and it should be interpreted as the aggregate productivity of all firms in the fringe. In our model, the market share of each small firm is infinitesimally small compared to any superstar firm. However, the fringe as a whole has a positive market share, which is allowed to be larger in aggregate than the market share of some (or all) superstar firms producing a differentiated good. This implies that superstar firms in our model are characterized by higher productivity and larger market shares than every single small firm in equilibrium, but collectively, the small firms matter.[19]

Entry and Exit of Superstar Firms At any time t, each small firm k in the competitive fringe can generate a Poisson arrival density X_{kjt} of entry into superstar firms when $N_{jt} < \bar{N}$. The associated R&D cost is given by

$$R_{kjt}^{e} = \nu X_{kjt}^{\epsilon} Y_t. \tag{8}$$

where ϵ is the small firm R&D cost convexity parameter with $\epsilon > 1$, and $\nu > 0$ is the scale parameter.

Because all small firms are homogeneous within an industry, they all perform the same level of innovation in equilibrium. We can rewrite the industry level Poisson rate of innovation $X_{jt} = \int X_{kjt} dk = m_{jt} X_{kjt}$ and the industry level R&D expenditures of small firms $R_{jt}^{e} = m_{jt} R_{kjt}^{e}$.

[19] See Section J in the Revision Appendix for a more detailed discussion of the competitive fringe assumption.

When a small firm successfully innovates, it becomes a superstar, and the number of superstar firms in the industry is increased by one unless the number of firms in the industry is already equal to $\bar{N}$, in which case entry is not allowed. The new entrant is assumed to enter as the smallest superstar firm in the industry, i.e., it is assumed to have a productivity level $\bar{n}$ steps below the leader. In this sense, entry into superstars in our model should be interpreted as a small firm becoming more productive and large enough to strategically interact with other superstars.[20] In particular, our assumptions imply that firms more than $\bar{n}$ steps below the leader in any industry are not large enough to be considered as having any meaningful strategic interaction with other firms.[21] Consistent with this interpretation of our model, a firm endogenously loses its superstar firm status when it is $\bar{n}$ steps below the industry leader and the leader innovates. In that case, the number of superstar firms in the industry decreases by one.

Entry and Exit of Small Firms We also introduce entry into and exit from the competitive fringe, which allows for a realistic mapping between small firm entry in the model and new business creation in the data. This is different from the existing endogenous growth literature where entrants can immediately become the leader in their industry, or large enough to interact strategically with the current leader. In this sense, our model features a realistic firm life-cycle, where thousands of small firms enter, but only a select few become superstars, several years or even decades after entry. We assume an exogenous exit rate of small firms equal to $\tau > 0$. Regarding entry, there is a mass one of entrepreneurs who can pay a cost $\psi e_t^2 Y_t$ to generate a Poisson rate e_t of starting a new small firm. The new firms are randomly allocated to the competitive fringe of an industry, which implies $m_{jt} = m_t$ for all industries j. In order to keep the mass of entrepreneurs unchanged, we assume that they sell their firm on a competitive market at its full value and remain in the set of entrepreneurs.[22]

2.2 Equilibrium

Consumer's problem Household lifetime utility maximization delivers the standard Euler equation:

$$\frac{\dot{C}_t}{C_t} = r_t - \rho. \tag{9}$$

Final Good Producers The final good is produced competitively. The representative final good producer chooses the quantity of each variety in each industry which maximizes profit:

$$\max_{[\{y_{ijt}\}_{i=1}^{N_{jt}}, \tilde{y}_{cjt}]_{j=0}^{1}} \exp\left(\frac{\eta}{\eta-1}\int_0^1 \ln\left[\sum_{i=1}^{N_{jt}} y_{ijt}^{\frac{\eta-1}{\eta}} + \tilde{y}_{cjt}^{\frac{\eta-1}{\eta}}\right] dj\right) - \int_0^1 \left(\sum_{i=1}^{N_{jt}} p_{ijt} y_{ijt} + p_{cjt}\tilde{y}_{cjt}\right) dj. \tag{10}$$

[20] A breakthrough innovation by small firms represents a jump in individual productivity. However, the resultant increase in productivity is different from λ, which naturally calls for different R&D cost functions for small firms and the superstars. See Section J in the Revision Appendix for a discussion of small firms' productivity.

[21] In the estimated model, $\bar{n}$ is chosen large enough such that the largest superstars that we stop keeping track of are strictly smaller than 4/10,000 of the leader in terms of revenue and profit in all industry-states. Keeping track of these firms would decrease the profits of the remaining superstars by strictly less than 3/10,000, and this would not noticeably change the results.

[22] This is without loss of generality. Alternatively, we could have assumed that, upon new business creation, successful entrepreneurs are replaced by new entrepreneurs. This alternative assumption delivers exactly the same observed equilibrium, because all entrepreneurial profits accrue to the representative household, which would remain the same regardless of which assumption is chosen.

where p_{ijt} (p_{cjt}) is the price of variety i (the competitive fringe variety) in industry j at time t. This leads to the following inverse demand function:

$$p_{ijt} = \frac{y_{ijt}^{-\frac{1}{\eta}} Y_t}{\sum_{k=1}^{N_{jt}} y_{kjt}^{\frac{\eta-1}{\eta}} + \tilde{y}_{cjt}^{\frac{\eta-1}{\eta}}} \tag{11}$$

and

$$\frac{y_{ijt}}{y_{kjt}} = \left(\frac{p_{kjt}}{p_{ijt}}\right)^{\eta} \tag{12}$$

where y_{ijt} should be replaced by $\tilde{y}_{cjt}$ for the competitive fringe.

Variety Producers We assume that superstar firms within the same industry compete *à la* Cournot. Each firm maximizes profit:

$$\max_{y_{ijt}} \; p_{ijt} y_{ijt} - w_t l_{ijt} = \max_{y_{ijt}} \; \frac{y_{ijt}^{\frac{\eta-1}{\eta}} Y_t}{\sum_{k=1}^{N_{jt}} y_{kjt}^{\frac{\eta-1}{\eta}} + \tilde{y}_{cjt}^{\frac{\eta-1}{\eta}}} - \frac{w_t y_{ijt}}{q_{ijt}}. \tag{13}$$

This delivers the following best response functions for superstar firms:

$$y_{ijt} = \left[\frac{\eta-1}{\eta} q_{ijt} \frac{\sum_{k\neq i} y_{kjt}^{\frac{\eta-1}{\eta}} + \tilde{y}_{cjt}^{\frac{\eta-1}{\eta}}}{\left[\sum_{k=1}^{N_{jt}} y_{kjt}^{\frac{\eta-1}{\eta}} + \tilde{y}_{cjt}^{\frac{\eta-1}{\eta}}\right]^2} \frac{Y_t}{w_t}\right]^{\eta} \tag{14}$$

$$= \frac{\eta-1}{\eta} q_{ijt} \frac{\sum_{k\neq i} \left(\frac{y_{kjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}} + \left(\frac{\tilde{y}_{cjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}}}{\left[\sum_{k=1}^{N_{jt}} \left(\frac{y_{kjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}} + \left(\frac{\tilde{y}_{cjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}}\right]^2} \frac{Y_t}{w_t} \tag{15}$$

The total production of the competitive fringe is given by:

$$\tilde{y}_{cjt} = q_{cjt} \frac{\frac{Y_t}{w_t}}{\sum_{k=1}^{N_{jt}} \frac{y_{kjt}}{\tilde{y}_{cjt}}^{\frac{\eta-1}{\eta}} + 1} \tag{16}$$

Relative production between each superstar variety and the competitive fringe within the industry can then be written as:

$$\left(\frac{y_{ijt}}{y_{kjt}}\right)^{\frac{1}{\eta}} = \frac{q_{ijt}}{q_{kjt}} \frac{\sum_{l\neq i} \left(\frac{y_{ljt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}} + \left(\frac{\tilde{y}_{cjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}}}{\sum_{l\neq k} \left(\frac{y_{ljt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}} + \left(\frac{\tilde{y}_{cjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}}} \tag{17}$$

$$\left(\frac{y_{ijt}}{\tilde{y}_{cjt}}\right)^{\frac{1}{\eta}} = \frac{\eta-1}{\eta}\frac{q_{ijt}}{q_{cjt}}\frac{\sum_{l\neq i}\left(\frac{y_{ljt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}} + \left(\frac{\tilde{y}_{cjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}}}{\sum_{l=1}^{N_{jt}}\left(\frac{y_{ljt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}} + \left(\frac{\tilde{y}_{cjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}}}. \tag{18}$$

For each industry j, this is a system of N_{jt} equations and N_{jt} unknown production ratios which can be solved given relative productivities within the industry.

We can further derive variety prices (p_{ijt}) and profits before R&D expenditures (π_{ijt}) which only depend on relative productivities within the industry:

$$p_{ijt} = \frac{\eta}{\eta-1}\frac{\sum_{k=1}^{N_{jt}}\left(\frac{y_{kjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}} + \left(\frac{\tilde{y}_{cjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}}}{\sum_{k\neq i}\left(\frac{y_{kjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}} + \left(\frac{\tilde{y}_{cjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}}}\frac{w_t}{q_{ijt}} \tag{19}$$

$$\pi_{ijt} = \frac{Y_t}{\left[\sum_{k=1}^{N_{jt}}\left(\frac{y_{kjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}} + \left(\frac{\tilde{y}_{cjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}}\right]^2}\frac{\eta + \sum_{k\neq i}\left(\frac{y_{kjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}} + \left(\frac{\tilde{y}_{cjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}}}{\eta} \tag{20}$$

Firms charge varying markups (M_{ijt}) over marginal cost that depend on the number of competitors as well as their relative productivities:

$$M_{ijt} = \frac{\eta}{\eta-1}\frac{\sum_{k=1}^{N_{jt}}\left(\frac{y_{kjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}} + \left(\frac{\tilde{y}_{cjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}}}{\sum_{k\neq i}\left(\frac{y_{kjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}} + \left(\frac{\tilde{y}_{cjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}}} \tag{21}$$

Superstar Value Function and R&D Decision Static production decision, markups, and profits within each industry only depend on the number of superstars, and the distribution of their relative productivities. In other words, the relevant state variables for a firm i in industry j at time t can be summarized by the vector of the number of productivity steps between superstar firm i and every other superstar firm $k \in \{(1, 2, ..., N_{jt})\backslash\{i\}\}$ in the industry. Letting $n_{ijt}^k \in \{-\bar{n}, -\bar{n}+1, ..., \bar{n}-1, \bar{n}\}$ be the number of steps by which firm i in industry j leads firm k at time t, the relevant state variables for firm i in industry j at time t are given by the vector $\mathbf{n}_{ijt} = \{n_{ijt}^k\}_{k\neq i}$ and $N_{jt} = |\mathbf{n}_{ijt}| + 1$.[23]

Henceforth, we drop the time subscripts unless otherwise needed. A superstar firm i chooses an

[23] We can rewrite the relative productivity of firm i and k as $\frac{q_{ijt}}{q_{kjt}} = (1+\lambda)^{n_{ijt}^k}$.

innovation rate (z_i) to maximize the value of the firm given by:

$$\begin{aligned} rV(\mathbf{n}_i, N) = \quad & \max_{z_i} \pi(\mathbf{n}_i, N) - \chi z_i^{\phi} Y \\ & + z_i \left[V(\mathbf{n}_i \backslash \{n_i^k = \bar{n}\} + \mathbf{1}, N - |\{n_i^k = \bar{n}\}|) - V(\mathbf{n}_i, N) \right] \\ & + \sum_{k: n_i^k = -\bar{n}} z_{kj} (0 - V(\mathbf{n}_i, N)) \\ & + \sum_{k: n_i^k \neq -\bar{n}} z_{kj} \left[V(\mathbf{n}_i \backslash \{n_i^k\} \cup \{n_i^k - 1\} \backslash \{n_i^l = \bar{n} + n_i^k\}, N - |\{n_i^l = \bar{n} + n_i^k\}|) - V(\mathbf{n}_i, N) \right] \\ & + X_j \left[V(\mathbf{n}_i \cup \{\min\{\bar{n}, \bar{n} + \min(\mathbf{n}_i)\}\}, \min(N+1, \bar{N})) - V(\mathbf{n}_i, N) \right] + \dot{V}(\mathbf{n}_i, N) \end{aligned} \tag{22}$$

The first line is the flow profit minus the cost of R&D. The second line is the change in firm value due to a successful innovation by firm i which happens with Poisson rate z_i. If firm i innovates, it increases its lead over any other firm by one. Any firm $\bar{n}$ productivity steps below firm i exits the set of superstars. The third line corresponds to the change in value due to endogenous exit which arises if one of the industry leaders is $\bar{n}$ steps ahead of firm i and innovates.

The fourth line comes from any other firm (not leading i by $\bar{n}$) innovating. In that case, the lead of firm i with respect to the innovating firm decreases by one. In addition, if the innovating firm k is also leading any other firm l by $\bar{n}$ (which happens if $n_i^l - n_i^k = \bar{n}$), firm l exits. The first term in the fifth line is the effect of firm entry on the value of firm i. In that case, the entrant starts $\bar{n}$ productivity steps below the industry leader. The second term on the same line is the growth in firm value.

We can guess and verify that, in a balanced growth path (BGP), $V(\mathbf{n}_i, N) = v(\mathbf{n}_i, N)Y$. In that case, $\dot{V}(\mathbf{n}_i, N) = gv(\mathbf{n}_i, N)Y$ (where g is the growth rate of Y). Using equation (9), we can write:

$$\begin{aligned} \rho v(\mathbf{n}_i, N) = \quad & \max_{z_i} \frac{\pi(\mathbf{n}_i, N)}{Y} - \chi z_i^{\phi} \\ & + z_i \left[v(\mathbf{n}_i \backslash \{n_i^k = \bar{n}\} + \mathbf{1}, N - |\{n_i^k = \bar{n}\}|) - v(\mathbf{n}_i, N) \right] \\ & + \sum_{k: n_i^k = -\bar{n}} z_{kj} (0 - v(\mathbf{n}_i, N)) \\ & + \sum_{k: n_i^k \neq -\bar{n}} z_{kj} \left[v(\mathbf{n}_i \backslash \{n_i^k\} \cup \{n_i^k - 1\} \backslash \{n_i^l = \bar{n} + n_i^k\}, N - |\{n_i^l = \bar{n} + n_i^k\}|) - v(\mathbf{n}_i, N) \right] \\ & + X_j \left[v(\mathbf{n}_i \cup \{\min\{\bar{n}, \bar{n} + \min(\mathbf{n}_i)\}\}, \min(N+1, \bar{N})) - v(\mathbf{n}_i, N) \right]. \end{aligned} \tag{23}$$

The optimal level of innovation is given by:

$$z_i = \left\{ \frac{\left[v(\mathbf{n}_i \backslash \{n_i^k = \bar{n}\} + \mathbf{1}, N - |\{n_i^k = \bar{n}\}|) - v(\mathbf{n}_i, N) \right]}{\chi \phi} \right\}^{\frac{1}{\phi - 1}}. \tag{24}$$

Notice that the optimal innovation z_i depends on the innovation choices of all other firms in the same industry across all possible future histories. The innovation choices of other firms affect the value function determined by the Hamilton-Jacobi-Bellman equation (23). The value function, in turn, determines the innovation choice z_i. This introduces dynamic strategic interactions, where innovation policies of firms are functions of others' innovation policies. We focus on the Markov Perfect equilibrium at the intersection of all firms' best response functions, as is common in step-by-step innovation models such as Aghion, Harris, Howitt, and Vickers (2001).

Small Firm Innovation and Entry into Superstar Firms Since relative sales and innovation policies only depend on pairwise productivity differences and the number of firms in the industry, we can define $\Theta = (N, \vec{n})$ as the state of the industry with $N \in \{1, ..., \bar{N}\}$ being the number of superstars in the industry and $\vec{n} \in \{0, ..., \bar{n}\}^{N-1}$ denoting the number of steps followers are behind the leader (in ascending order). We let $f(\Theta) = \frac{1}{\eta-1} \ln \left(\sum_{i=1}^{N_j} \left(\frac{y_{ij}}{y_{cj}}(\Theta) \right)^{\frac{\eta-1}{\eta}} + 1 \right)$ and define $p_{li}(\Theta)$ as the arrival rate of a leader innovation and $p(\Theta, \Theta')$ as the instantaneous flows from state Θ to Θ'. In each industry j (with $N_j < \bar{N}$), each small firm in the competitive fringe can perform R&D. All small firms within an industry are symmetric and choose R&D investment to maximize:

$$\begin{aligned} rV^e(\Theta_j) \quad &= \quad \max_{X_{kj}} X_{kj} V(\{\tilde{\mathbf{n}}_j - \bar{n}\} \cup \{-\bar{n}\}, N_j + 1) - \tau V^e(\Theta_j) - \nu X_{kj}^{\epsilon} Y \\ &\quad + \sum_{\Theta'} p(\Theta_j, \Theta')(V^e(\Theta') - V^e(\Theta_j)) + \dot{V}^e(\Theta_j) \end{aligned} \tag{25}$$

where $V^e(\Theta_j)$ is the value of a small firm in industry j and $\tilde{\mathbf{n}}_j = \mathbf{n}_{kj}$ where k denotes a productivity leader in industry j.[24]

Guessing and verifying that, in a BGP, $V^e(\Theta_j) = v^e(\Theta_j)Y$, we can rewrite:

$$\begin{aligned} (\rho + \tau)v^e(\Theta_j) \quad &= \quad \max_{X_{kj}} X_{kj} v(\{\tilde{\mathbf{n}}_j - \bar{n}\} \cup \{-\bar{n}\}, N_j + 1) - \nu X_{kj}^{\epsilon} \\ &\quad + \sum_{\Theta'} p(\Theta_j, \Theta')(v^e(\Theta') - v^e(\Theta_j)) \end{aligned} \tag{26}$$

The optimal innovation intensity by a small firm in industry j is then:

$$X_{kj} = \left(\frac{v(\{\tilde{\mathbf{n}}_j - \bar{n}\} \cup \{-\bar{n}\}, N_j + 1)}{\nu \epsilon} \right)^{\frac{1}{\epsilon - 1}} \tag{27}$$

Plugging in the optimal solution, the normalized value of a small firm is calculated as

$$v^e(\Theta_j) = \frac{1}{\rho + \tau} \left[\left(1 - \frac{1}{\epsilon}\right) \frac{v(\{\tilde{\mathbf{n}}_j - \bar{n}\} \cup \{-\bar{n}\}, N_j + 1)^{\frac{\epsilon}{\epsilon - 1}}}{(\nu\epsilon)^{\frac{1}{\epsilon - 1}}} + \sum_{\Theta'} p(\Theta_j, \Theta')(v^e(\Theta') - v^e(\Theta_j)) \right] \tag{28}$$

Entrepreneurs and Entry into the Competitive Fringe There is a mass one of entrepreneurs in the economy who can pay a cost $\psi e^2 Y$ to create a Poisson rate e of becoming a small firm in a randomly selected industry. We assume that a successful entrepreneur immediately sells the small firm in a competitive market. The expected selling price of the new small firm of a successful entrepreneur is equal to:

[24] Note that we use $\int_{k=i} V_k^e(\Theta_j) dk = 0$ in the first term, i.e., the value of the small firm is insignificant compared to the value of the superstar firm it becomes, since it is of mass zero in the competitive fringe.

$W = \sum_{\Theta} V^e(\Theta)\mu(\Theta)$, where $\mu(\Theta)$ is the mass of industries of type Θ.[25] The value of being an entrepreneur (S) can be written as:

$$\rho S = \max_{e} -\psi e^2 Y + eW \tag{29}$$

Guessing and verifying that, in a BGP, $S = sY$, we obtain that:

$$e = \frac{W}{2\psi Y} = \frac{\sum_{\Theta} v^e(\Theta)\mu(\Theta)}{2\psi} \tag{30}$$

which implies:

$$s = \frac{\left[\sum_{\Theta} v^e(\Theta)\mu(\Theta)\right]^2}{4\psi\rho} \tag{31}$$

In a stationary equilibrium, entry into the competitive fringe equals exit from the competitive fringe which means:

$$e = \tau m \tag{32}$$

Combining equations (30) and (32), we get an equation that pins down the value of m as:

$$m = \frac{\sum_{\Theta} v^e(\Theta)\mu(\Theta)}{2\psi\tau} \tag{33}$$

Equilibrium Definition We focus on the unique Markov-Perfect Equilibrium of our economy. An equilibrium is defined by a set of allocations $\{C_t, Y_t, y_{ijt}, y_{ckjt}\}$, policies $\{l_{ijt}, l_{ckjt}, z_{ijt}, X_{kjt}, e_t\}$, prices $\{p_{ijt}, p_{cjt}, w_t, r_t\}$, the number of superstars in each industry N_{jt}, a mass of small firms m_t, a set of vectors $\{\mathbf{n}_{ijt}\}$ that denote the relative productivity distance between firm i and every other firm in the same industry j at time t, such that, $\forall t \geq 0$, $j \in [0,1]$, $i \in \{1, ..., N_{jt}\}$:

(i) Given prices, final good producers maximize profit.

(ii) Given $\mathbf{n}_{ij}$ and N_{jt}, superstars choose y_{ijt} to maximize profit.

(iii) Given prices, small firms in the competitive fringe choose y_{ckjt} to maximize profit.

(iv) Superstar firms choose innovation policy z_{ijt} to maximize firm value.

(v) Small firms choose innovation policy X_{kjt} to maximize firm value.

(vi) Entrepreneurs choose e_t to maximize profit.

(vii) The real wage rate w_t clears the labor market.

(viii) Aggregate consumption C_t grows at rate $r_t - \rho$.

(ix) Resource constraint is satisfied: $Y_t = C_t + \int_0^1 \sum_{i=1}^{N_{jt}} \chi z_{ijt}^{\phi} Y_t \, dj + \int_0^1 m_t \nu X_{kjt}^{\epsilon} Y_t \, dj + \psi e_t^2 Y_t$.

[25] We can show that the expected value of $\sum_{\Theta'} p(\Theta, \Theta')(V^e(\Theta') - V^e(\Theta))$ in a stationary equilibrium is equal to zero (see Proposition 1 in Appendix A.2). W is thus equal to $\frac{1-\frac{1}{\epsilon}}{\rho+\tau} \frac{\int_0^1 V(\{\tilde{\mathbf{n}}_j - \bar{n}\} \cup \{-\bar{n}\}, N_j+1)^{\frac{\epsilon}{\epsilon-1}} \, dj}{(\nu\epsilon)^{\frac{1}{\epsilon-1}}}$.

Growth Rate and Balanced Growth Path We can derive the growth rate of the economy at time t (g_t) as:[26]

$$g_t = -g_{\omega,t} + \sum \left[p_{lit}(\Theta)\ln(1+\lambda)\right]\mu_t(\Theta) + \sum_{\Theta}\sum_{\Theta'}\left[f_t(\Theta') - f_t(\Theta)\right] p_t(\Theta,\Theta')\mu_t(\Theta) \quad (34)$$

where $g_{\omega,t}$ is the growth rate of the relative wage, $\omega_t = \frac{w_t}{Y_t}$, the second term comes from the growth rate of the industry leaders, and the third term accounts for production reallocation as industries move between states. In a balanced growth path with time-invariant distribution over Θ, $g_{\omega,t} = 0$, $\mu_t(\Theta) = \mu(\Theta)$ and:

$$g = \sum \left[p_{li}(\Theta)\ln(1+\lambda)\right]\mu(\Theta) \quad (35)$$

which corresponds to the expected growth rate of industry leaders' log productivity. In other words, long-run growth is driven by industry leaders pushing the technological frontier in their industry. Other firms' (superstars and small firms) decisions influence the growth rate of the economy by affecting the incentives for industry leaders to invest in innovation, $p_{li}(\Theta)$, and the stationary distribution across industry states, $\mu(\Theta)$, which is a function of the innovation decisions by all firms in all industries through their contribution to the instantaneous flow matrix $p(\Theta,\Theta')$.[27]

2.3 Non-Stationary Equilibria and Transitional Dynamics

Most of the discussion above has focused on stationary (balanced growth path) equilibria. Despite the richness of our model in terms of heterogeneity and dynamics, it is possible to solve for non-stationary equilibria as well. Computing a non-stationary equilibrium is more complex than in standard macroeconomic models. The relevant state variables to keep track of are (1) the average log productivity level of industry leaders $Q_t = \int \ln q_{jt}^{leader} dj$, (2) the mass of small firms m_t, and (3) the industry-state distribution $\mu_t(\Theta)$, which is time-varying in a non-stationary equilibrium. Given our choices for $\bar{n}$ and $\bar{N}$ in our baseline estimation, this means solving for a non-stationary equilibrium starting from arbitrary initial conditions $(Q_0, m_0, \mu_0(\Theta))$ requires finding the complete time paths of 86 continuous state variables under rational expectations, which might seem daunting at first.[28] Despite the complexity, the tractability of our framework and our choice to use a continuous-time setting render the computation of non-stationary equilibria feasible. This is accomplished without any deviation from the assumption of rational expectations. For brevity, the full details including the algorithm are relegated to Section A.9 of the Appendix.

We can distinguish two categories of decisions in the model. For a given industry state Θ, within-industry relative labor demand, production, prices, markups, and profits are time-invariant due to the static nature of product market competition in a Markov Perfect equilibrium. So it suffices to solve for every possible industry equilibrium once. However, the innovation decisions of both superstars and small firms, as well as the new business creation decision of the entrepreneurs, depend on firm values, which in turn

[26]See Appendix A.1 for the full derivation.

[27]We provide an algorithm that can be used to solve for stationary equilibria in Section A.8 of the Online Appendix, which further elucidates the relationship between $p(\Theta,\Theta')$ and $\mu(\Theta)$ in a stationary equilibrium.

[28]For comparison, computing non-stationary equilibria in the canonical neoclassical growth model requires finding the complete time path of a single continuous state variable, the capital stock K_t.

rely on the future predicted time paths of Q_t, m_t, and $\mu_t(\Theta)$. These endogenous objects, in turn, depend on the optimal innovation and business creation decisions at every point in time. Their joint determination is what pins down the unique transition path towards the stationary equilibrium implied by the parameter values.

Of particular interest are the properties of non-stationary equilibria that initially start from a stationary equilibrium, and arise as a result of shocking the values of a subset of parameters. In such non-stationary equilibria, the speed at which important variables converge to their new stationary equilibrium values is important. In our quantitative experiments, we observe that firm value functions and innovation policies are quick to respond, in many cases jumping to the immediate vicinity of their long-run values. The state variables m_t and $\mu_t(\Theta)$ are the slow-moving objects. As discussed earlier, the growth rate g_t of the economy given in equation (34) depends both on leader innovation rates $p_{lit}(\Theta)$, and the distribution of industries across industry states $\mu_t(\Theta)$. The prior reacts immediately, but the latter adjusts slower, since endogenous changes to the market structure take time. As a consequence, the growth rate of the economy can jump to a new level in response to shocks in parameter values, but it can take a while to reach its eventual stationary value.[29]

2.4 Computing Social Welfare

To calculate welfare, we need to compute the consumption stream of the representative household. In a non-stationary equilibrium, the time path of consumption needs to be directly calculated.[30] In a balanced growth path equilibrium, two components are sufficient: the growth rate of consumption g, and the initial consumption level C_0. This, in turn, requires us to compute initial output Y_0 and aggregate spending on R&D and new business creation. The level of initial output Y_0 is given by:

$$\ln(Y_0) = \int_0^1 \ln q_{j0}^{\text{leader}}\, dj + \ln\zeta - \ln\omega + \sum f(\Theta)\mu(\Theta) \tag{36}$$

All terms are time-invariant except for the average log productivity level of the industry leaders at time 0, given by $Q_0 = \int_0^1 \ln q_{j0}^{\text{leader}} dj$. When comparing welfare across economies, we can fix this term to be equal to zero in all economies without loss of generality.[31] Next, initial consumption C_0 is given by

$$C_0 = Y_0 \frac{C_0}{Y_0} = Y_0 \left(1 - \int_0^1 \sum_{i=1}^{N_{j0}} \chi z_{ij0}^{\phi}\, dj - \int_0^1 m_0 \nu X_{kj0}^{\epsilon}\, dj - \psi e_0^2 \right) \tag{37}$$

where the second factor is the share of output left for consumption after R&D and entrepreneur entry costs are subtracted. Then, the welfare of the representative household in a BGP equilibrium can be calculated as:

$$W = \int_0^\infty e^{-\rho t} ln(C_t)dt = \frac{ln(C_0)}{\rho} + \frac{g}{\rho^2} \tag{38}$$

[29]The first and the third terms in equation (34) are also slow-moving, but their quantitative significance is dwarfed by that of the second term by several orders of magnitude.

[30]See Section A.9.1 in the Appendix for details.

[31]In other words, we keep the initial frontier technology level the same across counterfactual economies.

The model allows for a closed-form decomposition of changes in welfare across two economies as follows:

$$\Delta W = \frac{1}{\rho}\left[\Delta \ln \zeta - \Delta \ln \omega + \Delta \sum f(\Theta)\mu(\Theta) + \Delta \ln\left(\frac{C}{Y}\right)\right] + \frac{1}{\rho^2}\Delta g \tag{39}$$

For two economies A and B, we can define a consumption equivalent welfare measure (ϖ) which corresponds to the percentage increase in lifetime consumption that an agent in economy A would need to be indifferent between being in economy A and B:

$$W_B = \frac{\ln(C_0^A(1+\varpi))}{\rho} + \frac{g^A}{\rho^2} \tag{40}$$

Solving for ϖ, we get:

$$\varpi = \exp\left(\left(W_B - \frac{g^A}{\rho^2}\right)\rho - \ln(C_0^A)\right) - 1 \tag{41}$$

2.5 Social Planner's Problem

Our model features several inefficiencies that interact, such as markup dispersion, R&D externalities, and business stealing. Consequently, decentralized equilibria are generically suboptimal. One important question is if the static inefficiencies due to oligopolistic competition and positive markups are quantitatively more or less important than the dynamic inefficiencies due to the positive knowledge spillovers and negative business stealing effects inherent in innovation decisions. This is a quantitative question that necessitates solving the dynamic social planner's problem, so that we can compare decentralized equilibria with allocations that shut down static and/or dynamic inefficiencies in the economy.

The social planner's problem can be solved in two parts. First, for a given distribution over industry states, the social planner chooses the optimal labor allocation that maximizes output: i.e., the allocation that would be obtained in the absence of market power and heterogeneous markups in equilibrium. The second part of the planner's problem consists of optimally choosing dynamic innovation allocations for all small firms and superstars in every industry state, and the new business creation rate for entrepreneurs. In doing so, the planner also chooses the implied stationary distribution over industry states $\mu(\Theta)$ and the stationary value of the mass of small firms m, while taking knowledge externalities into account. Since innovation and new business creation both use the final good in the economy, there is also a trade-off between how much resources to spend on improving productivity growth versus current consumption.

In Section A.5 of the Appendix, we provide a detailed description of the social planner's problem as well as its solution. We simplify this complex problem substantially, but the last step still requires solving for the optimal values of a large vector of positive scalars. Nevertheless, solving the social planner's problem is still feasible. We solve and discuss the planner's allocation for our estimated model in Section 4.4.

3 Data and Estimation

In this section, we start by discussing the relationship between competition and innovation in the data and our model. Next, we estimate our model using moments for the whole sample going from 1976 to 2004. We describe our estimation procedure, and use this estimated model to highlight its properties in Section 4. We also split our full sample into two periods (referred to as early and late periods) by choosing the end of 1994 as the mid-point, and repeat the same estimation using these two subsamples. The two estimated stationary equilibria are used in the quantitative application of our model in Section 5 which attempts to disentangle the economic mechanisms underlying the various macroeconomic trends observed in the United States during this time period.

3.1 Relationship Between Competition and Innovation in the Data

Theoretically, the relationship between competition and innovation is ambiguous. Intense competition from peers can encourage a firm to innovate to escape competition, which would imply a positive relationship. At the same time, competition pushes down profitability, which discourages innovation – the so-called Schumpeterian effect.

Our theoretical framework allows the relationship between competition and innovation to be non-monotonic within and across industries. Our model can generate a nonlinear relationship between market concentration and industry innovation, as well as market share and firm innovation without any ex-ante heterogeneity.

Thanks to this new feature of our framework, we can require our model to replicate the empirically-observed relationship between competition and innovation, which helps us discipline the counterfactual model implications for innovation, economic growth, and welfare. In other words, using our framework, we can replicate the whole nonlinear relationship between competition and innovation within and across industries, which creates a much tighter link between the model and the data compared to only matching the aggregate growth rate and R&D intensity.

We conduct an empirical analysis to investigate the aforementioned non-linear relationship between competition and innovation within and across industries.[32] We use the USPTO Utility Patent Grant Data obtained from the NBER Patent Database Project, which covers the years 1976-2006, and rely on Compustat North American Fundamentals Annual for financial statement information of US-listed firms for the same years. Following a dynamic assignment procedure, we link the two data sets. We relegate the details to Section A.3 in the appendix. The stylized facts can be summarized as follows:

1. **Market concentration and industry innovation:** As previously documented in Aghion, Bloom, Blundell, Griffith, and Howitt (2005), the empirically-observed relationship between industry-level innovation and competition can exhibit an inverted-U shape. We confirm this finding using our data, where we document a robust inverted-U shape relationship between market concentration and several different innovation metrics.

[32] We should stress that we do not claim that a causal relationship exists. This exercise documents correlation patterns that we would like our model to be able to reproduce. Market concentration and innovation are both endogenous variables in our model as well.

2. **Relative sales and firm innovation:** We also investigate the within-industry relationship between competition and innovation. We document a robust inverted-U relationship between firm-level innovation and relative sales. This is consistent with other studies such as Hashmi and Biesebroeck (2016).

To establish the robustness of our results, we use several different metrics to capture innovation (patent count, patent quality, tail innovations, originality, generality, R&D expenses) as well as investments that are potentially correlated with innovation (physical capital investment, advertising), and direct measures of firm growth (sales growth, employment growth, asset growth). We confirm the robustness of our results with all the variables across numerous specifications outlined in the appendix. We also conduct a hypothesis test developed by Lind and Mehlum (2010) to test for the existence of an inverted U, which is summarized in Section A.3.4.

3.2 Estimation

Ten parameter values must be determined: $\rho, \lambda, \eta, \chi, \nu, \zeta, \phi, \epsilon, \psi, \tau$. The consumer discount rate ρ is set to 0.04, which implies a real interest rate of 6% when the growth rate is 2%.[33] The rest are structurally estimated following a simulated method of moments approach.[34] Aggregate moments in the model are mapped to aggregate moments in the data. Since the Compustat database consists of large US-listed firms, we map the moments obtained using the Compustat sample to moments obtained from superstar firms in our model. In this section, we discuss the data moments we use to discipline the parameter values, and provide the relevant data sources for each of these moments.[35]

The success of the SMM estimation depends on model identification, which requires that we choose moments that are sensitive to variations in the structural parameters. All these parameters are jointly estimated to match the following targeted data moments: the growth rate of real GDP per capita, aggregate R&D intensity, labor share, firm entry rate, (sales-weighted) average markup,[36] within-year standard deviation of markups, the linear term and the top point obtained from the intra-industry regression of a firm's innovation on its relative sales,[37] average profitability, average relative quality of the leader, and its standard deviation across industries. We discuss them below:

1. **Growth rate:** To discipline output growth in our model, we obtain the annual growth rate of real GDP per capita from the US Bureau of Economic Analysis, and calculate the geometric averages for each sub-sample.

[33] We target a relatively high real interest rate to remain conservative. For instance, a lower real interest rate of 4% would halve the implied discount rate to $\rho = 0.02$. This would double the welfare contribution of the output growth rate relative to that from the initial consumption level, significantly amplify the dynamic welfare gains, and further strengthen our findings.

[34] See Section A.4.2 in the Online Appendix for the objective function (equation (46)) and the algorithm used for estimation.

[35] A more detailed description of the data moments and the estimation procedure is provided in Appendix A.4. In particular, we clarify the details regarding each moment, as well as the data sources the moments are obtained from, in Section A.4.1. In Section A.4.2, we discuss which moments help identify which parameters.

[36] In Section 7.5, we re-estimate the model using cost-weighted markups from Edmond, Midrigan, and Xu (2023), and the results are found to be similar. Motivated by Bond, Hashemi, Kaplan, and Zoch (2021), we also conduct another re-estimation that does not rely on any markup-based moments obtained through the De Loecker and Warzynski (2012) methodology. This strengthens our results.

[37] Matching the documented inverted U relationship is crucial. In the Revision Appendix, we illustrate that failing to capture this relationship (for instance, by producing a U-shaped curve instead of an inverted U) can reverse the growth and welfare results in the counterfactual experiments.

2. **Labor share:** We obtain the labor share estimates from Karabarbounis and Neiman (2013); in particular the time series for corporate labor share (OECD and UN). For capital share, we rely on the data from Barkai (2020).

3. **R&D intensity:** The data for aggregate R&D intensity is taken from the National Science Foundation, who report total R&D expenditures divided by GDP.

4. **Level and dispersion of markups:** To discipline markups, we target the sales-weighted average markup and the sales-weighted standard deviation of markups found in De Loecker, Eeckhout, and Unger (2020).

5. **Relationship between firm innovation and relative sales:** As discussed earlier, replicating the observed inverted-U relationship between competition and innovation helps us firmly discipline the counterfactual implications of the model regarding economic growth and social welfare. To achieve this, we target the relationship between firm innovation and relative sales. Innovation in the model is measured as the Poisson arrival event of quality improvement, whereas it is measured as average patent citations for each firm in the data. We normalize both by subtracting their means and dividing by their standard deviation.

6. **Average profitability:** In the model, average profitability is calculated as static profit flow minus R&D expenses divided by sales. In the data, it is defined as operating income before depreciation divided by sales (OIBDP/SALE in Compustat.)

7. **Level and dispersion of leader quality:** We target the average relative quality of the leader in an industry, and its standard deviation across all industries. In the model, quality is known. In the data, we proxy quality by calculating the stock of past patent citations.

8. **Firm entry:** In our model, firm entry rate is defined as the entry rate of new small firms. We obtain the data counterpart – the entry rate of new businesses – from the Business Dynamics Statistics (BDS) database compiled by the US Census.

Panel A of Table 1 reports the values of the parameters, whereas Panel B provides an overview of the values of the targeted moments in the data and the estimated model. The model tightly matches the eleven data moments. The Jacobian matrix of the model moments with respect to the model parameters in percentage terms is displayed in Table A3.

4 Model Properties

In this section, we use our estimated model for the whole sample to discuss its properties, and how it relates to the typical Schumpeterian framework in the literature. We discuss how our model can generate the empirically-observed inter-industry inverted-U relationship between industry innovation and market concentration (untargeted), and the within-industry inverted-U relationship between firm innovation and relative sales (targeted). We also discuss the new features of our model that deliver empirically-consistent firm and industry dynamics, firm life cycles, industry-level market share distributions, and market concentration.

TABLE 1: BASELINE MODEL PARAMETERS AND TARGET MOMENTS

A. Parameter estimates

Parameter	*Description*	*Whole sample*	*Early sub-sample*	*Late sub-sample*
λ	innovation step size	0.3126	0.3006	0.3255
η	elasticity within industry	6.6800	19.9413	6.6290
χ	superstar cost scale	120.5659	107.9854	72.7328
ν	small firm cost scale	3.4046	1.3946	2.4152
ζ	competitive fringe ratio	0.5912	0.6054	0.5306
ϕ	superstar cost convexity	3.8711	3.8367	3.6366
ϵ	small firm cost convexity	2.6594	2.8329	2.3525
τ	exit rate	0.1151	0.1144	0.0964
ψ	entry cost scale	0.0149	0.0088	0.0213

B. Moments

	Whole sample		Early sub-sample		Late sub-sample	
Target moments	Data	Model	Data	Model	Data	Model
growth rate	2.20%	2.20%	2.19%	2.19%	2.31%	2.31%
R&D intensity	2.43%	2.02%	2.40%	2.07%	2.50%	2.50%
average markup	1.3498	1.3462	1.3014	1.3014	1.4442	1.4441
std. dev. markup	0.346	0.387	0.306	0.325	0.421	0.452
labor share	0.652	0.628	0.656	0.628	0.644	0.610
firm entry rate	0.115	0.115	0.114	0.114	0.096	0.096
β(innovation, relative sales)	0.629	0.726	0.449	0.683	0.631	0.783
top point (intra-industry)	0.505	0.448	0.443	0.462	0.515	0.448
average profitability	0.144	0.176	0.136	0.162	0.152	0.210
average leader relative quality	0.749	0.642	0.751	0.607	0.746	0.678
std. dev. leader relative quality	0.223	0.161	0.224	0.140	0.222	0.165

Notes: The estimation is done with the simulated method of moments. Panel A reports the estimated parameters. Panel B reports the simulated and actual moments.

4.1 Policy Functions

Figure 1 displays the optimal innovation policy functions followed by the firms in the estimated equilibrium obtained by targeting data moments for the whole sample. The left panel of Figure 1 depicts the innovation policy of a firm in an industry with two superstar firms. The relevant state variable is how many steps the current firm is ahead of its competitor, where negative numbers indicate that the current firm is lagging behind the competing firm. We see that the incentive to innovate is increasing from -5 to -1, and it is decreasing from -1 to 5. This means that a two-firm industry experiences the highest amount of superstar innovation when the two competing firms are very close to each other in terms of productivity (neck-and-neck and one step difference.)

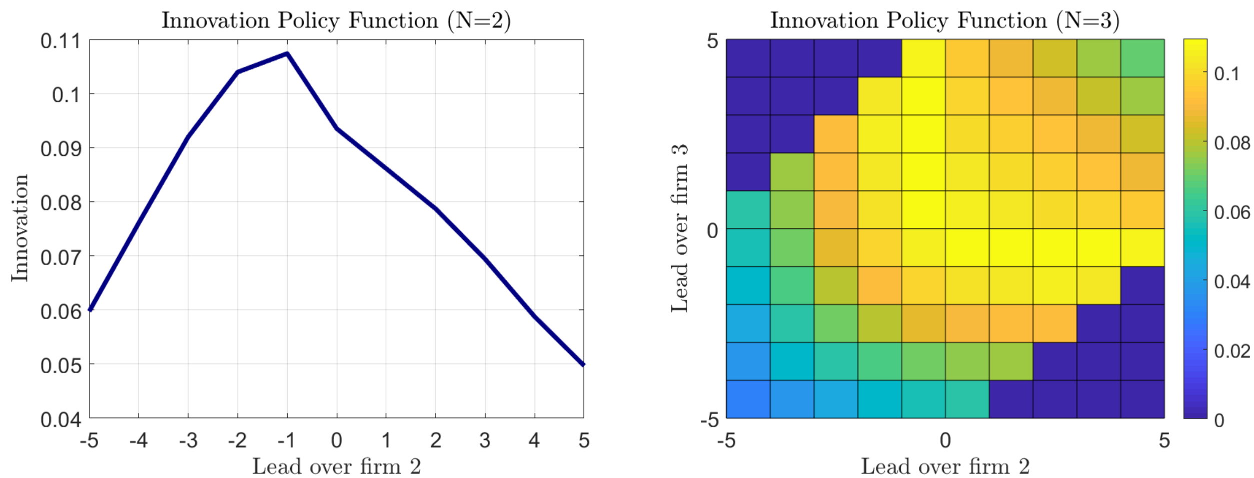

FIGURE 1: INNOVATION POLICY FUNCTION

Notes: This figure displays the optimal innovation policy functions followed by the firms in the estimated equilibrium obtained by targeting data moments for the whole sample. The left panel of this figure depicts the innovation policy of a firm in an industry with two superstar firms ($N = 2$). The right panel of this figure does the same for a firm in an industry with three superstar firms ($N = 3$). The lead is defined as the number of steps the current firm is ahead of its competitor, where negative numbers indicate that the current firm is lagging behind the competing firm.

The right panel of Figure 1 does the same for a firm in an industry with three superstar firms. This time the state variable is two-dimensional, and the innovation decision of the firm is a function of how many steps the current firm is ahead of both of its competitors. Since competitors that have the same relative distance in terms of quality are identical, the two-dimensional surface is symmetric along the antidiagonal. There are also some illegal states which cannot happen as we assume the maximum number of steps between two firms cannot exceed $\bar{n}$. These correspond to the two blue triangles at the top left and bottom right corner, and should be ignored. Overall, the policy function shares some properties with the case for a two-firm industry. In particular, along the antidiagonal where the two competitors have the same quality, the innovation policy is once again increasing until the firms are close to being neck-and-neck, and then decreasing.

Figure B1 in the Appendix is the case for a firm in an industry with four superstars. In this case, the state variable is three-dimensional. Therefore, we split the innovation policy function into ten separate subfigures constructed similar to the right panel of Figure 1, where each subfigure corresponds to the fourth competitor being a certain number of steps behind the current firm. Note that there are more illegal cases that arise whenever $n_4 \neq 0$ which are once again colored blue. Similar properties along the antidiagonal are observed.

4.2 Model-Implied Relationship Between Innovation and Competition

One key feature of our model compared to other models with endogenous markups is its ability to deliver a hump-shaped relationship between innovation and competition at the industry level, which is shown in the left panel of Figure 2.[38] Each marker on the plot corresponds to the innovation choice of a firm given a firm state. The horizontal axis depicts the relative sales of the firm compared to the total sales of all superstars in the same industry. The legend for the figure clarifies the number of superstar firms in the industry the observation is coming from. The blue curve is the quadratic fit to the observations. We see the observations constitute an inverted-U shape both when they are all considered at the same time, and also separately conditioning on a certain number of superstar firms. The innovation choices are normalized by demeaning and dividing by the standard deviation. Each observation is assigned weights based on their occurrence dictated by the time-invariant distribution of industry types $\mu(\Theta)$. Likewise, the right panel of Figure 2 depicts the same for R&D spending on relative sales. The overall shape looks similar, but the differences are magnified as the innovation level increases, owing to the convexity of the superstar innovation cost function.

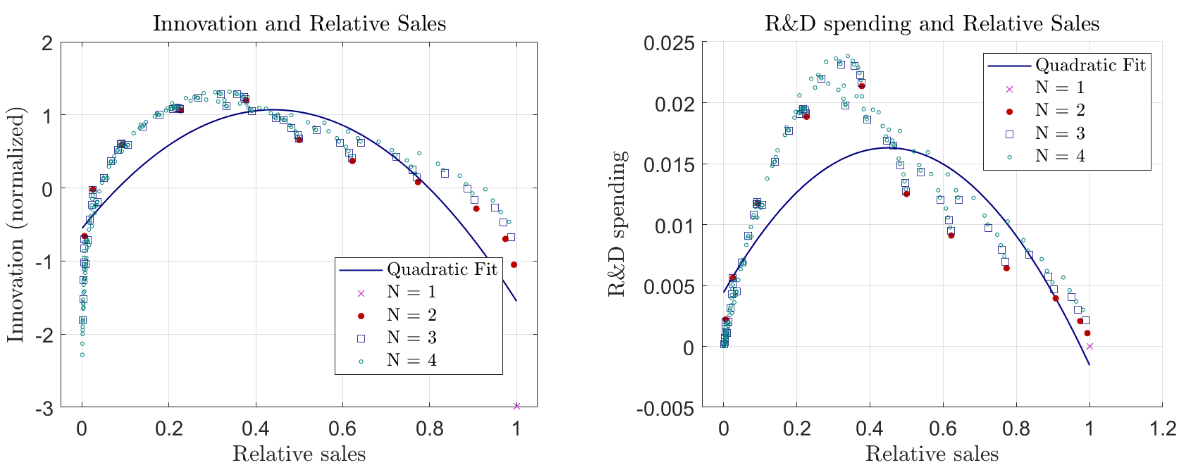

FIGURE 2: INNOVATION, R&D EXPENSES, AND FIRM RELATIVE SALES

Notes: The left panel of this figure illustrates the model-implied relationship between firms' innovation and relative sales, while the right panel shows the model-implied relationship between R&D spending and relative sales. The figure is based on the whole sample. The horizontal axis depicts the relative sales of the firm compared to the total sales of all superstars in the same industry. Each marker on the plot represents the innovation choice (left panel) or R&D spending choice (right panel) of a firm for a specific firm state. The innovation choices are normalized by demeaning and dividing by the standard deviation. N denotes the number of superstar firms in the industry the observation is coming from. The blue curve is the quadratic fit to the observations.

Figure 3 shows the model-implied relationship between competition and innovation across industries. We use the Herfindahl-Hirschman Index to measure market concentration.[39] The figure shows that there is an inverted-U relationship between an industry's market concentration and its total innovation. The vertical blue line corresponds to the top point of the quadratic polynomial fitted to the model-generated

[38] It is worth noting that the model is flexible enough to generate the opposite relationship under alternative parameter values, i.e., a U-shape for the policy function, as well as a U-shape relationship between innovation and competition within and across industries. Therefore, the inverted-U relationship is an estimation result rather than a model implication.

[39] Note that the market share percentage of each individual small firm in the competitive fringe is zero.

data, whereas the vertical red line corresponds to the average HHI of industries in the economy, weighted by their shares in the time-invariant industry state distribution $\mu(\Theta)$. Note that this average is lower than the top point. This reveals that the Schumpeterian creative destruction effect dominates the escape competition effect for most industries in the estimated economy. This foreshadows our finding that a change that increases market concentration would increase innovation overall. We would expect the opposite to be true if the red line was to the right of the blue line.

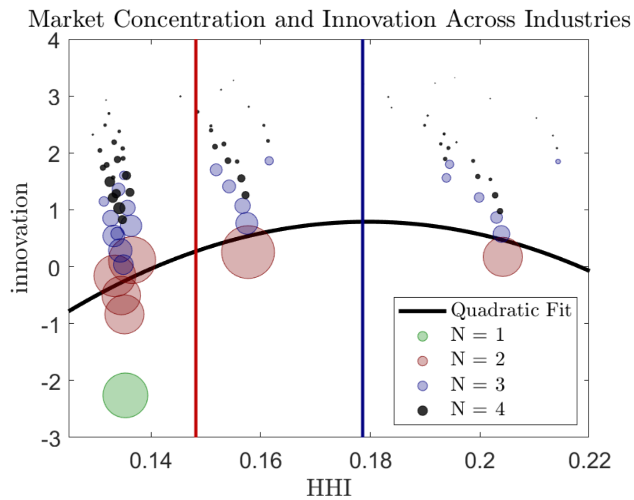

FIGURE 3: INNOVATION AND HHI

Notes: This figure illustrates the model-implied relationship between competition and innovation across industries. The figure is based on the whole sample. We measure market concentration using the Herfindahl-Hirschman Index. The figure reveals an inverted-U relationship between an industry's market concentration and its overall innovation. Each circle represents an industry state. The circle's color indicates the number of superstars in the industry, while its size signifies the share of that industry state in the baseline invariant distribution. N denotes the number of superstar firms in the industry the observation is coming from. The black curve provides a quadratic fit to the observations. The vertical blue line marks the top point of the quadratic polynomial fitted to the model-generated data. In contrast, the vertical red line corresponds to the average HHI of industries in the economy, weighted by their shares in the time-invariant industry state distribution.

It is worth mentioning that our model is able to generate this second relationship without explicitly targeting the quadratic relationship in the estimation. Furthermore, our model generates the inverted U without introducing any exogenous heterogeneity.[40] This is different from recent models that endogenize markups. In Schumpeterian models with a single active firm, all observations would be clustered at 1 for relative sales and HHI, implying no relationship. In step-by-step innovation models with perfect substitution and Bertrand competition, there would be two points at 0.5 and 1 for relative sales, and 0.5 and 1 for HHI, which would imply a linear relationship. In models with exogenous productivity evolution, we would get a flat line (no relationship). Finally, in models with innovation only at entry, there would be a monotone relationship between competition and innovation at entry, the shape of which depends on that of the cost function. After entry, there would be no relationship due to no incumbent innovation. Our model's ability to match this relationship aids in disciplining the counterfactual behavior of aggregate productivity growth in response to endogenous changes in market concentration, which helps discipline the estimated social welfare implications of increasing markups.

[40] The original inverted-U paper by Aghion, Bloom, Blundell, Griffith, and Howitt (2005) achieves this through the introduction of exogenous collusion heterogeneity across industries.

4.3 Model-Implied Market Share Distributions and Firm Dynamics

Though not targeted, our model generates realistic relative sales and concentration numbers unlike Schumpeterian models with Bertrand competition and homogeneous goods. Industries in the United States consist of thousands of firms on average. While superstar firms command a disproportionate share of the market, the remaining small firms still collectively account for a significant portion of the market. Existing Schumpeterian models feature at most two firms per industry and hence generate CR4 ratios of 100% in every industry. Our model avoids this issue through two of our contributions: (1) allowing an endogenous number of superstar firms and (2) introducing a competitive fringe that can collectively account for a large market share in each industry, even though each fringe firm is infinitesimally small. Even in industries with a single superstar firm, the competitive fringe ensures that the superstar captures only a fraction of the market. Our estimated model generates an average CR4 ratio across industries of 46.7%, with a realistic distribution across industries – 37.6% for the 25^{th} percentile and 54.6% for the 75^{th} percentile.[41]

In addition, the employment and sales growth profiles of individual firms in our model evolve slowly over time, which fits the data better than the instantaneous jumps between 0%, 50%, and 100% of the whole market observed in many models. Such abrupt changes in market shares are counterfactual. Our model delivers realistic firm life cycles in comparison. Entrepreneurs create thousands of new small businesses. Only a very small fraction of these small firms eventually succeed in becoming a superstar firm. Those that become a superstar firm start out $\bar{n}$ steps behind the industry leader in terms of productivity, making them the smallest superstar in the industry. As time passes, if they succeed in their innovation efforts, they can rise up in the ranking slowly over time. Their market share tracks their productivity compared to their competitors, and it also changes only gradually as a result of their own innovation, or that of their competitors. Figure 2 gives an indication of the large degree of market share heterogeneity that our model can deliver. Finally, the introduction of entrepreneurs in the model also adds to the realistic life cycle of firms and allows our model to generate an entry rate of small firms that is directly comparable to the way entry rate is usually measured using BDS data.

4.4 Social Planner's Allocations

In Section 2.5, we had discussed the social planner's static and dynamic problems, and how they could be solved. In this section, we apply the developed solution methods to solve for the social planner's static and dynamic allocations using the parameter values obtained from the whole sample estimation of our model.[42]

The static problem involves finding the allocation of labor that maximizes (log) output in the economy, given the productivity distribution $[\{q_{ijt}\}_{i=1}^{N_{jt}}, q_{cjt}]_{j=0}^{1}$. We take the initial productivities implied by the stationary industry-state distribution $\mu(\Theta)$ in the decentralized equilibrium, and solve this problem. The social planner simply eliminates all markups observed in the decentralized equilibrium, which increases initial output by 25.4% of its value. This large number is comparable to those reported in Baqaee and Farhi (2020) and Edmond, Midrigan, and Xu (2023).

[41]These number match very well against the empirical CR4 estimates we obtain by combining Compustat data with industry sales information from the Bureau of Economic Analysis. See Table L22 in the Revision Appendix for the exact figures.

[42]A full description of the results can be found in Appendix A.5.4.

We then solve the (unconstrained) dynamic social planner's problem, again using the same parameter values. This time, the social planner not only eliminates all markups, but also chooses the small firm entry rate, as well as every innovation decision by every firm in every industry simultaneously, taking all the externalities into account, and cognizant of their effect on the stationary mass of small firms m and the industry state distribution $\mu(\Theta)$. The optimal allocation yields a growth rate of 5.6% per year, which is more than double what is observed in the decentralized equilibrium. The consumption-equivalent welfare gain obtained from switching to the first-best is found to be 115%, which is four times that of the gains from eliminating static misallocation of labor due to markups. This means the dynamic inefficiency in the estimated economy is much more severe than the static efficiency due to oligopolistic competition.

Finally, we study a constrained optimal allocation, where the planner cannot choose positive R&D for large firms in single-superstar industries. In a decentralized equilibrium, single superstars do not perform R&D as they have no incentive to do so. As a result, implementing the first-best allocation with taxes and subsidies would require a 100% subsidy of R&D by large firms in single-superstar industries. Our constraint is motivated by the severe implementation problems this would entail. Our results show this constrained optimal allocation delivers a very large share of the welfare difference between the unconstrained first-best and the decentralized equilibrium allocations, at 97.6%.

5 Application: Understanding the Structural Transition and the Rise in Markups

As a quantitative application of our framework, we rely on the early (1976-1994) and late (1995-2005) subsample estimations conducted in Section 3, and use the two estimated stationary equilibria to disentangle the economic mechanisms underlying the various macroeconomic trends observed in the United States during this time period.[43] The estimated parameter values in the two sub-samples are quite different, which captures the structural changes in the US economy throughout this time period. These estimates are reported in Table 1.

5.1 Changes in Structural Parameters

The elasticity of substitution within an industry, η, decreases from 19.94 to 6.63.[44] This parameter primarily governs the degree of static product market competition between superstar firms, and the significant decrease in its value indicates that superstar firms enjoy higher market power in the late sub-sample. This is in line with the documented increase in market power over the last decades (see for instance, De Loecker, Eeckhout, and Unger (2020)). At the same time, the ratio of the competitive fringe's productivity relative to the industry leader, ζ, decreases from 0.605 to 0.531. This parameter captures the product market competition from non-superstar firms, and a decrease in its value indicates that superstar firms can charge higher markups thanks to reduced competition. One potential factor behind this trend

[43] We extend the analysis to 2006-2016 in Section 7.1.

[44] In our model, the industry structure is endogenous, and therefore η is primarily identified by the dispersion of firm-level markups rather than the average, along with average profitability. In monopolistic competition models, the reported change in the value of η would imply an 11.9% increase in the average markup, whereas the change is only 0.24% in our model, as shown in Section 5.2.1.

could be the slowdown in technology diffusion from market leaders to followers as discussed in Akcigit and Ates (2023).

If we turn to parameters that govern the innovation cost function of the superstars, we observe a decrease in the scale parameter χ from 107.99 to 72.73, and the convexity ϕ from 3.84 to 3.64. The first change reduces the cost of superstar innovation, whereas the second change allows innovation to be more concentrated across firms, as it reduces the diminishing returns to innovation within a firm. Given other parameter values, the decrease in ϕ also increases innovation costs on average. These changes in χ and ϕ imply that the cost of a given level of innovation z is higher in the late period if $z < 0.139$, which is the case for all firms in the model. The innovation scale parameter for small firm innovation, ν, increases from 1.39 to 2.42, and the convexity ϵ decreases from 2.83 to 2.35. Both changes increase the cost of innovation for small firms, which reduces the entry rate of new superstar firms. Overall, these results are consistent with ideas getting harder to find over time as highlighted in Bloom, Jones, Van Reenen, and Webb (2020). Finally, the cost of new business entry rises as ψ increases from 0.88% to 2.13%. This rise in the entry cost of entrepreneurs and the associated decline in the entry rate of small firms are in line with the overall decline in business dynamism documented, for instance, in Decker, Haltiwanger, Jarmin, and Miranda (2016).

5.2 Disentangling the Structural Transition

In order to better understand and disentangle the effects of the transition implied by the estimated parameter values in the two sub-samples, we conduct counterfactual exercises where we investigate the effects of each change separately. To do so, we compare the estimated late sub-sample economy against counterfactual economies in which we set the values of selected parameters to their early sub-sample estimates. These exercises illustrate how the economy would look like if there were no structural change in a particular mechanism. This, in turn, allows us to understand which mechanisms are the primary drivers of the time trends in important quantities such as the average markup, the labor share, output growth, as well as their implications for social welfare. In other words, we use our model as a tool to uncover how the structural parameters change over time, and establish which mechanisms induce the observed macroeconomic changes, in what magnitude, and in which direction.

The results are presented in Table 2, where the first column displays the benchmark values of chosen model moments in the late sub-sample. The last two columns of the bottom panel show the results of changing all the parameters to their early period values to provide context. All the remaining columns show how the moments change in each exercise. Below, we discuss these results in detail.

5.2.1 Competition from Superstars vs. Small Firms

We first look at what happens when we set the elasticity of substitution within an industry, η, to its early period value. Even though the direct effect of the change is a decrease in the market power of the superstar firms, average markup increases slightly by 0.24% of its value, due to the general-equilibrium changes in the stationary distribution $\mu(\Theta)$ towards higher markup industries. The reduction in market power relatively favors the industry leaders who have the highest productivity. Consequently, the average number of superstars per industry falls by 10.3%, and so does the initial output by 7.5%. There is a drop

TABLE 2: DISENTANGLING THE STRUCTURAL TRANSITION

	Benchmark	Early η	% change	Early ζ	% change	Early ν, ϵ	% change
growth rate	2.31%	2.07%	-10.39%	1.83%	-20.99%	2.74%	18.28%
R&D intensity	2.50%	2.37%	-5.07%	1.55%	-37.68%	3.32%	33.02%
average markup	1.444	1.448	0.24%	1.320	-8.56%	1.450	0.39%
std. dev. markup	0.452	0.425	-6.01%	0.381	-15.74%	0.438	-3.25%
labor share	0.610	0.604	-1.06%	0.653	6.98%	0.605	-0.92%
entry rate	0.096	0.096	0.00%	0.096	0.00%	0.096	0.00%
β(innov, relative sales)	0.783	0.706	-9.94%	0.795	1.42%	0.838	7.01%
top point (intra-industry)	0.448	0.435	-2.91%	0.443	-1.01%	0.466	4.16%
avg. profitability	0.210	0.219	4.35%	0.166	-21.04%	0.209	-0.49%
avg. leader relative quality	0.678	0.720	6.18%	0.728	7.42%	0.569	-16.02%
std. dev. leader rel. quality	0.165	0.176	6.78%	0.181	9.40%	0.130	-21.06%
superstar innovation	0.169	0.145	-13.86%	0.129	-23.48%	0.230	36.16%
small firm innovation	0.019	0.011	-41.25%	0.011	-41.58%	0.052	175.60%
output share of superstars	0.516	0.549	6.25%	0.429	-16.89%	0.547	6.04%
avg. superstars per industry	2.090	1.874	-10.33%	1.868	-10.59%	2.819	34.89%
mass of small firms	1.000	0.719	-28.13%	0.667	-33.28%	1.387	38.71%
initial output	0.793	0.733	-7.49%	0.819	3.32%	0.811	2.23%
CE Welfare change		-12.76%		-7.60%		12.65%	

	Benchmark	Early χ, ϕ	% change	Early ψ, τ	% change	All	% change
growth rate	2.31%	2.39%	3.14%	2.46%	6.37%	2.19%	-5.40%
R&D intensity	2.50%	2.47%	-0.97%	2.72%	9.05%	2.07%	-17.01%
average markup	1.444	1.444	0.02%	1.446	0.14%	1.301	-9.88%
std. dev. markup	0.452	0.451	-0.31%	0.448	-0.91%	0.325	-28.06%
labor share	0.610	0.610	-0.07%	0.609	-0.29%	0.628	2.87%
entry rate	0.096	0.096	0.00%	0.114	18.65%	0.114	18.65%
β(innov, relative sales)	0.783	0.748	-4.55%	0.775	-1.06%	0.683	-12.82%
top point (intra-industry)	0.448	0.447	-0.28%	0.452	0.98%	0.462	3.17%
avg. profitability	0.210	0.211	0.40%	0.210	0.01%	0.162	-22.74%
avg. leader relative quality	0.678	0.668	-1.48%	0.649	-4.31%	0.607	-10.44%
std. dev. leader rel. quality	0.165	0.164	-0.70%	0.154	-6.72%	0.140	-15.27%
superstar innovation	0.169	0.177	4.85%	0.184	9.24%	0.180	6.75%
small firm innovation	0.019	0.021	11.64%	0.024	27.35%	0.028	46.31%
output share of superstars	0.516	0.519	0.55%	0.525	1.74%	0.483	-6.46%
avg. superstars per industry	2.090	2.149	2.82%	2.239	7.16%	2.412	15.41%
mass of small firms	1.000	1.074	7.38%	1.438	43.84%	1.000	0.00%
initial output	0.793	0.794	0.19%	0.798	0.65%	0.769	-3.03%
CE Welfare change		2.05%		4.18%		-5.59%	

Notes: The table reports the changes in model moments when setting the parameters of interest back to their estimated levels in the early sub-sample while keeping other parameters fixed at their estimated values in the late sub-sample. The benchmark is the late-period stationary equilibrium.

by 13.9% in superstar innovation, whereas small firm innovation drops significantly more by 41.3% of its value. The consequent decrease in the profitability of small firms reduces their mass by 28.1%. As a combined effect of the changes in innovation, there is a significant decrease in the growth rate by 10.39% of its value, which translates into a welfare loss of 12.76% in consumption-equivalent terms.[45]

This experiment reveals that the increase in the estimated market power of superstar firms results in increased economic growth due to better incentives to innovate and the resultant changes in the industrial structure. If the elasticity of substitution had not fallen, there would be less incentive to innovate to become superstar firms, which implies industries being dominated by an even smaller number of superstar firms. These superstar firms, in turn, would not innovate as much, since the "escape competition" effect is weaker – fewer superstar firms means less peer competition. Therefore, the average markup would stay virtually the same, and the static welfare gains would be limited. The dynamic losses from reduced growth overshadow slightly higher markups, resulting in lower welfare.

Next, we look at what happens when we set the competitive fringe's productivity relative to the industry leader, ζ, to its early period value. Unlike η, changes in this parameter capture the competition from small firms. This time, the average markup falls from 1.44 to 1.32. Given that the average markup in the early sample is 1.30, the change in ζ can explain nearly all of the change in the average markup across time. While the reduction in markups improves static efficiency, increasing initial output by 3.32% and the labor share by 6.98%, average profitability falls by 21.0%. Consequently, firms have less incentive to innovate, reducing small firm innovation by 41.6% and superstar innovation by 23.5%. Combined together, this reduces the output growth rate from 2.31% to 1.83%. Despite the gains in static efficiency, social welfare drops by 7.60%.[46]

There are two main takeaways from this experiment. First, our model suggests that the overall increase and the polarization in markups which were observed after 1975 owe mostly to a reduction in competition from small firms, rather than a reduction in competition between superstar firms, or the changing costs of innovation. This is consistent with the previous findings in the literature regarding the decline in business dynamism and "winner-takes-most" dynamics as in Decker, Haltiwanger, Jarmin, and Miranda (2016) and Autor, Dorn, Katz, Patterson, and Van Reenen (2020) among others. The output share of superstars increases because the relative productivity of small firms is lower. Second, our model's implications regarding the social costs of increasing markups are quite different from other papers in the literature which focus on static efficiency gains alone. Our model suggests that the increase in markups can be welfare-improving when the dynamic effects on productivity growth are taken into account. Since this is a key difference of our model, we discuss the static vs. dynamic effects of increasing markups in richer detail in Section 5.2.4.

What could be the reason behind the estimated decline in the relative productivity of small firms? Several potential mechanisms exist, such as increasing globalization (access to international markets and inputs), advantages of the superstars in acquiring and exploiting data, changes in advertising and brand

[45]Note that this welfare number should be interpreted with caution, since the change in η can be considered a change in consumer preferences rather than a change in production technology. Therefore, the drop in initial consumption might not indicate a welfare loss. However, the significant drop in the growth rate still means that a hypothetical consumer with unchanged preferences would be worse off.

[46]While welfare for the representative household drops by 7.60%, we show in Appendix A.6 that these losses are unevenly distributed between hypothetical pure workers and pure capitalists. Capital owners experience a much larger welfare loss from decreased markups than workers.

value, increasing costs of imitation (technological as well as legal), or a decline in knowledge spillovers. Our results highlight a direction for future research, suggesting that the key to understanding the increase in markups lies behind uncovering the factors that led to the decline in the relative productivity of small firms, and their relative contributions.

5.2.2 Costs of Superstar vs. Small Firm Innovation

In our third experiment, we set the parameters that govern the innovation cost function of small firms, ν and ϵ, to their early period values. Unsurprisingly, there is a very significant direct effect on small firm innovation which increases by 175.6% of its value. The average number of superstar firms per industry increases by 34.9%, which is roughly double the total difference between the early and late periods. Average profitability is virtually the same, so the Schumpeterian creative destruction effect does not change. On the other hand, the increased number of superstars boosts the "escape competition" effect, and superstar innovation increases by 36.2% of its value. Hence output growth increases from 2.31% to 2.74%. Coupled with a slight gain in initial output of 2.23%, social welfare improves by 12.65%.

Recall that in our model we define entry into superstars not as newly established firms. Through successful innovation, small firms become superstar firms. Whereas the decrease in the competitive fringe's productivity ζ captures the declining product market competition from small firms, the increase in the cost of small firm innovation reduces the frequency at which small firms can join the ranks of superstar firms. Therefore, our estimates suggest that the number of small firms with high growth potential ("gazelles") has gone down, which allows industries to be dominated by fewer superstar firms. This finding is consistent with the recent evidence by Sterk, Sedláček, and Pugsley (2021), who find that both the share of such "gazelles" among all firms, and their average growth rates have fallen over time after 1975. As the experiment suggests, this leads to a weakening of the incentives to innovate by the remaining superstar firms, which hurts productivity growth.

Our fourth experiment reverts the parameters of the superstar innovation cost function, χ and ϕ, to their early period values. Superstar innovation increases by 4.85% in response. However, this increase is more than matched by a simultaneous increase in small firm innovation by 11.6%. Average profitability and the output share of superstars do not change by much, leading to virtually unchanged average markup and initial output. This means static efficiency remains comparable to the benchmark, and the 3.1% increase in output growth translates to a slight 2.05% increase in welfare.

This experiment relates directly to the literature on the increasing costs of innovation ("ideas are getting harder to find"). As noted in Bloom, Jones, Van Reenen, and Webb (2020), the number of researchers employed in research and development of new ideas has been constantly increasing over the time period. Looking at the determinants of output growth in our model, the reduced competition from superstars and small firms push for higher growth, whereas increased innovation costs offset some of the competition channel, resulting in the observed increase in productivity growth. In this aspect, our findings differ from the popular hypotheses regarding how the decline in competition might have led to a productivity slowdown. There can be many reasons behind the decline in research productivity, ranging from the reduced arrival rate of general purpose technologies (Gordon (2012)) to stronger intellectual property protection (Han (2018)), increased patent litigation (Galasso and Schankerman (2014)), protective patenting (Argente, Baslandze, Hanley, and Moreira (2023)), or increasing misallocation of talent in innovation due to increasing wealth inequality (Celik (2023)). Our results suggest future research directed at identifying the reasons behind the decline in research productivity might hold the key to understanding the most recent decline in productivity growth. Comparing the results of the third and fourth experiments also highlight the disproportionate role the decline in the R&D efficiency of small firms played in slowing down productivity growth. In comparison, the increasing costs of innovation for superstar firms have a significant, but smaller effect.

5.2.3 Entrepreneurship and Firm Entry

In our final experiment, we consider the effects of the decline in the firm entry rate, and the changes to the costs of founding new businesses. To this purpose, we set the scale parameter of the entrepreneur cost function, ψ, and the exit rate, τ, to their early period values. From the early period to the late period, the exit rate τ goes down, and the scale parameter ψ goes up. The prior increases the survival rate of small firms, therefore increasing the expected value obtained from founding a new business. The latter increases the cost of doing so. Therefore, the two changes push the equilibrium mass of small firms in opposite directions. The total effect is an increase of 43.84%. This increase boosts small firm innovation by 27.35%, which also encourages more innovation by superstars at 9.24%. Initial output is virtually the same, and the growth rate increases by 6.37% of its value. The combined welfare effect is calculated as 4.18%.

The increasing cost of entrepreneurship and the decline in firm exit rate act to partially offset each other. If we consider their effects separately, reverting the value of ψ by itself leads to an increase in growth by 9.00% and welfare by 5.98%, whereas repeating the same exercise for τ reduces growth and welfare by 3.81% and 2.40% only, respectively. For average markups and market concentration, the relative productivity of small firms seems to matter much more than their total number. For productivity growth, the changes in costs of innovation dominate the changes in firm entry and the cost of founding new businesses.

5.2.4 Static vs. Dynamic Costs of Higher Markups

The second counterfactual experiment in Section 5.2.1 where the relative productivity of the competitive fringe ζ was reverted to its early period value revealed that increased competition from small firms reduced the average markup to nearly its early period value, yet the dynamic change in welfare was a loss of 7.6%. To better understand why this happens, it is useful to decompose the change in welfare into its constituent parts using equation (39). At the same time, our finding differs significantly from static analyses that focus on the efficiency gains from reduced markups without taking the implications for productivity growth and endogenous industry dynamics into account. This motivates us to also perform the decomposition where we only consider the static changes from reduced markups, and the static changes plus the endogenous change in the distribution of industries, while keeping the innovation policies and the output growth rate the same. The results are presented in Table 3.

The first column of Table 3 calculates and decomposes the change in welfare if we ignore the dynamics completely. The dynamics come into play through three model components: (1) the growth rate, (2) the change in consumption to output ratio due to R&D spending and the cost of new business entry, and (3) the changes in the relative wage rate ω and the production by superstar firms $\sum f(\Theta)\mu(\Theta)$ which both

TABLE 3: STATIC VS. DYNAMIC COSTS OF HIGHER MARKUPS

	Static		Static+New Distribution		Dynamic	
	ΔW	CEWC	ΔW	CEWC	ΔW	CEWC
competitive fringe productivity	3.297	14.10%	3.297	14.10%	3.297	14.10%
relative wage	-1.594	-6.18%	-1.687	-6.53%	-1.687	-6.53%
output of superstar firms	-0.691	-2.73%	-0.794	-3.13%	-0.794	-3.13%
consumption/output	0.000	0.00%	0.000	0.00%	0.243	0.98%
output growth	0.000	0.00%	0.000	0.00%	-3.034	-11.43%
total	1.012	4.13%	0.816	3.32%	-1.975	-7.60%

Notes: The table decomposes the change in welfare into its constituent parts using equation (39).

depend on the distribution of industries $\mu(\Theta)$. To calculate the static effects, we keep the growth rate, R&D spending, new business creation, and $\mu(\Theta)$ at the late period levels. There is a large increase in output and welfare as a direct effect of increasing ζ. Increased production increases labor demand, which pushes the relative wage ω up, the effect of which is negative. Finally, increased production by small firms results in reduced supply from superstar firms, so the third component is negative as well. The direct effect dominates these endogenous (static) responses, and welfare is increased. Column 2 shows the change in consumption-equivalent welfare by each individual component. The combined effect is a significant 4.13% gain in welfare.

The third column repeats the same exercise as in column one with a single difference: we use the distribution of industries $\mu(\Theta)$ implied by the dynamic long-run change in response to the increase in ζ rather than retaining the late sample values. This changes the effects from the relative wage and the output of superstars slightly, and the gain in welfare remains close but lower at 3.32%. This means ignoring the long term effects on industrial structure – i.e., the number and relative qualities of superstars – would result in overestimating the welfare gains by 24%, which is not insignificant.

The fifth column displays the welfare decomposition for the full dynamic response. As alluded to in the second exercise, the difference in the initial output (and hence the static efficiency) is limited. There is a small welfare gain from the consumption to output ratio term due to reduced R&D spending and new business entry. However, there is a very significant decline due to reduced growth that wipes out both the static welfare gains from reduced markups and the gains from higher initial consumption. Therefore, the final tally is a loss of 7.60%, as the dynamic losses from reduced productivity growth valued at 11.43% of consumption completely dominate the 3.32% gain from improved static efficiency. A static model that does not endogenize productivity growth would not be able to obtain this result, and highly overestimate the cost of increased markups in the US during this time period.[47]

6 Model Validation

Beyond its ability to replicate the (untargeted) inverted-U shape relationship between market concentration and innovation (see Figure 3) and to generate realistic market shares, concentration ratios, and firm life cycles, our estimated model also delivers several other predictions that can be tested, especially

[47]Note that our baseline analysis assumes a representative household. In Appendix A.6, we show that the gains from higher markups might be unevenly distributed between different types of agents, e.g., workers and owners of capital.

related to trends in productivity, market concentration, and the labor share. In particular, we show that our model correctly predicts the increase in productivity dispersion documented by Barth, Bryson, Davis, and Freeman (2016) and the negative correlation between productivity dispersion and the labor share across industries highlighted in Gouin-Bonenfant (2022). In addition, our model is in line with several facts related to changes in the labor share documented in Autor, Dorn, Katz, Patterson, and Van Reenen (2020).

6.1 Increase in Productivity Dispersion

Recent studies emphasize the role played by increased between-firm dispersion in explaining trends in income inequality. For instance, Song, Price, Guvenen, Bloom, and Von Wachter (2018) find that two-thirds of the rise in the variance of earnings is due to increased between-firm wage dispersion. In addition, Barth, Bryson, Davis, and Freeman (2016) document a significant rise in the variance of between-firm productivity (log revenue per worker) between 1977 and 2007.[48] In our model, productivity dispersion both within and across industries arises endogenously as the outcome of innovation. In particular, our quantitative exercise allows us to determine how productivity dispersion responds to the structural transition of the economy. Using the same measure as in Barth, Bryson, Davis, and Freeman (2016) (the variance of log revenue per worker), our estimation predicts a rise in productivity dispersion by 39.27% between the early and late subsamples (i.e., from 1976 to 2005) which is very close to the estimated 32.6% increase in Barth, Bryson, Davis, and Freeman (2016) between 1977 and 2007.

6.2 Productivity Dispersion, Value-Added, and the Labor Share

Building on the evidence from Barth, Bryson, Davis, and Freeman (2016), Gouin-Bonenfant (2022) further shows that an exogenous increase in productivity dispersion could explain the observed fall in the labor share in a model with labor market monopsony power. Testing one of the main predictions of his model, he shows, using Canadian data, that the industry-level labor share is negatively correlated with dispersion in productivity. In our model, both labor share and productivity dispersion are endogenously determined. Productivity dispersion within an industry depends on innovation activity whose returns, in turn, shape the productivity distribution. Given the rich structure of our model, we can also compute the correlation between the industry-level labor share and productivity dispersion. Thanks to our oligopolistic structure based on Atkeson and Burstein (2008), our model features a negative association between the industry-level labor share and the dispersion of productivity (TFPQ) in both sub-samples, in line with the findings in Gouin-Bonenfant (2022).

Furthermore, as in Autor, Dorn, Katz, Patterson, and Van Reenen (2020) and Kehrig and Vincent (2021), Gouin-Bonenfant (2022) also documents a negative relationship between (log) firm-level labor share and (log) value-added. We repeat this regression in our model, and obtain a coefficient of -0.0883, which is very close to the value -0.112 that he documents for Canada.

[48] Faggio, Salvanes, and Van Reenen (2010) provide evidence for a similar increase in productivity dispersion in the UK since 1980. Andrews, Criscuolo, and Gal (2016) also highlight an increase in productivity difference between frontier and laggard firm in 24 OECD countries.

6.3 Market Concentration and the Labor Share

Using US firm-level data, Autor, Dorn, Katz, Patterson, and Van Reenen (2020) also highlight how the decrease in the labor share has been associated with several dimensions of the rise in market concentration and, in particular, the rise of superstar firms. First, they show that industry sales increasingly concentrate in a small number of superstar firms. In our model, the market share of superstar firms increases by 6.9% between the early and late sub-samples. This result is comparable to those found in Autor, Dorn, Katz, Patterson, and Van Reenen (2020), if slightly lower, and is closest to the rise in manufacturing. This untargeted change in our model is directly related to the competition between superstars and small firms which we have shown to be decreasing over time and which is the main driver of the observed increase in markups.

Regressing the change in the 4-digit industry labor share on the change in market concentration (measured by the market share of top 4 and top 20 firms as well as the Herfindahl index), they further show that industries in which market concentration rose the most also experienced the sharpest decline in their labor share. Our model delivers the same prediction between our two sub-samples. In particular, focusing on the corresponding two relevant measures of concentration in our model (i.e., top 4 market share and the Herfindahl index), we also find a negative association between the change in market concentration and in the labor share.[49] This negative correlation can be obtained in our model thanks to our oligopolistic market structure based on Atkeson and Burstein (2008). From a quantitative perspective, the estimates from those regressions fall in the ballpark of those reported in Autor, Dorn, Katz, Patterson, and Van Reenen (2020). Our regression based on the top 4 share of sales delivers a coefficient of -0.196 (the estimates in Autor, Dorn, Katz, Patterson, and Van Reenen (2020) range between -0.146 and -0.339). For the Herfindahl index regression, our estimate is -0.531 (between -0.213 and -0.502 in Autor, Dorn, Katz, Patterson, and Van Reenen (2020)). These quantitative results in line with what is observed in the data can be seen as additional external validation of our model.

7 Robustness of Quantitative Results

It is important to test whether our model's sharp growth and welfare predictions are driven by the modeling assumptions or the data used for estimation. In this section, we present and discuss several robustness checks to show that this is not the case.

7.1 Extending the Estimation to 2006-2016

This section extends the analysis to the most recent period going from 2006 to 2016. While this period has some data limitations and covers the Great Recession, our re-estimation and associated counterfactual results confirm the main results from the early and late subsamples.[50] Most of the trends that were apparent during the transition from the early to the late period continue until the most recent decade

[49] In particular, we compute the change in market concentration and in the labor share for industries with the same number of firms and the same productivity step distribution in the early and late subsamples.

[50] We rely on patent citation data from UVA Darden Global Corporate Patent Dataset to construct the innovation-related data moments (Bena, Ferreira, Matos, and Pires (2017)). We exclude the years 2007-2009 when constructing the data moment for the output growth rate to exclude the effect of the Great Recession.

for which we have data. Markups and their dispersion have continued to rise significantly, R&D intensity kept increasing, the labor share experienced a further decrease, and so did the entry rate of new firms (see Table B13 in Appendix B). The main difference compared to the trends between the early and late subsamples relates to productivity growth. While productivity growth increased between the early and late subsamples, the latest period experienced a significant decrease in productivity growth.

The results of our experiments remain unchanged when comparing the early and later subsamples. Innovation costs are still on the rise as ideas keep getting harder to find. Most of the change in markups is attributed to the decrease in the relative productivity of the fringe ζ, and the rise in markups is associated with an increase in welfare by 8.36%, stemming from dynamic gains from the response of firms' innovation to the increase in market concentration, which more than offset the static losses from higher markups. All the welfare effects are now either amplified or remain very similar: -13.56% vs. -12.76% for η, -8.36% vs. -7.60% for ζ, 12.38% vs. 12.65% for small firm R&D efficiency, and 9.59% vs. 2.05% for large firm R&D efficiency. The full set of results from our counterfactual experiments for the period 2006-2016 can be found in Table B14 in Appendix B.

7.2 Non-Stationary Dynamics and the Welfare Costs of the Transition

The welfare results in Section 5.2 are obtained by comparing the actual and hypothetical stationary equilibria. One might be concerned that a comparison across stationary equilibria might be insufficient to capture the full differences in welfare across the separate scenarios, since some effects of the changes are instantaneous (e.g., the effects on static product market competition), whereas the rest take more time to manifest in full (e.g., the changes to the industry-state distribution $\mu_t(\Theta)$.) To capture the welfare differences from the point of view of an agent at $t = 0$, one needs to take the transitional dynamics in non-stationary equilibria into account.

Before we repeat our quantitative experiments, it is worth discussing how fast several economic variables of interest move during the transitions between the early and late period economies, as well as between the late period and the most recent period of 2006-2016. Figures B2 and B3 display the time paths of selected variables for these two transitions, respectively.

As already discussed in Section 2.3, firms' value and innovation policy functions respond immediately by jumping to the immediate vicinity of their new stationary values, with little change over the transition. Likewise, given an industry state Θ, the static industry equilibria respond immediately to the changes in parameters. However, the distribution over industry states $\mu_t(\Theta)$, the mass of small firms m_t, and aggregate moments adjust more slowly. Panel (f) of Figure B2 shows the evolution of the distribution of the number of firms per industry over time, which summarizes the change in $\mu_t(\Theta)$.[51] Average markups and the labor share converge to their new steady state very fast after an initial jump that very slightly overshoots their new long-run values (see Panels (c) and (e) of Figure B2). Firm entry is also relatively slower at converging to its balanced growth path level, experiencing an initial large drop followed by a gradual increase as can be seen in Panel (d) of Figure B2).

Economic growth initially rises significantly above its new long-term value, followed by a slow convergence to its new stationary value, producing a spike in productivity growth, followed by a progressive

[51]We show this summary measure, since $\mu_t(\Theta)$ is an 84-dimensional object under our choices for $\bar{n}$ and $\bar{N}$.

slowdown. R&D intensity follows a similar trajectory (see Panels (a) and (b) of Figure B2). In other words, our model delivers transitional dynamics that are characterized by a temporary surge in productivity growth – as has been observed in the data in the late 1990s and early 2000s – followed by a slowdown. The model is therefore able to replicate this pattern in the data even though we do not target the transition path between the two periods in our estimation.

In the transition between the late and later periods, productivity growth slows down considerably (see Panel (a) of Figure B3) which can be linked to the recent productivity slowdown discussed for instance in Gordon (2012, 2014). R&D intensity slowly rises up towards its stationary value. The decline in the small firm entry rate is much more gradual, falling down initially by one third of the total amount, and then slowly converging towards the lower stationary value. The average markup once again responds almost immediately.

What about the counterfactual experiment results? In Section A.9.2 of the Appendix, we conduct the equivalents of the decomposition experiments, where we require all economies to start from the early period stationary equilibrium, and converge over time to the late stationary equilibrium. The welfare numbers are also recomputed taking the full transition into account, and are presented in Table A7. The welfare difference between the realized transition and the counterfactual of remaining in the early steady-state in perpetuum is now calculated to be -11.43%, as opposed to -5.59% in the baseline analysis. This shows that taking the non-stationary dynamics into account does not change the direction of the welfare impact of the structural transition in the US. The structural change that contributes the most to the loss in welfare is the change in the elasticity of substitution, the impact of which is calculated as 8.04%, which is not very different from the 12.76% found in the baseline. Finally, we find that the dynamic gains in welfare associated with higher markups still dominate the static losses in efficiency, but the total welfare gain is smaller (1.63%). This is because the increase in aggregate productivity growth takes time to fully manifest due to the time it takes for the industry-state distribution $\mu_t(\Theta)$ to converge to its stationary value, whereas the static losses from a less productive competitive fringe are instantaneous.

7.3 Lower Elasticity of Intertemporal Substitution

The period utility function in the baseline model is the natural logarithm, which implies an elasticity of intertemporal substitution (EIS) of 1. This is done for tractability, as it allows an intuitive decomposition of welfare, and the computation of stationary and non-stationary equilibria is simpler since the discount term in the firm value functions, $r - g$, is equal to the discount factor of the representative household ρ. Conducting a survey of 1429 studies, Havranek, Horvath, Irsova, and Rusnak (2015) report that the average estimate of the EIS for the US is 0.594, with a standard error of 0.036. Since this number is quite low in comparison, one might worry that our choice regarding the preferences might exaggerate the dynamic welfare gains from increased growth. To establish the robustness of our results, we change the preferences given in equation (1) to

$$U = \int_0^\infty e^{-\rho t} \frac{C_t^{1-\theta}}{1-\theta}\, dt \tag{42}$$

where θ is the constant relative risk aversion parameter. To remain conservative, we impose $\theta = 2$, which implies an EIS of 0.5; lower than the mean estimate for the US. We repeat the estimation with the new preferences, and the results are displayed in Table B15. To remain consistent with the baseline, we change the value of ρ to 0.02, so that the implied real interest rate under 2% growth is the same across estimations. Using the new estimation results, we repeat the counterfactual exercises in Section 5.2, and report the results in Table B16. In general, the results remain quite comparable. In particular, the dynamic losses from welfare still dominate the static gains in the experiment where markups are reduced to their early period level. The total welfare impact is still negative at -2.94%. We conclude that our choice of utility function does not drive our results.

7.4 Capital Accumulation

In the baseline model, production only uses labor as an input. This is done for tractability and ease of comparison to other work on endogenous markups discussed earlier. In this section, we show that extending our model to include endogenous physical capital accumulation does not change the results significantly. The details of this extended model can be found in Appendix A.7.

Unlike the previous two robustness checks, we do not re-estimate the model. This is because all targeted moments except R&D intensity remain exactly the same after the introduction of capital accumulation. However, consumption equivalent welfare changes are affected, since output is now also used for investment in physical capital. Table B17 presents the results of repeating the counterfactual exercises in Section 5.2. It is seen that the direction of welfare changes are maintained. In particular, reverting the value of ζ to its early period value still results in a welfare loss of 6.53%. We conclude that abstracting away from capital accumulation does not drive our results.

7.5 Sensitivity to Markup Estimates

In our baseline estimation, we use the sales-weighted average markup estimates from De Loecker, Eeckhout, and Unger (2020) as an estimation target. Concerns have been raised regarding whether the cost- or sales-weighted average markup should be the focus of attention, given that it is the cost-weighted average markup that summarizes the distortions in allocative efficiency in commonly-used theoretical frameworks.[52] Despite our choice to target the sales-weighted average markup, our model delivers lower values for the cost-weighted average markup in all three samples. This is consistent with what is observed in the data.

To further establish the robustness of our results, we re-estimate the model using cost-weighted average markup targets obtained from Edmond, Midrigan, and Xu (2023) to hit them precisely, and repeat our counterfactual experiments. Table B18 displays the results of re-estimation. The rise in markups is still welfare-enhancing at 3.60%.

More recently, Bond, Hashemi, Kaplan, and Zoch (2021) have raised concerns over the consistency of average markup estimates obtained using the De Loecker and Warzynski (2012) methodology, which both De Loecker, Eeckhout, and Unger (2020) and Edmond, Midrigan, and Xu (2023) follow. This raises the question of whether our results hinge on markup estimates that might potentially be biased. Fortunately, given the rich structure of our setting that ties several aggregate moments together, we can still estimate our model even if we do not explicitly target any markup-based moments. As seen in the decomposition

[52] See Edmond, Midrigan, and Xu (2023) among others.

exercise in Table 2, the relative productivity of small firms ζ which governs the average markup, is also responsible for the majority of the change in the labor share. Therefore, even if we do not explicitly target the average markup, the labor share can be used to identify the value of ζ, which in turn can identify the implied average markup through indirect inference. Following this line of reasoning, we re-estimate the model after dropping the average markup and the standard deviation of markups from the set of targeted moments.[53] Table B20 displays the results of re-estimation. The indirect inference suggests that the average markup has increased by 8.35% of its value between the early and late periods, which is similar to the 10.99% increase in the baseline. The average markup level in the early period is found to be at 1.29, very close to the 1.30 estimate in De Loecker, Eeckhout, and Unger (2020). The positive welfare effects associated with the increase in markups are still present: the welfare impact of keeping the relative productivity of small firms ζ the same as in the early sub-sample would decrease welfare by 4.53%. Not relying on markup estimates obtained through the De Loecker and Warzynski (2012) methodology does not change our conclusions on the direction of the effect of rising markups on welfare.

7.6 Further Robustness Checks

We have built several other model extensions, and conducted more robustness checks, estimations, and counterfactual experiments beyond those reported thus far. For brevity, these are relegated to the Revision Appendix. These not only demonstrate the robustness of our quantitative results, but also showcase the flexibility of our new unified framework, which we hope will be of use to future researchers. We briefly list some of these below:

- **Allowing small firms to have positive profits either through decreasing returns to scale in production or through collusion.**

  We extend the model and derive the static equilibrium conditions and the level of output for two alternative models: (i) decreasing returns to scale in production technology (for both small firms and superstars), and (ii) letting the small firms in the competitive fringe collude, and thereby act as if they were a superstar with the productivity of the fringe ζq_{jt}^{leader}. In both cases, we depart from the baseline model in which small firms were making zero profits. Letting small firms earn positive profits do not change our main results. More details about these extensions are relegated to the Revision Appendix Section C.

- **Extending the model to allow for multi-product firms.**

  We extend the model to allow for multi-product firms as in Klette and Kortum (2004). In this extension, on top of all the dynamics present in the baseline framework, firms can also grow by entering new markets. Superstar firms are now collections of several "product lines" in different markets, which can be interpreted as products/industries/locations/industry-location pairs. Each product line of a superstar firm operates exactly as it did in the baseline model. However, the superstar firms can now conduct "expansion" R&D/radical innovation. Conditional on success, they enter a new product line, and their initial productivity is $\bar{n}$ steps behind the product line leader, identical to that of a new superstar that could emerge in the same product line as a result of small

[53] This is feasible since our model is already overidentified (9 parameters vs. 11 targets.)

firm innovation. Expansion R&D lets firms grow horizontally across product lines, and these product lines can be lost endogenously through the same mechanism that caused firms to lose their superstar status in the baseline model. The difference is that such an event only destroys the superstar's presence in that particular product line, but a superstar with more than a single product line still survives. The extended model shows that embedding a Klette and Kortum (2004) superstructure in our model does not eliminate its tractability. See Section D of the Revision Appendix for more details.

- **Modeling competition à la Bertrand.**

  We assume Cournot competition in our baseline analysis due to its ability to generate more variation in markups and more realistic market share distributions consistent with what is observed for large firms in the United States, but the fact remains that most of our results go through regardless of the specific assumption on whether firms compete in prices or quantities. We have re-estimated the model and performed a robustness check on our results with differentiated Bertrand competition instead. More details about this extension are relegated to the Revision Appendix Section E.

- **Using a broader intangible investment definition in the estimation.**

  In the baseline estimation, we map firm-year-level innovation to firm-year-level average patent citations, and aggregate spending on innovation to aggregate spending on R&D in the data. We examine the robustness of our quantitative results by conducting an alternative estimation for the early and late subsamples in which we target (1) a measure of aggregate intangible investment to GDP rather than R&D to GDP alone, and (2) use a firm-year-level measure of total intangible investment to measure innovation, as opposed to relying on patent data, when obtaining the data moments that let us replicate the inverted-U relationship between innovation and relative sales. Our quantitative results remain robust. More details about this robustness check are relegated to the Revision Appendix Section F.

- **Increasing the maximum number of productivity steps between superstars ($\bar{n}$) as well as the maximum number of superstars per industry ($\bar{N}$).**

  In setting up the model, we had to make a choice on the upper bound for the maximum number of productivity steps ($\bar{n}$) and the maximum number of superstar firms within an industry ($\bar{N}$) so that the equilibria are computable. While in theory these numbers could be arbitrarily large, increasing any of them rapidly increases the number of potential firm and industry states, which slows down the numerical solution of the model. We have chosen the baseline values as a good compromise between allowing the model to have rich enough heterogeneity within and across industries and the time needed for estimation and computing counterfactual equilibria. In Section G of the Revision Appendix, we show that our quantitative results are virtually unchanged when we increase the value of $\bar{N}$. Increasing $\bar{n}$ requires re-estimation, but the quantitative results likewise remain robust.

- **Imposing a quadratic R&D cost function to both small and large firms.**

  While part of the existing literature uses quadratic R&D cost functions (see for instance Akcigit and Kerr (2018)), we have decided to estimate the convexity of the R&D cost functions for both small and superstar firms in our baseline analysis. As a robustness check, we have also re-estimated the model with quadratic R&D cost functions. It is worth noting that, when exogenously imposing quadratic costs, the model has a hard time matching the level of R&D intensity and the growth rate of the economy simultaneously. Despite the fact that the fit of the model is significantly worse than under our baseline calibration, setting R&D convexity to 2 does not significantly change the results of our counterfactual experiments. More details about this extension are relegated to the Revision Appendix Section H.

- **Examining the robustness of the results assuming a non-quadratic entrepreneur cost function.**

  We choose a quadratic entrepreneur cost function in the baseline due to the lack of available data that could be used to separately identify the scale (ψ) and convexity parameters. Unlike incumbent firms, for which we can empirically observe their innovation inputs (R&D expenditures) and outputs (patents, citations, sales growth, productivity growth, ...), we are not aware of any representative micro-data on business creation costs of entrepreneurs, which should ideally include not only the material costs of founding a new business, but also the opportunity cost of the entrepreneur(s). To mitigate the concerns that our quantitative results may be sensitive to the assumption about the entrepreneur cost function, we conduct two separate robustness checks. In the first one, we assume a higher convexity value of 3 instead of 2 for the entrepreneurs. In the second one, we assume a linear cost (i.e., free entry). No re-estimation is needed in either case, and the moment match is identical to that in the baseline. We repeat the counterfactual experiments. Our quantitative results remain robust in both settings. More details about these extensions are relegated to the Revision Appendix Section I.

8 Conclusion

We propose a new model of Schumpeterian growth in which firms strategically compete with other firms and dynamically choose their innovation strategies. Our model can account for an arbitrarily high number of firms in an industry, with endogenous entry and exit, and can generate non-degenerate sales, employment, markup, and innovation distributions within industries. This approach departs from much of the previous literature on endogenous growth studying markups, competition, and innovation, in which researchers use models featuring degenerate firm distributions with Bertrand competition in the product market. It also can generate endogenous industry dynamics in rich detail, and replicate the observed inverted-U relationship between innovation and competition both within and across industries.

We use the estimated model to gauge whether increasing markups boost or hinder aggregate innovation and economic growth. The findings reveal that while the increase in average markups causes a significant static welfare loss, this loss is overshadowed by the dynamic welfare gains from increased innovation in response to higher profit opportunities. Overall, our results suggest that the dynamic effects of increasing market concentration on innovation and productivity should not be ignored when trying to understand the transformation in the US in the last four decades; and the rise of superstar firms and markups is not necessarily detrimental to welfare.

Our results also highlight an increase in the costs of innovation over time. If the costs of innovation (for both small and superstar firms) were set back to their earlier levels, it would generate a further increase in productivity growth. These results point towards “the ideas are getting harder to find” hypothesis studied in Gordon (2012) and Bloom, Jones, Van Reenen, and Webb (2020).

We view our research as a starting point for understanding the aggregate implications of strategic interactions among heterogeneous firms. While our model shows the importance of using a model with non-degenerate firm distribution and realistic product market competition to gauge the welfare impact of rising markups, it can also be used in different settings where heterogeneous firms compete statically in the product market, and dynamically to improve their relative market shares. The model is highly tractable and could be easily extended to study the implications of various kinds of government policies, such as size-dependent R&D tax credits, subsidizing new business entry, and corporate taxation. We expect future studies along these lines to be both promising and fruitful.

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Online Appendices:

Are Markups Too High? Competition, Strategic Innovation, and Industry Dynamics[†]

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A Appendix

A.1 Growth rate

This section derives the growth rate of the economy.

$$\begin{aligned}
\ln(Y_t) &= \int_0^1 \frac{\eta}{\eta-1} ln\left[\sum_{i=1}^{N_{jt}} y_{ijt}^{\frac{\eta-1}{\eta}} + \tilde{y}_{cjt}^{\frac{\eta-1}{\eta}}\right] dj \\
&= \int_0^1 \frac{\eta}{\eta-1} \ln\left[y_{cjt}^{\frac{\eta-1}{\eta}}\left(\sum_{i=1}^{N_{jt}}\left(\frac{y_{ijt}}{\tilde{y}_{cjt}}\right)^{\frac{\eta-1}{\eta}} + 1\right)\right] dj \\
&= \int_0^1 \left[\ln\left(\tilde{y}_{cjt}\right) + \frac{\eta}{\eta-1}\ln\left(\sum_{i=1}^{N_{jt}}\left(\frac{y_{ijt}}{\tilde{y}_{cjt}}\right)^{\frac{\eta-1}{\eta}} + 1\right)\right] dj \\
&= \int_0^1 \left[\ln\left(\frac{1}{\omega_t}\frac{q_{cjt}}{\sum_{i=1}^{N_{jt}}\left(\frac{y_{ijt}}{\tilde{y}_{cjt}}\right)^{\frac{\eta-1}{\eta}} + 1}\right) + \frac{\eta}{\eta-1}\ln\left(\sum_{i=1}^{N_{jt}}\left(\frac{y_{ijt}}{\tilde{y}_{cjt}}\right)^{\frac{\eta-1}{\eta}} + 1\right)\right] dj \\
&= \int_0^1 \left[\ln\left(\frac{q_{cjt}}{\omega_t}\right) + \frac{1}{\eta-1}\ln\left(\sum_{i=1}^{N_{jt}}\left(\frac{y_{ijt}}{\tilde{y}_{cjt}}\right)^{\frac{\eta-1}{\eta}} + 1\right)\right] dj \\
&= \int_0^1 \ln\left(\frac{q_{cjt}}{\omega_t}\right) dj + \sum f_t(\Theta)\mu_t(\Theta) \qquad (43) \\
\ln(Y_{t+\Delta t}) - \ln(Y_t) &= -\ln(\omega_{t+\Delta t}) + \ln(\omega_t) + \sum p_{lit}(\Theta)\Delta t \ln(1+\lambda)\mu_t(\Theta) \\
&\quad + \sum_{\Theta}\sum_{\Theta'}\left[f_t(\Theta') - f_t(\Theta)\right] p_t(\Theta,\Theta')\mu_t(\Theta)\Delta t + o(\Delta t) \qquad (44) \\
g_t &= -g_{\omega,t} + \sum p_{lit}(\Theta)\ln(1+\lambda)\mu_t(\Theta) \\
&\quad + \sum_{\Theta}\sum_{\Theta'}\left[f_t(\Theta') - f_t(\Theta)\right] p_t(\Theta,\Theta')\mu_t(\Theta) \qquad (45)
\end{aligned}$$

A.2 Proposition 1

Let $\hat{\Theta}$ denote the set of all industry-states Θ. Let $h : \hat{\Theta} \to \mathbb{R}$ be a function. Let $p(\Theta, \Theta')$ denote the instantaneous flow from industry-state Θ to Θ'. Then, in a stationary equilibrium:

$$\begin{aligned}
\mathbb{E}\left[\sum_{\Theta'} p(\Theta,\Theta')(h(\Theta') - h(\Theta))\right] &= \sum_{\Theta}\sum_{\Theta'} p(\Theta,\Theta')(h(\Theta') - h(\Theta))\mu(\Theta) \\
&= \sum_{\Theta}\sum_{\Theta'} p(\Theta,\Theta')h(\Theta')\mu(\Theta) - \sum_{\Theta}\sum_{\Theta'} p(\Theta,\Theta')h(\Theta)\mu(\Theta) \\
&= \sum_{\Theta'} h(\Theta')\sum_{\Theta} p(\Theta,\Theta')\mu(\Theta) - \sum_{\Theta} h(\Theta)\sum_{\Theta'} p(\Theta,\Theta')\mu(\Theta) \\
&= \sum_{\Theta'} h(\Theta')\mu(\Theta') - \sum_{\Theta} h(\Theta)\mu(\Theta) \\
&= \mathbb{E}\left[h(\Theta')\right] - \mathbb{E}\left[h(\Theta)\right] \\
&= 0
\end{aligned}$$

A.3 Empirical Appendix

Unlike other recent models that study the effects of rising markups, we require our model to be consistent with the observed empirical relationship between competition and innovation, which helps us discipline the counterfactual model implications for innovation, economic growth, and welfare. In this section, we present and reconfirm the empirical regularities between competition and innovation across and within industries.

A.3.1 Variable Construction

Data Sources: We use the patent grant data obtained from NBER Patent Database Project which covers the years 1976-2006, and rely on Compustat North American Fundamentals for financial statement information of US-listed firms for the same years. Following a dynamic assignment procedure, we link the two data sets.

Patent Citations: Our first measure of innovation is the number of citations a patent received as of 2006. We use the truncation correction weights devised by Hall, Jaffe, and Trajtenberg (2001) to correct for systematic citation differences across different technology classes and for the fact that earlier patents have more years during which they can receive citations (truncation bias).

Tail Innovations: In order to distinguish disruptive patents from ordinary ones, we declare a patented innovation as a tail innovation if it is among the top 10% patents according to citations received among all patents applied for in the same year. The tail innovation index is constructed as the fraction of tail innovations among all granted patents of the firm in a given year, similar to Acemoglu, Akcigit, and Celik (2022). Tail count is likewise defined as the total number of tail innovations a firm receives in a given year. The first variable is scale-free, whereas the second one is scale-dependent.

Originality: We use the originality index devised by Hall, Jaffe, and Trajtenberg (2001). Let $i \in I$ denote a technology class and $s_{ij} \in [0,1]$ denote the share of citations that patent j makes to patents in technology class i (with $\sum_{i \in I} s_{ij} = 1$). Then for a patent j that makes positive citations, we define: $\text{Originality}_j = 1 - \sum_{i \in I} s_{ij}^2$. This index thus measures the dispersion of the citations made by a patent in terms of the technology classes of cited patents. Greater dispersion of citations is interpreted as a sign of greater originality, since the patented innovation combines information from a diverse range of technological fields. The patent classes used in the baseline analysis are the 36 two-digit technological subcategories defined in Hall, Jaffe, and Trajtenberg (2001).[54] The average originality of a firm's innovation in a given year is the average originality of all the patents for which the firm applied in that year. Originality count is constructed by summing the originality scores of all patents the firm applied for. The first variable is scale-free, whereas the second one is scale-dependent.

Generality: Similar to originality, we use the generality index devised by Hall, Jaffe, and Trajtenberg (2001). Let $i \in I$ denote a technology class and $s_{ij} \in [0,1]$ denote the share of citations that patent j receives from patents in technology class i (with $\sum_{i \in I} s_{ij} = 1$). Then for a patent j that receives positive citations, we define: $\text{Generality}_j = 1 - \sum_{i \in I} s_{ij}^2$. This index measures the dispersion of the citations made to a patent in terms of the technology classes of citing patents. Greater dispersion of citations is interpreted

[54] For robustness, the same measures are recalculated using the three-digit International Patent Classification categories, and the US Patent Class categories assigned internally by the USPTO.

as a sign of greater generality, since the patented innovation contributes to the creation of patents in a diverse range of technological fields. Average generality and generality count are constructed in the same way as average originality and originality count.

R&D Spending: Patent-based measures capture a successful innovation outcome. However, it might also be worthwhile to look at the amount of resources spent by a firm to conduct innovation regardless of success, as this captures the firm's intent. For this purpose, we use the R&D spending reported in Compustat. We have two variables, *log R&D spending* and *log R&D spending 2*. The first one excludes firms when the variable value is missing, whereas the second one replaces missing values with zeroes.

Other Variables: While many inventions are patented, firms can choose not to patent some inventions and keep them as trade secrets. Alternatively, a firm might improve its productivity through methods that are not considered novel enough to warrant a patent by the patent authorities. In such cases, it might be better to look at other firm outcomes that are likely to be correlated with productivity improvements. To do so, we consider investment in advertising and physical capital on the cost side. In addition, we directly look at the measured growth rates of firms' sales, employment, and total assets.[55]

A.3.2 Industry Innovation and Market Concentration

As documented in Aghion, Bloom, Blundell, Griffith, and Howitt (2005), innovation and market concentration have a non-linear relationship. While higher competition provides incentives for firms to improve their relative productivity compared to their peers ("escape competition") to improve their market share and profits, increased competition can also reduce the incentives to innovate as it pushes down profits in the whole market, which makes R&D investment less worthwhile. Depending on which effect dominates in a particular market, increased competition can reduce or increase overall innovation.

We document the same empirical regularity using our own sample of firms from the US. We are interested in the relationship between our innovation variables and market concentration. Table A1 contains the results of this exercise. In Panel A, we regress total patent count, total citations, tail innovation count, and originality- and generality-weighted patent counts on market concentration as captured by the Herfindahl-Hirschman Index (HHI) of SIC4 industries. All columns control for the number of firms in the industry, as well as year and SIC2 industry fixed effects. As expected, the linear term has a strong positive coefficient, whereas the quadratic term has a strong negative coefficient, replicating the inverted-U relationship.[56] Panel B repeats the same regressions where the innovation variables are constructed as the average values for all firms in an SIC4 industry instead of the total across the industry. The results are quite similar with the exception of originality-weighted patent count, which has the same signs, but the coefficients are not statistically significant. Panel C regresses the average of firm-level average innovation quality metrics for each industry. The results with these scale-independent innovation quality measures are similar to the previous specifications.

[55]The growth rates are defined as in Davis, Haltiwanger, and Schuh (1996). This bounds the growth rates in the interval $[-2, +2]$, addressing concerns regarding outliers.

[56]We should stress that we do not claim that a causal relationship exists. This exercise documents correlation patterns that we would like our model to be able to reproduce. Market concentration and innovation are both endogenous variables in our model as well.

TABLE A1: INDUSTRY INNOVATION AND MARKET CONCENTRATION (HHI) – BASELINE SPECIFICATION

Panel A: Total Innovation by Industry

	patent count	total citations	tail count	original count	general count
HHI	256.792	57.539	52.281	43.730	28.863
	(73.620)***	(13.716)***	(11.841)***	(16.929)***	(13.850)**
HHI sq.	-184.904	-39.240	-34.843	-29.048	-20.688
	(58.133)***	(10.692)***	(9.399)***	(13.384)**	(10.914)*
number of firms	5.303	0.943	0.921	1.185	0.812
	(0.619)***	(0.104)***	(0.095)***	(0.145)***	(0.101)***
R^2	0.22	0.22	0.23	0.22	0.22
N	11,305	11,305	11,305	11,305	11,305

Panel B: Industry Average of Total Innovation by Firms

	patent count	total citations	tail count	original count	general count
HHI	10.632	191.412	195.286	196.976	274.055
	(5.295)**	(67.088)***	(65.003)***	(128.902)	(94.877)***
HHI sq.	-10.377	-180.346	-181.419	-171.365	-247.240
	(4.395)**	(54.975)***	(54.239)***	(111.115)	(81.058)***
number of firms	-0.076	-0.694	-0.543	-1.714	-1.675
	(0.015)***	(0.179)***	(0.151)***	(0.338)***	(0.295)***
R^2	0.07	0.06	0.06	0.06	0.10
N	11,305	11,305	11,305	11,305	11,305

Panel C: Industry Average of Average Innovation Quality by Firms

	avg. citations	tail innov	avg. originality	avg. generality
HHI	2.015	2.333	3.248	3.046
	(0.483)***	(0.725)***	(0.972)***	(0.965)***
HHI sq.	-1.791	-1.962	-2.275	-2.907
	(0.464)***	(0.746)***	(1.057)**	(1.007)***
number of firms	0.008	0.010	-0.003	-0.006
	(0.001)***	(0.001)***	(0.001)**	(0.002)***
R^2	0.32	0.15	0.34	0.35
N	11,305	11,305	11,305	11,305

Notes: Robust asymptotic standard errors are reported in parentheses. The sample period is from 1976 to 2004 at the annual frequency. All regressions control for year dummies, and a full set of two-digit SIC industry dummies. $***p < 0.01, **p < 0.05, *p < 0.1$.

A.3.3 Firm Innovation and Relative Sales

Different from earlier studies, we also consider the relationship between the relative market share of a firm and its innovation. In this specification, we have *firm* $\times$ *year* level observations. Table A2 documents our main findings. In Panel A, we regress average citations, tail innovations, average originality, and

Table A2: Firm Innovation and Relative Sales – Baseline Specification

Panel A

	avg. citations	tail innov. (10%)	avg. originality	avg. generality
relative sales	7.919	6.861	8.396	17.845
	(1.192)***	(1.374)***	(1.789)***	(1.903)***
relative sales sq.	-7.851	-6.793	-6.271	-15.182
	(1.435)***	(1.803)***	(2.208)***	(2.363)***
R^2	0.15	0.10	0.26	0.25
N	104,911	104,911	104,911	104,911

Panel B

	log total patents	log total citations	log R&D spending	log R&D spending 2
relative sales	2.144	3.582	1.331	0.966
	(0.197)***	(0.307)***	(0.093)***	(0.080)***
relative sales sq.	-1.462	-2.691	-1.169	-0.896
	(0.269)***	(0.402)***	(0.119)***	(0.104)***
R^2	0.57	0.50	0.96	0.94
N	104,911	104,911	61,186	104,911

Panel C

	log(xad)	log(capx)	sales growth	employment growth	asset growth
relative sales	10.702	12.054	0.254	0.194	0.269
	(0.333)***	(0.228)***	(0.020)***	(0.016)***	(0.021)***
relative sales sq.	-10.034	-11.145	-0.236	-0.183	-0.248
	(0.436)***	(0.297)***	(0.025)***	(0.020)***	(0.025)***
R^2	0.73	0.68	0.12	0.12	0.13
N	37,779	103,558	102,726	96,718	103,598

Notes: Robust asymptotic standard errors reported in parentheses are clustered at the firm level. The sample period is from 1976 to 2004 at the annual frequency. All regressions control for profitability, leverage, market-to-book ratio, log R&D stock, firm age, the coefficient of variation of the firm's stock price, year dummies, and a full set of four-digit SIC industry dummies. $***p < 0.01, **p < 0.05, *p < 0.1$.

average generality on relative sales of the firm in its SIC4 industry and its square. The control variables include profitability, leverage, market-to-book ratio, log R&D stock, firm age, the coefficient of variation of the firm's stock price, and a full set of year and SIC4 industry fixed effects.[57] Robust standard errors are clustered at the firm level. In all four columns, we observe a strong inverted-U relationship between a firm's relative sales and its innovation output measures.[58] As a firm's market share increases, it invests more resources into innovation. However, there are diminishing returns. As the firm becomes more dominant in its industry and enjoys a larger market share, it begins to lower its investment in innovation.

Panel B repeats the same regressions where the scale-free innovation measures are replaced with scale-dependent ones such as log total patents, log total citations, log R&D spending where missing values

[57] Table B1 reports the same regressions with year and industry fixed effects alone.
[58] This is consistent with the findings in other studies such as Hashmi and Biesebroeck (2016).

are dropped, and log R&D spending 2 which replaces missing R&D values with zeroes. The inverted-U relationship is still present when we look at the totals instead of the average innovation quality. Panel C replaces the independent variable with other firm moments that are expected to be positively related to productivity improvements. The same non-linear relationship holds if we consider log advertising spending, log physical capital expenditures, firm sales growth, firm employment growth, or firm asset growth. Since these firm moments are not reliant on the patent data to calculate, these results reassure us that the regularities we document are not driven by the properties of patent-based innovation metrics.

Our results do not depend on our choice to look at the relative market share of a firm. Tables B2 and B3 replicate the results in Table A2 where relative sales is replaced with relative employment and relative total assets respectively. The same inverted-U relationship is present in all specifications.

We further test the robustness of our results by replacing the SIC4 fixed effects with SIC3, SIC2, and firm fixed effects. The results can be found in Tables B4, B5, and B6, respectively. The results with SIC2 and SIC3 fixed effects are quite similar to the baseline specification. Although the last specification with firm fixed effects is much more demanding, the inverted-U relationship is still present in all seven regressions, with some loss of significance in columns 2 and 3 in Panel A.

We confirm that our findings hold for both tradable and nontradable industries. Table B7 presents the results obtained when we restrict the sample to firms operating in tradable industries, whereas Table B8 presents the same for firms operating in nontradable industries.[59] A robust inverted-U relationship is detected in both samples.

Finally, we consider how the relationship changes across time. Tables B9 and B10 split the sample into two periods: early (1976-1994) and late (1995-2005). Although the quantitative magnitudes are different, the inverted-U relationship is present in both samples.

A.3.4 Robustness of the Inverted-U Relationship

Thus far, we have sought to identify an inverted-U relationship between competition and innovation across and within industries using the standard approach found in numerous economic studies investigating such non-linear relationships. This involves running regressions with linear and quadratic terms, establishing the significance of their coefficients, and showing that the extremum lies within the data range. However, the sufficiency of this methodology to establish the existence of a genuinely U or inverted-U shape relationship has been the subject of some debate. In light of these concerns, Lind and Mehlum (2010) develop a hypothesis test for the existence of U- and inverted-U-shape relationships.[60] To further establish the robustness of our results, we conduct the hypothesis test proposed in Lind and Mehlum (2010) for all specifications in Tables A1 and A2, where the null hypothesis is the lack of an inverted-U relationship. This involves testing whether or not the slope of the curve is positive at the start and negative at the end of the interval of the variable of interest. Correspondingly, Tables B11 and B12 report the t- and p-values at the lower and upper bounds of the interval of the explanatory variable. The null hypothesis is

[59] Tradable industries consist of agriculture, forestry, and fishing (01-09), mining (10-14), and manufacturing (20-39), where the numbers in parentheses refer to the two-digit SIC codes. Nontradable industries consist of all remaining industries except non-classifiable establishments (99).

[60] Arcand, Berkes, and Panizza (2015), Rodrik (2016), Bazzi, Gaduh, Rothenberg, and Wong (2016), Kesavan, Staats, and Gilland (2014), Tan and Netessine (2014), and Batt and Terwiesch (2017) among others use the test proposed in Lind and Mehlum (2010) to establish the existence of U- and inverted-U-shape relationships.

firmly rejected in all 27 specifications but one. The inverted-U relationships that we have identified pass the formal test of existence, with p-values below 1% in the vast majority of cases.[61]

A.4 Estimation Procedure

We pick the maximum number of superstars in an industry $\bar{N} = 4$ and the maximum number of productivity steps between any two superstar firms $\bar{n} = 5$, which delivers 84 unique industry states Θ.[62] The model has ten parameters to be determined: $\rho, \lambda, \eta, \chi, \nu, \zeta, \phi, \epsilon, \psi, \tau$. The consumer discount rate ρ is set to 0.04, which implies a real interest rate of 6% when the growth rate is 2%.[63] The remaining nine parameters are structurally estimated following a simulated method of moments approach. In this section, we discuss the data moments we use to discipline the parameter values, provide the relevant data sources for each of these moments, and discuss which moments help identify which parameters. The associated Jacobian matrix is presented in Table A3.

A.4.1 Data Moments and Sources

1. **Growth rate:** To discipline output growth in our model, we obtain the annual growth rate of real GDP per capita from the US Bureau of Economic Analysis, and calculate the geometric averages for each sub-sample.

2. **Labor share:** We obtain the labor share estimates from Karabarbounis and Neiman (2013); in particular the time series for corporate labor share (OECD and UN). For capital share, we rely on the data from Barkai (2020). For both time series, we calculate the averages across all years for each sub-sample. In our baseline model, there is no capital. Therefore, the model-generated labor share $\omega L = wL/Y$ corresponds to the share of labor income among labor income plus profits. For comparability, we multiply this number by $(1-\kappa)$ where κ is the (exogenous) capital share, following Akcigit and Ates (2023).[64]

3. **R&D intensity:** The data for aggregate R&D intensity is taken from the National Science Foundation, who report total R&D expenditures divided by GDP.

4. **Level and dispersion of markups:** To discipline markups, we target the sales-weighted average markup and the sales-weighted standard deviation of markups found in De Loecker, Eeckhout, and Unger (2020). In Section 7.5, we re-estimate the model using cost-weighted markups from Edmond, Midrigan, and Xu (2023), and the results are found to be similar. Motivated by Bond,

[61] Out of the 27 specifications, the null hypothesis is rejected at 1%-level at both bounds in 19 cases, 5%-level in 4 cases, and 10%-level in 3 cases. The p-value is 13.8% in the only setting without significance.

[62] The results do not significantly change if we increase $\bar{n}$ or $\bar{N}$. The estimated value of λ adjusts to absorb the choice of a different $\bar{n}$. The relative productivity of the competitive fringe ζ adjusts to absorb the changes in $\bar{N}$. In the estimated model, $\bar{n}$ is chosen large enough such that the largest superstars that we stop keeping track of are strictly smaller than 4/10,000 of the leader in terms of revenue and profits in all industry-states. Keeping track of these firms would decrease the profits of the remaining superstars by strictly less than 3/10,000, and this would not noticeably change the results.

[63] We target a relatively high real interest rate to remain conservative. For instance, a lower real interest rate of 4% would halve the implied discount rate to $\rho = 0.02$. This would double the welfare contribution of the output growth rate relative to that from the initial consumption level, significantly amplify the dynamic welfare gains, and further strengthen our findings.

[64] In Section 7.4, we explicitly add physical capital to the model, and calculate labor share without any correction, and the labor share in the model becomes $(1-\kappa)\omega L$, justifying this correction.

Hashemi, Kaplan, and Zoch (2021), we also conduct another re-estimation that does not rely on any markup-based moments obtained through the De Loecker and Warzynski (2012) methodology. This leads to similar results.

5. **Relationship between firm innovation and relative sales:** As discussed earlier, replicating the observed inverted-U relationship between competition and innovation helps us firmly discipline the counterfactual implications of the model regarding economic growth and social welfare. To achieve this, we target the relationship between firm innovation and relative sales. Innovation in the model is measured as the Poisson arrival event of quality improvement, whereas it is measured as average patent citations for each firm in the data. We normalize both by subtracting their means and dividing by their standard deviation. In the data, we regress average citations on relative sales of the firm in its SIC4 industry and their square. The control variables include profitability, leverage, market-to-book ratio, log R&D stock, firm age, the coefficient of variation of the firm's stock price, the number of firms in the industry, and a full set of year and SIC4 industry fixed effects. We target the linear and quadratic terms of a regression of (standardized) average citations on relative sales.

6. **Average profitability:** In the model, average profitability is calculated as static profit flow minus R&D expenses divided by sales. In the data, it is defined as operating income before depreciation divided by sales (OIBDP/SALE in Compustat.)

7. **Level and dispersion of leader quality:** We target the average relative quality of the leader in an industry, and its standard deviation across all industries. In the model, quality is known. In the data, we proxy quality by calculating the stock of past patent citations. The relative quality of the leader is defined as the quality of the leader divided by the sum of the qualities of the top four firms in an industry (SIC4 in the data.)

8. **Firm entry:** In our model, firm entry rate is defined as the entry rate of new small firms. We obtain the data counterpart – the entry rate of new businesses – from the Business Dynamics Statistics (BDS) database compiled by the US Census.

A.4.2 Identification and the Estimation Algorithm

The model is highly nonlinear, and all parameters affect all the moments. Nevertheless, some parameters are more important for certain statistics. The success of the SMM estimation depends on model identification, which requires that we choose moments that are sensitive to variations in the structural parameters. We now describe and rationalize the moments that we choose to match.

Table A3 reports the Jacobian matrix associated with the estimation of the baseline model. Each entry of the matrix reports the percentage change in each moment given one percent increase in each parameter. This table gives us some indication on which data moments are most informative in helping us identifying each parameter: (i) The productivity step size parameter λ is mainly identified by the output growth rate. A higher λ implies a higher increase in firm productivity upon successful innovation, which leads to higher output growth rate; (ii) Average profitability and the standard deviation of markups are most helpful in identifying the elasticity of substitution between superstar firms η. Larger η implies higher substitution among industry varieties, which leads to lower market power, profitability, and heterogeneity

Table A3: Identification: Jacobian Matrix

	λ	η	χ	ν	ζ	ϕ	ϵ	ψ	τ
growth rate	0.328	-0.397	-0.332	-0.120	-1.327	3.143	0.921	-0.100	-0.174
R&D intensity	-0.850	-0.651	-0.264	-0.182	-3.348	1.867	1.440	-0.152	-0.264
average markup	-0.033	-0.061	0.002	-0.003	-0.650	-0.010	0.023	-0.003	-0.005
std. dev. markup	0.148	-0.177	-0.004	0.014	-1.341	-0.010	-0.111	0.012	0.020
labor share	0.052	0.041	-0.002	0.005	0.493	0.009	-0.038	0.004	0.007
entry rate	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	1.000
β(innovation, rel. sales)	0.435	0.065	-0.154	0.014	-0.040	0.365	0.034	0.012	0.021
top point (intra-industry)	0.088	0.000	-0.016	-0.025	-0.105	0.088	0.217	-0.021	-0.036
avg. profitability	-0.144	-0.116	0.039	-0.002	-1.881	-0.240	0.010	-0.002	-0.003
avg. leader rel. quality	0.644	0.316	-0.014	0.099	0.524	-0.165	-0.788	0.082	0.143
std. dev. leader rel. quality	0.431	0.291	-0.027	0.139	0.660	0.182	-0.940	0.116	0.202

Notes: The table shows the Jacobian matrix associated with the estimation of the baseline model. Each entry of the matrix reports the percentage change in each moment given one percent increase in each parameter.

in markups across firms; (iii) An increase in either superstar innovation cost scale parameter χ or small firm innovation cost scale parameter ν reduces the aggregate R&D intensity and output growth rate. Since superstar innovation has a direct effect on the growth rate of the economy, the effect of χ on the output growth rate relative to its effect on R&D intensity is larger, whereas ν has a larger impact on R&D intensity. In addition, χ and ν have opposite implications for the level and dispersion of leader quality. Overall, larger χ tends to reduce the innovation of superstar firms, narrowing the quality gaps between the industry leader and other superstar firms. While higher ν increases the R&D cost of small firms, which reduces their innovation, leading to a reallocation of market share to superstar firms and a higher heterogeneity in qualities among superstar firms; (iv) The relative productivity of small firms ζ is identified very precisely by matching the average markup and the labor share as we extensively discuss in Sections 5.2.1 and 7.5. Lower ζ implies reduced competition from small firms and a within-industry market share reallocation to superstar firms, which generates a higher average markup and lower labor share; (v) As innovation policies in our estimated model are below unity, an increase in the R&D cost convexity parameters ϕ and ϵ reduces the innovation cost, which increases R&D intensity and the growth rate. These two parameters, however, have different implications for inverted-U relationship between innovation and market shares. While ϕ strongly influences the linear coefficient of the innovation-market share regression, changes in ϵ almost exclusively govern the location of the top point of the inverted-U relationship. The two parameters' impacts on average profitability and the standard deviation of leader relative quality are also opposite. (vi) τ is directly identified by targeting the entry rate of new businesses, since firm entry rate equals firm exit rate in a stationary equilibrium. (vii) Given all other parameter values, the value of ψ is set to normalize the measure of small firms m_t to one. Its exact value hinges on the average value of small firms, which itself is determined by the values of all other parameters. In particular, setting $m = 1$, we can rewrite equation (33) to get $\psi = \frac{\sum_\Theta v^e(\Theta)\mu(\Theta)}{2\tau}$.

SMM proceeds in the following way: For an arbitrary value of parameter vector $\theta = \{\lambda, \eta, \chi, \nu, \zeta, \phi, \epsilon, \tau\}$, the dynamic problem is solved and the policy functions are generated. Then we use the policy functions to calculate the model moments. The simulated moments estimator is defined as the solution to the minimization of:

$$\hat{\theta} = \arg\min_{\theta} \sum_{k=1}^{11} \left[\frac{\text{moment}_k^{model}(\theta) - \text{moment}_k^{data}}{\text{moment}_k^{data}} \right]^2 \hat{\Omega}_k \tag{46}$$

where moment_k^{model} is the value of moment k in the model and moment_k^{data} is the value of the moment in the data with weight $\hat{\Omega}_k$.

We use a simulated annealing algorithm for minimizing the objective function. This starts with a predefined first and second guess. For the third guess onward, it takes the best prior guess and randomizes from this to generate a new set of parameter guesses. That is, it takes the best-fit parameters and randomly "jumps off" from this point for its next guess. Over time the algorithm "cools", so that the variance of the parameter jumps falls, allowing the estimator to fine-tune its parameter estimates around the global best fit. We restart the program with different initial conditions to ensure the estimator converges to the global minimum.

A.5 Social Planner's Problem

There are several distortions in the decentralized equilibrium of the economy in our model. On the static side, the superstar firms use their market power to increase their profits through charging positive markups. On the dynamic side, the superstars, small firms, as well as entrepreneurs all ignore their effects on the rest of the economy: the positive contribution of their innovation to productivity growth, as well as the negative contribution of their investments that result in business-stealing. Due to these reasons, it is useful to solve the social planner's problem so that we can compare the inefficient decentralized equilibrium allocation to the Pareto-efficient allocation. In this section, we solve the problem in steps, and undertake the comparison.

A.5.1 The Complete Social Planner's Problem

The goal of the social planner is to maximize the lifetime utility of the representative household subject to technological constraints. Given the initial conditions, $\mu_0(\Theta), m_0$, and Q_0, the full problem can be stated as follows:

$$\max_{[[\{l_{ijt},z_{ijt}\}_{i=1}^{N_{jt}},[l_{kcjt},X_{kjt}]_{k=0}^{m_t}]_{j=0}^{1},e_t]_{t=0}^{\infty}} \int_0^\infty e^{-\rho t}\ln(C_t)dt, \text{ such that} \tag{47}$$

$$C_t + R_t Y_t \leq Y_t \tag{48}$$

$$R_t = \int \left(\sum_{i=1}^{N_{jt}} \chi z_{ijt}^{\phi} + \int \nu X_{kjt}^{\epsilon} dk\right) dj + \psi e_t^2 \tag{49}$$

$$\ln(Y_t) = \int_0^1 \ln(y_{jt})dj \tag{50}$$

$$y_{jt} = \left(\sum_{i=1}^{N_{jt}} y_{ijt}^{\frac{\eta-1}{\eta}} + \tilde{y}_{cjt}^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \tag{51}$$

$$\tilde{y}_{cjt} = \int y_{kcjt}dk \tag{52}$$

$$y_{ijt} = q_{ijt}l_{ijt} \tag{53}$$

$$y_{kcjt} = q_{cjt}l_{kcjt} \tag{54}$$

$$\int \left(\sum_{i=1}^{N_{jt}} l_{ijt} + \int l_{kcjt}dk\right) dj \leq L = 1 \tag{55}$$

$$q_{jt}^{leader} = \max\{q_{1jt}, ..., q_{N_{jt}jt}\} \tag{56}$$

$$q_{cjt} = \zeta q_{jt}^{leader} \tag{57}$$

$$\{q_{1jt}, ..., q_{N_{jt}jt}\} = \left\{q_{jt}^{leader}, \frac{q_{jt}^{leader}}{(1+\lambda)^{\vec{n}_{jt}(1)}}, ..., \frac{q_{jt}^{leader}}{(1+\lambda)^{\vec{n}_{jt}(N_{jt}-1)}}\right\} \tag{58}$$

$$\Theta_{jt} = (N_{jt}, \vec{n}_{jt}) \tag{59}$$

$$Q_t = \int \ln(q_{jt}^{leader})dj \tag{60}$$

$$\frac{\dot{Q}_t}{Q_t} = \ln(1+\lambda)\sum_{\Theta} p_{lit}(\Theta)\mu_t(\Theta) \tag{61}$$

$$\dot{\mu}_t(\Theta) = \sum_{\Theta'} p_t(\Theta',\Theta)\mu_t(\Theta') - \sum_{\Theta'} p_t(\Theta,\Theta')\mu_t(\Theta) \tag{62}$$

$$\sum_{\Theta} \mu_t(\Theta) = 1 \tag{63}$$

$$\dot{m}_t = e_t - \tau m_t \tag{64}$$

where equation (48) is the resource constraint, and equation (49) is the total R&D and business creation investment as a share of GDP. Equations (50) and (51) are respectively the final good and industry output production functions. Equation (52) is the production of the competitive fringe in industry j at time t. Equations (53) and (54) are the production functions of superstars and small firms. Equation (55) is the aggregate labor feasibility constraint. Equation (56) defines the productivity of industry j leader at time t (q_{jt}^{leader}) as the highest firm-level productivity in the industry. Equation (57) imposes that the productivity of each small firm in the competitive fringe in industry j is a fraction ζ of the industry leader's productivity at any time. Equation (58) defines the vector of productivity in industry j at time t for an industry with N_{jt} superstar firms where $\vec{n}_{jt}$ is the vector of productivity steps between each firm in the industry and the leader. Θ_{jt} in equation (59) is the state of industry j at time t, which can be summarized by the number of superstars in the industry (N_{jt}) and the number of productivity steps between each firm and the industry leader $\vec{n}_{jt}$. Q_t is the average (log) productivity of leaders across industries at time t (equation (60)). The growth rate of Q_t is given by equation (61) where $\mu_t(\Theta)$ is the mass of industries in state Θ and $p_{lit}(\Theta)$ is the arrival rate at which one of the industry leaders innovates. This arrival rate in turn depends on the mass of small firms and innovation policies. Equation (62) is the law of motion of the industry distribution. The first term corresponds to inflows into state Θ while the second term represents outflows. $p(\Theta, \Theta')$ is the instantaneous flow from state Θ to Θ' which depends on the innovation policies and the mass of small firms in the economy. Equation (63) states that the mass of industries has to sum to one. Finally, equation (64) is the law of motion of the mass of small firms.

The social planner maximizes welfare by choosing the labor allocation to every superstar firm i in industry j at time t (l_{ijt}) and to every small firm k in industry j at time t (l_{kcjt}). The social planner also chooses the R&D innovation policies for every superstar firm (z_{ijt}) and small firm (X_{kjt}) as well as the entry policy of the entrepreneurs (e_t). Since small firms within the fringe of a given industry are symmetric, we can write the total labor allocation to small firms in industry j at time t as $l_{cjt} = m_t l_{kcjt}$ and the Poisson rate of innovation by small firms as $X_{jt} = m_t X_{kjt}$.

This is a large problem to solve. However, it can be split into a static problem and a dynamic problem. First, note that the final good and labor feasibility constraints (equations (48) and (55)) must bind with equality. This is because the preferences of the representative household are increasing in consumption C_t, and there is no disutility of labor. As a consequence, given the productivity distribution $[\{q_{ijt}\}_{i=1}^{N_{jt}}, q_{cjt}]_{j=0}^{1}$, the social planner's optimal solution must maximize total output Y_t for all t, subject to the production technologies outlined in equations (50) to (54) and the labor feasibility constraint (55). We solve this static output maximization problem in the next subsection.

A.5.2 Static Output Maximization

Given the productivity distribution $[\{q_{ijt}\}_{i=1}^{N_{jt}}, q_{cjt}]_{j=0}^{1}$, the social planner's static (log-)output maximization problem at time t can be stated as follows:

$$\max_{[\{l_{ijt}\}_{i=1}^{N_{jt}}, l_{cjt}]_{j=0}^{1}} \quad \int_0^1 \frac{\eta}{\eta-1} \ln\left(\sum_{i=1}^{N_{jt}} \left(l_{ijt}q_{ijt}\right)^{\frac{\eta-1}{\eta}} + \left(l_{cjt}q_{cjt}\right)^{\frac{\eta-1}{\eta}}\right) dj, \text{ such that} \tag{65}$$

$$\int_0^1 \left(\sum_{i=1}^{N_{jt}} l_{ijt} + l_{cjt}\right) dj = 1 \tag{66}$$

This delivers the first order conditions

$$\frac{q_{ijt}^{\frac{\eta-1}{\eta}} l_{ijt}^{-\frac{1}{\eta}}}{\sum_{i=1}^{N_{jt}} \left(l_{ijt}q_{ijt}\right)^{\frac{\eta-1}{\eta}} + \left(l_{cjt}q_{cjt}\right)^{\frac{\eta-1}{\eta}}} = \omega_t, \forall i \in \{1, ..., N_{jt}\}, \forall j \in [0,1] \tag{67}$$

$$\frac{q_{cjt}^{\frac{\eta-1}{\eta}} l_{cjt}^{-\frac{1}{\eta}}}{\sum_{i=1}^{N_{jt}} \left(l_{ijt}q_{ijt}\right)^{\frac{\eta-1}{\eta}} + \left(l_{cjt}q_{cjt}\right)^{\frac{\eta-1}{\eta}}} = \omega_t, \forall j \in [0,1] \tag{68}$$

where ω_t is the Lagrange multiplier associated with the labor feasibility constraint (66). To find the exact labor allocations, first note the following:

$$\omega_t\left(\sum_{i=1}^{N_{jt}} l_{ijt} + l_{cjt}\right) = \frac{\sum_{i=1}^{N_{jt}}\left(l_{ijt}q_{ijt}\right)^{\frac{\eta-1}{\eta}} + \left(l_{cjt}q_{cjt}\right)^{\frac{\eta-1}{\eta}}}{\sum_{i=1}^{N_{jt}}\left(l_{ijt}q_{ijt}\right)^{\frac{\eta-1}{\eta}} + \left(l_{cjt}q_{cjt}\right)^{\frac{\eta-1}{\eta}}} = 1 \tag{69}$$

$$\int_0^1 \omega_t\left(\sum_{i=1}^{N_{jt}} l_{ijt} + l_{cjt}\right) dj = \int_0^1 1 dj \tag{70}$$

$$\omega_t = 1 \tag{71}$$

The first equation obtained by using equations (67) and (68). The last step uses the labor feasibility constraint (66). In turn, plugging $\omega_t = 1$ into the first equation delivers:

$$\sum_{i=1}^{N_{jt}} l_{ijt} + l_{cjt} = 1, \forall j \in [0,1] \tag{72}$$

This equation establishes that the total labor allocated to each industry j is always equal. Next, using equations (67) and (68), we establish:

$$\frac{l_{ijt}}{l_{kjt}} = \left(\frac{q_{ijt}}{q_{kjt}}\right)^{\eta-1}, \forall i,k \in \{1,...,N_{jt}\}, \forall j \in [0,1] \tag{73}$$

$$\frac{l_{ijt}}{l_{cjt}} = \left(\frac{q_{ijt}}{q_{cjt}}\right)^{\eta-1}, \forall i \in \{1,...,N_{jt}\}, \forall j \in [0,1] \tag{74}$$

Combined with (72), we have:

$$l_{ijt} = \frac{1}{\sum_{k=1}^{N_{jt}}\left(\frac{q_{kjt}}{q_{ijt}}\right)^{\eta-1} + \left(\frac{q_{cjt}}{q_{ijt}}\right)^{\eta-1}}, \forall i \in \{1,...,N_{jt}\}, \forall j \in [0,1] \tag{75}$$

$$l_{cjt} = \frac{1}{\sum_{k=1}^{N_{jt}}\left(\frac{q_{kjt}}{q_{cjt}}\right)^{\eta-1} + 1}, \forall j \in [0,1] \tag{76}$$

This concludes finding the optimal labor allocation that maximizes output. We plug in the optimal solution into the production function to calculate the implied log-output:

$$\begin{aligned}
\ln(Y_t) &= \int_0^1 \frac{\eta}{\eta-1} \ln\left[\sum_{k=1}^{N_{jt}} y_{kjt}^{\frac{\eta-1}{\eta}} + y_{cjt}^{\frac{\eta-1}{\eta}}\right] dj \\
&= \int_0^1 \ln(y_{cjt}) + \frac{\eta}{\eta-1} \ln\left[\sum_{k=1}^{N_{jt}} \left(\frac{y_{kjt}}{y_{cjt}}\right)^{\frac{\eta-1}{\eta}} + 1\right] dj \\
&= \int_0^1 \ln\left[\frac{q_{cjt}}{\sum_{k=1}^{N_{jt}} \left(\frac{q_{kjt}}{q_{cjt}}\right)^{\eta-1} + 1}\right] dj + \frac{\eta}{\eta-1} \int_0^1 \ln\left[\sum_{k=1}^{N_{jt}} \left(\frac{q_{kjt}}{q_{cjt}}\right)^{\eta-1} + 1\right] dj \\
&= \int_0^1 \ln(q_{cjt}) dj + \frac{1}{\eta-1} \int_0^1 \ln\left[\sum_{k=1}^{N_{jt}} \left(\frac{q_{kjt}}{q_{cjt}}\right)^{\eta-1} + 1\right] dj \\
&= \ln\zeta + \int_0^1 \ln q_{jt}^{leader} dj + \frac{1}{\eta-1} \sum_{\Theta} \ln\left[\sum_{k=1}^{N(\Theta)} \left(\frac{q_{kjt}}{q_{cjt}}\right)^{\eta-1} + 1\right] \mu_t(\Theta) \\
&= \ln\zeta + \int_0^1 \ln q_{jt}^{leader} dj + \sum_{\Theta} \tilde{f}_t(\Theta)\mu_t(\Theta) \\
&= \ln\zeta + Q_t + \sum_{\Theta} \tilde{f}_t(\Theta)\mu_t(\Theta)
\end{aligned}$$

The last line provides a closed-form solution, which establishes the efficient amount of log-output as a function of the average productivity level of the leaders Q_t, and the industry state distribution $\mu_t(\Theta)$.

TABLE A4: SOCIAL PLANNER'S PROBLEM: STATIC WELFARE GAINS

output DE	0.824
output SPP	1.033
CEWC	25.37%

Notes: This table reports the static gains from removing all markups for the whole sample. The first row shows the level of initial output in the decentralized equilibrium (DE). The second line reports initial output when all markups are removed (using the decentralized equilibrium industry distribution). The third row displays the static consumption-equivalent welfare gains from removing all markups.

Before we move on to the full dynamic problem of the social planner, we can first derive the static welfare gain that would be obtained by removing all markups but keeping the distribution and dynamic policies unchanged. Table A4 shows that the static welfare gains can be substantial (around 25% in consumption equivalent terms). It is interesting to note that our model does not underestimate the static cost of markups. If anything, our estimates are slightly larger than those found in Baqaee and Farhi (2020) and Edmond, Midrigan, and Xu (2023).

A.5.3 Dynamic Welfare Maximization

Given the results of the static output maximization, we can greatly simplify the complete problem of the social planner. As we would like to compare the results to the general equilibrium along a balanced growth path, we also focus on the stationary version of the social planner's problem, where the state variables $\mu_0(\Theta)$ and m_0 are initialized at their stationary values. The average productivity of the leaders,

Q_0, is initialized at 0 to remain consistent with Section 2.4. The dynamic welfare maximization problem of the social planner can therefore be re-stated as:

$$\max_{\left\{\left\{\{z_i(\Theta)\}_{i=1}^{N(\Theta)}, X_k(\Theta)\right\}_{\Theta}, e\right\}} \left\{\frac{\ln C_0}{\rho} + \frac{g}{\rho^2}\right\}, \text{such that} \tag{77}$$

$$C_0 = (1-R)Y_0 \tag{78}$$

$$\ln Y_0 = \ln \zeta + Q_0 + \sum_{\Theta} \tilde{f}(\Theta)\mu(\Theta) \tag{79}$$

$$g = \ln(1+\lambda) \sum_{\Theta} p_{li}(\Theta)\mu(\Theta) \tag{80}$$

$$R = \sum_{\Theta} \left(\sum_{i=1}^{N(\Theta)} \chi(z_i(\Theta))^{\phi} + \nu m (X_k(\Theta))^{\epsilon} \right) \mu(\Theta) + \psi e^2 \tag{81}$$

$$\dot{\mu}_t(\Theta) = \sum_{\Theta'} p(\Theta', \Theta)\mu(\Theta') - \sum_{\Theta'} p(\Theta, \Theta')\mu(\Theta) = 0, \forall \Theta \tag{82}$$

$$\sum_{\Theta} \mu_t(\Theta) = 1 \tag{83}$$

$$\dot{m}_t = e - \tau m = 0 \tag{84}$$

where initial consumption is equal to output minus R&D and business creation investment cost (equation 78), equation (79) is (log) initial output consistent with the optimal static allocation chosen by the social planner (see Section A.5.2), g is the growth rate of output in a balanced growth path (equation (80)), R is the share of output allocated to R&D and business creation (equation (81)), equation (82) is the law of motion of the industry distribution, equation (83) imposes that the mass of industries in the economy is equal to one and the dynamics of the mass of small firms is given by equation (84). In addition, those constraints require that g, $\mu_t(\Theta)$, and m_t remain constant in a balanced growth path.

The social planner chooses the innovation policy of superstar firms ($\{z_i(\Theta)\}_{i=1}^{N(\Theta)}$) and small firms ($X_k(\Theta)$) as well as investment in new business creation (e). We can notice that this formulation of the social planner's problem reduces the dimensionality of the maximization problem. Instead of solving for a continuum of continuous functions, we have reduced the problem to solving for a finite number of positive scalars, $\left\{\left\{\{z_i(\Theta)\}_{i=1}^{N(\Theta)}, X_k(\Theta)\right\}_{\Theta}, e\right\}$.

The dynamic social planner's problem, while greatly simplified, still requires 90 (=84+6) constraints to be satisfied. However, we can plug in all of the constraints into the objective function, and turn the problem into an unconstrained maximization problem (except for the non-negativity constraints for the choice variables.) It is trivial to see this is the case for equations (78) to (81), and equation (84). The inflow-outflow equations (82) and equation (83) that determine the stationary industry state distribution $\mu(\Theta)$ are less obvious.

First, note that given the choice variables, the values $p(\Theta, \Theta')$ are constants. Therefore, the 84 equations described by equation (82) constitute a system of 84 linear equations in $\mu(\Theta)$. One of these 84 equations is superfluous since the system is closed. Combined together with equation (83), they constitute a linear system of 84 equations in 84 unknowns. Rewrite this system of equations in matrix form as $A\vec{\mu} = b$, where A is an invertible square matrix, b is a column vector, and $\vec{\mu}$ is the industry state distribution written in vector form. Hence, we have $\vec{\mu} = A^{-1}b$. In other words, given the choice variables, the stationary industry state distribution $\mu(\Theta)$ can be obtained using matrix algebra, and the resulting values can be plugged into the objective function.

At the end, we are left with an unconstrained optimization problem where we need to determine the optimal values of 253 positive scalars. We solve this problem using global optimization methods. The results are discussed below.

A.5.4 Results from Social Planner's Optimization

TABLE A5: SOCIAL PLANNER'S PROBLEM: DYNAMIC WELFARE GAINS

	DE	SPP (unconstrained)	SPP ($z_1 = 0$)
growth rate	2.20%	5.60%	5.22%
initial output	0.824	1.009	1.021
CEWC		115.02%	97.61%

Notes: This table reports the results of the unconstrained and constrained social planner's problems for the whole sample. The first column reports the growth rate and initial output in the decentralized equilibrium (DE). The second column shows the results for the unconstrained social planner's problem (SPP) for the growth rate, initial output, and the consumption-equivalent welfare gain compared to the decentralized equilibrium. The third column displays the same information for the constrained social planner in which large firms in single-superstar industries perform no R&D.

In this section, we report the results of the unconstrained social planner's problem. First, the optimal (static) allocation corresponds to a decentralized equilibrium allocation with no markups. In addition, the social planner chooses an industry distribution which converges to a degenerate distribution with only one superstar firm which does all the R&D. Results comparing welfare and output between the optimal and the decentralized allocations can be found in Table A5 for the full sample. The optimal allocation consistently features higher initial output (static welfare gain) and growth rates (dynamic welfare gain) with large overall welfare gains. Table A5 shows that the static welfare gains from higher initial consumption are substantial (between 20% and 25%). It is interesting to note that our model does not underestimate the static cost of markups. If anything, our estimates are slightly larger than those found in Baqaee and Farhi (2020) and Edmond, Midrigan, and Xu (2023). However, despite the higher estimated static cost of markups, we find that the dynamic benefits from increased markups easily dominates the static gains.

A.5.5 Constrained Planner's Problem

In addition to the full social planner's problem discussed in Section A.5.4, in this section, we report the results from a constrained social planner's problem where we impose that there is no superstar innovation in industries with a single superstar firm. In the decentralized equilibrium, there is no innovation by superstars in single superstar industries since they have no incentive to do R&D. Under the optimal allocation, however, the distribution converges to a degenerate industry distribution with only single-superstar industries. In addition, only the incumbent superstar performs R&D. The objective of the following exercise is to analyze how much worse the social planner would do in terms of welfare if we impose that single superstars cannot perform R&D, i.e., we only allow the planner to allocate R&D resources to firms which perform R&D in the decentralized equilibrium. Results for the full sample can be found in Table A5. Once again, most of the welfare gains are due to removing dynamic inefficiencies (as is the case for the unconstrained optimal allocation). In addition, a large share of the welfare difference between the optimal and decentralized allocations can be achieved under the constrained optimal allocation with no R&D by large firms in single-superstar industries.

A.6 Distributional Implications of the Structural Transition

Throughout our analysis, we have opted to use a representative household. However, as the empirical findings indicate, the structural transition in the US over the last four decades decreased labor's share of income while driving up profits. Wealth is heavily concentrated in the US: According to the 2013 Survey of Consumer Finances, households in the bottom 50% of the wealth distribution hold only 1% of total wealth, whereas the top 5% hold 63% and top 1% hold 36%. Therefore, the gains from the increase in the profit share accrue to a very small portion of the population, whereas the decline in the labor share hurts most of the population who derive their income primarily from labor, not assets. This means the increase in markups and the decline in the labor share can have significant distributional implications, as studied in Boar and Midrigan (2019).

While adding a complete heterogeneous agent framework on the household side with credit market imperfections remains beyond the scope of our paper, there are less costly ways to uncover the first-order implications for inequality with little alteration to the model. We can separate the representative household into two types of consumers: (1) workers, who derive all of their income from labor, and cannot own any assets, (2) capitalists, who have no labor income, but own all assets in the economy and receive all entrepreneurial income. With this modification, the consumption of workers along the BGP is given by $C_t^{\text{worker}} = \omega Y_t$, and the consumption of capitalists is given by $C_t^{\text{capitalist}} = C_t - C_t^{\text{worker}}$. With these consumption streams, we can compute the consumption equivalent welfare change (CEWC) for both types of consumers. We can also compute how the relative consumption of the two groups, $C^{\text{capitalist}}/C^{\text{worker}}$ changes in each counterfactual exercise.

The results are shown in Table A6. The first row repeats the consumption equivalent welfare change for the representative consumer in Section 5.2, whereas the second and third rows report it for the workers and capitalists, respectively. The last row reports the percentage change in relative consumption. In most exercises, the welfare changes of the workers and capitalists go in the same direction, and are very close in magnitude. Naturally, the only exception to this is the exercise where markups are reduced. When the relative productivity of the competitive fringe, ζ, is reset to its early period value, the welfare of the capitalists drops by 27.65%, whereas that of the workers decreases slightly by 2.10%. The relative consumption of the capitalists goes down by 26.1%, reducing the inequality between the two groups.

Recall that the overall welfare loss was 7.6% for the representative household. This means that while increased competition from small firms would be detrimental for the overall economy, since the gains from innovation are not shared equally, consumers who primarily rely on labor income would be losers by a slight margin, whereas wealthy consumers would be the obvious losers. It is also interesting to note that workers' welfare is higher in the early period while the reverse is true for capitalists.

It means that higher markups are not the problem, but the unequal distribution of the gains from higher growth can be. Policies that aim to directly reduce markups through price controls or reducing market power might be detrimental to efficiency and economic growth. Redistributing the gains from innovation in a more equitable way through transfers can, therefore, be a more successful policy than a

Table A6: Distributional Implications of the Structural Transition

	Early η	Early ζ	Early ν, ϵ	Early χ, ϕ	Early ψ, τ	All
Benchmark CEWC	-12.76%	-7.60%	12.65%	2.05%	4.18%	-5.59%
Worker CEWC	-13.80%	-2.10%	12.58%	1.95%	4.12%	0.36%
Capitalist CEWC	-8.96%	-27.65%	12.88%	2.38%	4.39%	-27.30%
$\Delta\%\ C^{\text{capitalist}}/C^{\text{worker}}$	5.61%	-26.10%	0.27%	0.42%	0.25%	-27.56%

Notes: The table reports consumption-equivalent welfare change numbers for the representative consumer, as well as the idealized workers and capitalists in the counterfactual experiments.

direct reduction in markups if the aim is to improve the well-being of the average consumer.

A.7 Extended Model with Capital Accumulation

In this section, we introduce an extension of our baseline model with endogenous capital accumulation. In particular, we assume the following production function for the final good producer:

$$\ln(Y_t) = \int_0^1 \ln\left(y_{jt}^{1-\kappa} k_{jt}^{\kappa}\right) dj \tag{85}$$

where k_{jt} is capital which depreciates at a rate δ. Households own the stock of capital ($K_t = \int_0^1 k_{jt}\, dj$) and rent it to final good producers at a rental rate $R_t = r_t + \delta$. The final good is now used for consumption, investment in R&D, costs of new business entry, and investment in physical capital.

Profit maximization by final good producers implies:

$$k_{jt} = \frac{\kappa Y_t}{R_t} \tag{86}$$

$$p_{ijt} = \frac{(1-\kappa) y_{ijt}^{-\frac{1}{\eta}} Y_t}{\sum_{k=1}^{N_{jt}} y_{kjt}^{\frac{\eta-1}{\eta}} + \tilde{y}_{cjt}^{\frac{\eta-1}{\eta}}} \tag{87}$$

where κ is the capital share of income. This implies that, in a balanced growth path, the stock of capital grows at the same rate as aggregate output, Y_t. This delivers the same system of equations in unknown production ratios as in equation (17). Profits of superstars can be written as:

$$\pi_{ijt} = \frac{(1-\kappa) Y_t}{\left[\sum_{k=1}^{N_{jt}} \left(\frac{y_{kjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}} + \left(\frac{\tilde{y}_{cjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}}\right]^2} \frac{\eta + \sum_{k \neq i} \left(\frac{y_{kjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}} + \left(\frac{\tilde{y}_{cjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}}}{\eta} \tag{88}$$

Compared to the baseline model, the dynamic problem of superstar firms and small firms is only affected by the multiplicative constant $(1-\kappa)$ that appears in the profit function of the superstar firm. Final good production is equal to:

$$\ln(Y_t) = \int_0^1 \ln q_{jt}^{\text{leader}}\, dj + \ln\zeta - \ln\omega + \sum f(\Theta)\mu(\Theta) + \ln(1-\kappa) + \frac{\kappa}{1-\kappa}\left[\ln(\kappa) - \ln(R_t)\right] \tag{89}$$

The law of motion for capital is given by $\dot{K}_t = I_t - \delta K_t$, where I_t is investment in physical capital at time t. We can show that the investment-to-GDP ratio is equal to:

$$\frac{I_t}{Y_t} = \frac{\kappa(r_t - \rho + \delta)}{r_t + \delta} \tag{90}$$

which is constant in a balanced growth path. The equation for the growth rate remains the same. The level of initial consumption, C_0, becomes:

$$C_0 = Y_0 \frac{C_0}{Y_0} = Y_0 \left(1 - \int_0^1 \sum_{i=1}^{N_{j0}} \chi z_{ij0}^{\phi}\, dj - \int_0^1 \nu X_{j0}^{\epsilon}\, dj - \psi e_0^2 - \frac{I_0}{Y_0}\right) \tag{91}$$

A.8 Algorithm for Computing Stationary Equilibria

Define $\varphi = [\rho, \lambda, \eta, \zeta, \nu, \epsilon, \chi, \phi, \psi, \tau]$ as the vector of structural parameters of the economy. Select values for the maximum productivity steps allowed between superstars, $\bar{n} \in \mathbb{Z}_+$, and the maximum number of superstars per industry, $\bar{N} \in \mathbb{Z}_+$. Without loss of generality, normalize the initial average log productivity of the leaders $Q_t = \int_0^1 \ln q_{j0}^{\text{leader}} dj = 0$. The goal is to compute the unique stationary Markov-Perfect Equilibrium associated with the parameter vector φ using the baseline model (i.e., the unique balanced growth path equilibrium). Below, we describe an algorithm that can be used for this purpose.

1. Guess an initial value for the stationary mass of small firms $m_{old} > 0$.
2. Repeat the following until the stationary mass of small firms m converges; i.e., $\|m_{old} - m_{new}\| < \epsilon_m$ for some tolerance value $\epsilon_m > 0$:
   (a) Calculate the quantities, prices, static profit flows, and markups of all firms in each industry-state $\Theta \in \hat{\Theta}$ that arise as a result of static product market competition. (Solve a system of N non-linear equations given by equations 17 and 18 for each industry state, where N is the number of superstar firms.)
   (b) Guess an initial value for the normalized stationary superstar value function $v_{old}(\mathbf{n}_i, N)$.
   (c) Repeat the following value function iteration until the normalized stationary superstar value function $v(\mathbf{n}_i, N)$. converges; i.e., $\|v_{old}(.) - v_{new}(.)\| < \epsilon_v$ for some tolerance value $\epsilon_v > 0$:
      i. Using the guess for the normalized stationary superstar value function $v_{old}(.)$, use equation 24 to calculate the implied optimal level of superstar innovation $z_i(.)$ for all possible firm states.
      ii. Using the guess for the normalized stationary superstar value function $v_{old}(.)$, use equation 27 to calculate the implied optimal level of small firm innovation $X_{kj}(.)$ for all possible industry states. Multiply it with the stationary mass of small firms m_{old} to calculate total small firm innovation $X_j(.)$ for all possible firm states.
      iii. Using the guess for the normalized stationary superstar value function $v_{old}(.)$ and the calculated values for innovation policy functions $z_i(.)$ and $X_j(.)$, calculate a new guess for the normalized stationary superstar value function $v_{new}(.)$ using equation 23.

iv. Check if the normalized stationary superstar value function has converged ($\|v_{old}(.) - v_{new}(.)\| < \epsilon_v$). If true, save the normalized stationary superstar value function $v(.)$, and the associated innovation policy functions $z_i(.)$ and $X_j(.)$, and exit. If false, update $v_{old}(.) = \xi v_{new}(.) + (1-\xi)v_{old}(.)$, and go back to step (i).[65]

(d) Using the innovation policy functions $z_i(.)$ and $X_j(.)$, construct the instantaneous flow matrix $p(\Theta, \Theta')$ for all $\Theta, \Theta' \in \hat{\Theta}$.

(e) In a stationary equilibrium, the mass of industries of type Θ denoted as $\mu(\Theta)$ must be time-invariant. Using the system of linear equations defined by the instantaneous flow matrix $p(\Theta, \Theta')$, $\frac{d\mu(\Theta)}{dt} = 0, \forall \Theta \in \hat{\Theta}$, and $\sum_{\Theta} \mu(\Theta) = 1$, calculate the equilibrium value of $\mu(\Theta)$ for all industry states. Standard linear equation solvers work well.

(f) Using the normalized stationary superstar value function $v(.)$, calculate the normalized stationary small firm value function $v^e(.)$ for all possible industry states using equation 28.

(g) Using the normalized stationary small firm value function $v^e(.)$ and the stationary industry state distribution $\mu(.)$, calculate the implied stationary mass of small firms m_{new} using equation 33.

(h) Check if the stationary mass of small firms has converged ($\|m_{old} - m_{new}\| < \epsilon_m$). If true, the stationary equilibrium has been found; exit. If false, update m_{new} using a bisection algorithm, and go back to step (a).

3. Calculate other allocations and statistics of interest as necessary, given the equilibrium values of the stationary mass of small firms m and the stationary industry state distribution $\mu(.)$.

A.9 Non-Stationary Equilibria

In this section, we describe the algorithm used to compute non-stationary equilibria of our model, and repeat the quantitative experiments in Table 2 while taking transitional dynamics into account.

A.9.1 Algorithm for Computing Non-Stationary Equilibria

Define $\varphi_i = [\rho_i, \lambda_i, \eta_i, \zeta_i, \nu_i, \epsilon_i, \chi_i, \phi_i, \psi_i, \tau_i]$ as the vector of structural parameters of economy i. Suppose an economy is initially at its stationary equilibrium implied by the initial parameter values φ_b, where b stands for the beginning. Without loss of generality, normalize the initial average log productivity of the leaders $Q_t = \int_0^1 \ln q_{j0}^{\text{leader}} dj = 0$. At time $t = 0$, the vector of structural parameters of the economy changes from φ_b to φ_e, where e stands for the end. As $t \to \infty$, the economy will converge to the stationary equilibrium implied by the final parameter values φ_e. To compute the described non-stationary equilibrium, we need to characterize the full transition path of all prices and allocations in the economy. Below, we describe the algorithm used for this purpose.

1. Compute the stationary equilibrium implied by the initial parameter vector φ_b. Call the associated time-invariant mass of small firms m_b, and the time-invariant industry-state distribution $\mu_b(\Theta)$.

2. Compute the stationary equilibrium implied by the final parameter vector φ_e. Call the associated time-invariant mass of small firms m_e, and the time-invariant industry-state distribution $\mu_e(\Theta)$.

[65] $\xi \in (0, 1]$ is an update weight. $\xi = 0.05$ works well.

3. Define $T > 0$ as the amount of time (measured in years) for which the time paths of allocations and prices will be computed. T must be large enough such that the state variables $\mu_t(\Theta)$ and m_t are sufficiently close to their stationary values; i.e., $\|\mu_T(\Theta) - \mu_e(\Theta)\| < \epsilon_{\mu_e(\Theta)}$ and $\|m_T - m_e\| < \epsilon_{m_e}$ for some tolerance values $\epsilon_{\mu_e(\Theta)}, \epsilon_{m_e} > 0$ where $\|.\|$ denotes the sup-norm. Choose a value for T — e.g., $T = 1000$.

4. Divide the time interval $[0, T]$ into a uniform grid with step size $\Delta t > 0$. Denote the resultant (discrete) time grid as $\vec{t} = \{s\Delta t\}_{s=0}^{S}$ with $S = T/\Delta t$. A lower grid step size Δt increases approximation accuracy (discussed below) at the cost of computation time.

5. Create an initial guess for the time path of the mass of small firms over the time grid. A reasonable initial guess is a linear interpolation with boundary values $m_0 = m_b$ and $m_T = m_e$. Call this initial guess $\vec{m}_{old}$. Initialize a guess for the time path of the growth rate of C/Y, $\vec{g}_{C/Y,old}$, at zero.

6. Repeat the following until the time path of the mass of small firms $\vec{m}$ and $\vec{g}_{C/Y}$ converge; i.e., $\|\vec{m}_{old} - \vec{m}_{new}\| < \epsilon_{\vec{m}}$ and $\|\vec{g}_{C/Y,old} - \vec{g}_{C/Y,new}\| < \epsilon_{\vec{g}_{C/Y}}$ for some tolerance values $\epsilon_{\vec{m}}, \epsilon_{\vec{g}_{C/Y}} > 0$:

   (a) Calculate the quantities, prices, static profit flows, and markups of all firms in each industry-state $\Theta \in \hat{\Theta}$ that arise as a result of static product market competition. (Solve a system of N non-linear equations for each industry-state, where N is the number of superstar firms.)

   (b) Denote the time path of the normalized superstar firm value function as $\vec{v} = \{v_s(\mathbf{n}_i, N)\}_{s=0}^{S}$. Given the convergence of the state variables, $v_S(\mathbf{n}_i, N) \approx v_e(\mathbf{n}_i, N)$. Set $v_S(\mathbf{n}_i, N) = v_e(\mathbf{n}_i, N)$. We know $\dot{v}_t(\mathbf{n}_i, N) = \lim_{\Delta t \to 0} \frac{v_{t+\Delta t}(\mathbf{n}_i,N) - v_t(\mathbf{n}_i,N)}{\Delta t}$. Then $\dot{v}_t(\mathbf{n}_i, N) \approx \frac{v_{t+\Delta t}(\mathbf{n}_i,N) - v_t(\mathbf{n}_i,N)}{\Delta t}$, where the approximation is more accurate for smaller values of Δt. Plugging this approximate value into equation (22) yields an equation linking $v_s(.)$ to $v_{s+1}(.)$. Starting from $s = S$, use backward iteration to obtain the time path of the normalized superstar firm value function $\vec{v}$, along with the time paths of superstar innovation policy function $\vec{z} = \{z_{is}(\mathbf{n}_i, N)\}_{s=0}^{S}$ and small firm innovation policy function $\vec{X} = \{X_{kjs}(\Theta_j)\}_{s=0}^{S}$.

   (c) The initial value of the industry-state distribution at time $t = 0$ is determined by the value in the initial stationary equilibrium, $\mu_0(\Theta) = \mu_b(\Theta), \forall \Theta \in \hat{\Theta}$. Unlike a stationary equilibrium, the inflow and outflow rates for each industry-state Θ are now time-varying. Using the values obtained for $\vec{z}$ and $\vec{X}$, starting from $s = 0$, construct the time paths of inflows to $(\sum_{\Theta'} p_t(\Theta', \Theta)\mu_t(\Theta'))$ and outflows from $(\sum_{\Theta'} p_t(\Theta, \Theta')\mu_t(\Theta))$ each industry-state $\Theta \in \hat{\Theta}$ and the resultant time path of the industry-state distribution $\vec{\mu} = \{\mu_s(\Theta)\}_{s=0}^{S}$ using forward iteration. This yields $\mu_T(\Theta) = \mu_S(\Theta)$.

   (d) Using $\vec{v}$ and $\vec{X}$, construct the time path of expected profit flows of small firms in each industry-state Θ, denoted $\vec{\pi}^e = \{\pi_s^e(\Theta)\}_{s=0}^{S}$.

   (e) Denote the time path of the normalized small firm value function as $\vec{v}^e = \{v_s^e(\mathbf{n}_i, N)\}_{s=0}^{S}$. Given the convergence of the state variables, $v_S^e(\mathbf{n}_i, N) \approx v_e^e(\mathbf{n}_i, N)$. Set $v_S^e(\mathbf{n}_i, N) = v_e^e(\mathbf{n}_i, N)$. We know $\dot{v}_t^e(\mathbf{n}_i, N) = \lim_{\Delta t \to 0} \frac{v_{t+\Delta t}^e(\mathbf{n}_i,N) - v_t^e(\mathbf{n}_i,N)}{\Delta t}$. Then $\dot{v}_t^e(\mathbf{n}_i, N) \approx \frac{v_{t+\Delta t}^e(\mathbf{n}_i,N) - v_t^e(\mathbf{n}_i,N)}{\Delta t}$, where the approximation is more accurate for smaller values of Δt. Plugging this approximate value into equation (25) yields an equation linking $v_s^e(.)$ to $v_{s+1}^e(.)$. Starting from $s = S$, use backward iteration to obtain the time path of the normalized small firm value function $\vec{v}^e$.

(f) Using the time paths of the industry-state distribution $\vec{\mu}$ and the small firm value function $\vec{v^e}$, solve for the time path of the small business creation rate $\vec{e} = \{e_s\}_{s=0}^{S}$ consistent with entrepreneur profit maximization.

(g) The mass of small firms at time $t = 0$ is determined by the value in the initial stationary equilibrium, $m_0 = m_b$. Starting from $s = 0$, calculate the new time path of the mass of small firms $\vec{m}_{new}$ through forward iteration using $\vec{e}$ and the exogenous small firm exit rate τ.

(h) Calculate the time path of the relative wage rate $\vec{\omega}$ using the static product market competition results and $\vec{\mu}$.

(i) Calculate the time path of the output growth rate $\vec{g}$ using equation (34). Calculate the time path of the average log productivity level of industry leaders $\vec{Q}$ through forward iteration.

(j) Calculate other allocations and statistics of interest as necessary. To calculate lifetime utility of the representative consumer at time $t = 0$: Calculate the initial output Y_0, and the time path of aggregate output $\vec{Y}$ using $\vec{g}$. Calculate the time paths of aggregate R&D intensity and business creation costs to derive the time path of the consumption-to-output ratio. Using all, calculate the time path of aggregate consumption $\vec{C}$. Plug $\vec{C}$ into the utility function of the representative consumer to obtain lifetime utility at time $t = 0$. Update the guess for the time path of the growth rate of C/Y, $\vec{g}_{C/Y,new}$.

(k) Check if the time paths of the mass of small firms and $g_{C/Y}$ have converged. If true, exit. If false, update $\vec{m}_{old} = \xi\vec{m}_{new} + (1-\xi)\vec{m}_{old}$, $\vec{g}_{C/Y,old} = \vec{g}_{C/Y,new}$, and go back to step (a).[66]

7. Check if the state variables have converged ($\|\mu_T(\Theta) - \mu_e(\Theta)\| < \epsilon_{\mu_e(\Theta)}$ and $\|m_T - m_e\| < \epsilon_{m_e}$). If true, the non-stationary equilibrium is found. If false, either increase T or decrease Δt, and go back to step 5.

A.9.2 Disentangling the Structural Transition with Non-Stationary Dynamics

In this section, we repeat the quantitative experiments in Table 2 while taking transitional dynamics into account. Before we conduct the counterfactual experiments, we need to compute the realized non-stationary equilibrium between 1976 and 2005 in the US. To do so, we use the estimated parameter values from the early period sub-sample as φ_b, and those from the late period sub-sample as φ_e, and compute the baseline non-stationary equilibrium following the algorithm described in the preceding subsection. All consumption-equivalent welfare numbers in the following counterfactual experiments use the lifetime utility of the representative consumer at time $t = 0$ in this baseline economy as the yardstick.

As in Table 2, we conduct six separate counterfactual experiments. The initial vector of structural parameters φ_b is always the same, and uses the estimated parameter values from the early period sub-sample. The final vector of structural parameters φ_e is changed in each experiment. These are listed below in sequence:

1. Early η: $\varphi_e = [\rho, \lambda^{late}, \eta^{early}, \zeta^{late}, \nu^{late}, \epsilon^{late}, \chi^{late}, \phi^{late}, \psi^{late}, \tau^{late}]$
2. Early ζ: $\varphi_e = [\rho, \lambda^{late}, \eta^{late}, \zeta^{early}, \nu^{late}, \epsilon^{late}, \chi^{late}, \phi^{late}, \psi^{late}, \tau^{late}]$

[66] $\xi \in (0, 1]$ is an update weight. $\xi = 0.5$ works well.

3. Early ν, ϵ: $\varphi_e = [\rho, \lambda^{late}, \eta^{late}, \zeta^{late}, \nu^{early}, \epsilon^{early}, \chi^{late}, \phi^{late}, \psi^{late}, \tau^{late}]$
4. Early χ, ϕ: $\varphi_e = [\rho, \lambda^{late}, \eta^{late}, \zeta^{late}, \nu^{late}, \epsilon^{late}, \chi^{early}, \phi^{early}, \psi^{late}, \tau^{late}]$
5. Early ψ, τ: $\varphi_e = [\rho, \lambda^{late}, \eta^{late}, \zeta^{late}, \nu^{late}, \epsilon^{late}, \chi^{late}, \phi^{late}, \psi^{early}, \tau^{early}]$
6. Early all: $\varphi_e = [\rho, \lambda^{early}, \eta^{early}, \zeta^{early}, \nu^{early}, \epsilon^{early}, \chi^{early}, \phi^{early}, \psi^{early}, \tau^{early}]$

Table A7 presents the resultant consumption-equivalent welfare change numbers. First, looking at the final column, the welfare difference between the realized transition and the counterfactual of remaining in the early steady-state in perpetuum is now calculated to be -11.43%, as opposed to -5.59% in the baseline analysis.

Table A7: Disentangling the Structural Transition with Non-Stationary Dynamics

	Early η	Early ζ	Early ν, ϵ	Early χ, ϕ	Early ψ, τ	All
CEWC	-8.04%	-1.63%	2.82%	0.92%	0.64%	-11.43%

The individual experiments themselves also maintain the same signs as in Table 2, but the quantitative magnitudes change in some cases. In the first experiment, the welfare loss in the counterfactual economy with the early period elasticity of substitution η is now 8.04% instead of 12.76%. In the second experiment where the relative productivity of the competitive fringe ζ is held constant, we find that the dynamic gains in welfare associated with higher markups still dominate the static losses in efficiency, albeit with a smaller magnitude. This is because the increase in aggregate productivity growth takes time to fully manifest due to the time it takes for the industry-state distribution $\mu_t(\Theta)$ to converge to its stationary value, whereas the static losses from a less productive competitive fringe are instantaneous. The third column shows that restoring the R&D efficiency of small firms back to its early-period value is still welfare-enhancing, but the value is now lower at 2.82% as opposed to 12.65%. The impact of the decline in the R&D efficiency of superstar firms is calculated as 0.92%, which is slightly smaller than the 2.05% found in the baseline. Finally, the total effect of keeping new business creation costs and firm exit rate the same as in the early period sub-sample is now a wash on average, where the welfare barely moves at 0.64%, compared to the 4.18% value found in the baseline.

To summarize, the overall message remains the same: the primary driver that lies behind the observed increase in the average markup, the fall in the relative productivity of small firms, is still welfare-enhancing since the dynamic gains from improved productivity growth still dominate the static losses from lower static allocative efficiency.

B Additional Tables and Figures

TABLE B1: FIRM INNOVATION AND RELATIVE SALES – WITHOUT ADDITIONAL CONTROLS

Panel A

	avg. citations	tail innov. (10%)	avg. originality	avg. generality
relative sales	21.552	18.515	37.897	42.305
	(0.965)***	(1.058)***	(1.467)***	(1.588)***
relative sales sq.	-19.613	-16.749	-32.732	-37.443
	(1.165)***	(1.333)***	(1.881)***	(2.010)***
R^2	0.10	0.07	0.17	0.18
N	182,968	182,968	182,968	182,968

Panel B

	log total patents	log total citations	log R&D spending	log R&D spending 2
relative sales	5.605	9.209	13.053	8.114
	(0.240)***	(0.354)***	(0.323)***	(0.275)***
relative sales sq.	-4.719	-7.907	-11.966	-7.247
	(0.288)***	(0.430)***	(0.436)***	(0.329)***
R^2	0.34	0.31	0.57	0.45
N	182,968	182,968	99,482	182,968

Panel C

	log(xad)	log(capx)	sales growth	employment growth	asset growth
relative sales	14.843	17.343	0.017	0.056	0.085
	(0.290)***	(0.213)***	(0.015)	(0.012)***	(0.014)***
relative sales sq.	-13.732	-16.034	-0.037	-0.071	-0.104
	(0.417)***	(0.308)***	(0.019)*	(0.016)***	(0.019)***
R^2	0.65	0.58	0.04	0.04	0.04
N	64,091	180,076	165,392	150,097	168,734

Notes: Robust asymptotic standard errors reported in parentheses are clustered at the firm level. The sample period is from 1976 to 2004 at the annual frequency. All regressions control for year dummies and a full set of four-digit SIC industry dummies. $***p < 0.01, **p < 0.05, *p < 0.1$.

Table B2: Firm Innovation and Relative Employment

Panel A

	avg. citations	tail innov. (10%)	avg. originality	avg. generality
relative employment	6.821	5.908	7.094	16.539
	(1.110)***	(1.293)***	(1.718)***	(1.836)***
relative employment sq.	-6.692	-5.728	-5.346	-14.595
	(1.310)***	(1.642)***	(2.085)**	(2.241)***
R^2	0.15	0.10	0.26	0.25
N	101,853	101,853	101,853	101,853

Panel B

	log total patents	log total citations	log R&D spending	log R&D spending 2
relative employment	2.088	3.445	1.185	0.886
	(0.187)***	(0.294)***	(0.088)***	(0.076)***
relative employment sq.	-1.572	-2.766	-1.031	-0.812
	(0.229)***	(0.353)***	(0.110)***	(0.096)***
R^2	0.57	0.50	0.96	0.94
N	101,853	101,853	59,829	101,853

Panel C

	log(xad)	log(capx)	sales growth	employment growth	asset growth
relative employment	9.368	11.247	0.214	0.265	0.288
	(0.337)***	(0.228)***	(0.019)***	(0.016)***	(0.020)***
relative employment sq.	-8.715	-10.355	-0.193	-0.243	-0.263
	(0.434)***	(0.295)***	(0.024)***	(0.020)***	(0.025)***
R^2	0.71	0.68	0.13	0.12	0.13
N	36,987	100,605	99,912	96,718	100,690

Notes: This table replicates the results in Table A2 where relative sales is replaced with relative employment. Robust asymptotic standard errors reported in parentheses are clustered at the firm level. The sample period is from 1976 to 2004 at the annual frequency. All regressions control for profitability, leverage, market-to-book ratio, log R&D stock, firm age, the coefficient of variation of the firm's stock price, year dummies, and a full set of four-digit SIC industry dummies. $***p < 0.01, **p < 0.05, *p < 0.1$.

Table B3: Firm Innovation and Relative Total Assets

Panel A

	avg. citations	tail innov. (10%)	avg. originality	avg. generality
relative total assets	8.693	7.966	9.437	18.534
	(1.123)***	(1.338)***	(1.762)***	(1.854)***
relative total assets sq.	-8.555	-7.718	-7.669	-16.079
	(1.360)***	(1.754)***	(2.231)***	(2.325)***
R^2	0.15	0.10	0.26	0.25
N	104,911	104,911	104,911	104,911

Panel B

	log total patents	log total citations	log R&D spending	log R&D spending 2
relative total assets	2.149	3.631	1.381	0.967
	(0.192)***	(0.297)***	(0.091)***	(0.080)***
relative total assets sq.	-1.450	-2.717	-1.223	-0.896
	(0.271)***	(0.400)***	(0.117)***	(0.108)***
R^2	0.57	0.50	0.96	0.94
N	104,911	104,911	61,186	104,911

Panel C

	log(xad)	log(capx)	sales growth	employment growth	asset growth
relative total assets	9.855	12.073	0.272	0.257	0.407
	(0.329)***	(0.224)***	(0.019)***	(0.016)***	(0.021)***
relative total assets sq.	-9.092	-11.065	-0.259	-0.246	-0.381
	(0.429)***	(0.296)***	(0.024)***	(0.020)***	(0.026)***
R^2	0.72	0.69	0.12	0.12	0.13
N	37,779	103,558	102,726	96,718	103,598

Notes: This table replicates the results in Table A2 where relative sales is replaced with relative total assets. Robust asymptotic standard errors reported in parentheses are clustered at the firm level. The sample period is from 1976 to 2004 at the annual frequency. All regressions control for profitability, leverage, market-to-book ratio, log R&D stock, firm age, the coefficient of variation of the firm's stock price, year dummies, and a full set of four-digit SIC industry dummies. $***p < 0.01, **p < 0.05, *p < 0.1$.

Table B4: Firm Innovation and Relative Sales – SIC3 Fixed Effects

Panel A

	avg. citations	tail innov. (10%)	avg. originality	avg. generality
relative sales	5.879	4.976	7.035	15.018
	(1.162)***	(1.356)***	(1.788)***	(1.900)***
relative sales sq.	-6.422	-5.499	-5.508	-13.491
	(1.436)***	(1.783)***	(2.239)**	(2.398)***
R^2	0.14	0.10	0.25	0.24
N	104,911	104,911	104,911	104,911

Panel B

	log total patents	log total citations	log R&D spending	log R&D spending 2
relative sales	1.774	2.932	0.967	0.798
	(0.191)***	(0.299)***	(0.093)***	(0.080)***
relative sales sq.	-1.216	-2.258	-0.862	-0.773
	(0.264)***	(0.397)***	(0.121)***	(0.111)***
R^2	0.55	0.49	0.96	0.94
N	104,911	104,911	61,186	104,911

Panel C

	log(xad)	log(capx)	sales growth	employment growth	asset growth
relative sales	9.280	10.648	0.212	0.165	0.217
	(0.374)***	(0.259)***	(0.019)***	(0.016)***	(0.020)***
relative sales sq.	-8.942	-10.188	-0.200	-0.155	-0.207
	(0.491)***	(0.335)***	(0.024)***	(0.020)***	(0.026)***
R^2	0.68	0.65	0.12	0.11	0.12
N	37,779	103,558	102,726	96,718	103,598

Notes: Robust asymptotic standard errors reported in parentheses are clustered at the firm level. The sample period is from 1976 to 2004 at the annual frequency. All regressions control for profitability, leverage, market-to-book ratio, log R&D stock, firm age, the coefficient of variation of the firm's stock price, year dummies, and a full set of three-digit SIC industry dummies. $***p < 0.01, **p < 0.05, *p < 0.1$.

Table B5: Firm Innovation and Relative Sales – SIC2 Fixed Effects

Panel A

	avg. citations	tail innov. (10%)	avg. originality	avg. generality
relative sales	3.825	2.677	7.088	13.541
	(1.306)***	(1.513)*	(1.769)***	(1.982)***
relative sales sq.	-3.714	-2.455	-4.576	-11.063
	(1.613)**	(1.976)	(2.424)*	(2.705)***
R^2	0.13	0.08	0.23	0.22
N	104,911	104,911	104,911	104,911

Panel B

	log total patents	log total citations	log R&D spending	log R&D spending 2
relative sales	1.698	2.702	0.728	0.700
	(0.214)***	(0.332)***	(0.089)***	(0.078)***
relative sales sq.	-1.149	-1.983	-0.662	-0.695
	(0.333)***	(0.494)***	(0.114)***	(0.109)***
R^2	0.53	0.47	0.95	0.94
N	104,911	104,911	61,186	104,911

Panel C

	log(xad)	log(capx)	sales growth	employment growth	asset growth
relative sales	8.691	9.670	0.150	0.120	0.160
	(0.397)***	(0.295)***	(0.017)***	(0.015)***	(0.019)***
relative sales sq.	-8.341	-9.405	-0.140	-0.114	-0.154
	(0.524)***	(0.425)***	(0.023)***	(0.019)***	(0.024)***
R^2	0.63	0.58	0.11	0.11	0.12
N	37,779	103,558	102,726	96,718	103,598

Notes: Robust asymptotic standard errors reported in parentheses are clustered at the firm level. The sample period is from 1976 to 2004 at the annual frequency. All regressions control for profitability, leverage, market-to-book ratio, log R&D stock, firm age, the coefficient of variation of the firm's stock price, year dummies, and a full set of two-digit SIC industry dummies. $***p < 0.01, **p < 0.05, *p < 0.1$.

TABLE B6: FIRM INNOVATION AND RELATIVE SALES – FIRM FIXED EFFECTS

Panel A

	avg. citations	tail innov. (10%)	avg. originality	avg. generality
relative sales	6.181	4.026	6.030	11.929
	(1.817)***	(2.298)*	(2.701)**	(2.735)***
relative sales sq.	-5.310	-2.986	-3.321	-9.253
	(1.913)***	(2.485)	(3.098)	(2.952)***
R^2	0.44	0.34	0.52	0.50
N	104,911	104,911	104,911	104,911

Panel B

	log total patents	log total citations	log R&D spending	log R&D spending 2
relative sales	1.160	2.261	1.604	0.930
	(0.217)***	(0.357)***	(0.179)***	(0.132)***
relative sales sq.	-0.794	-1.665	-1.195	-0.729
	(0.244)***	(0.388)***	(0.207)***	(0.150)***
R^2	0.86	0.78	0.98	0.97
N	104,911	104,911	61,186	104,911

Panel C

	log(xad)	log(capx)	sale growth	employment growth	asset growth
relative sales	5.058	6.120	0.557	0.229	0.261
	(0.455)***	(0.285)***	(0.044)***	(0.035)***	(0.047)***
relative sales sq.	-3.973	-4.881	-0.475	-0.198	-0.226
	(0.537)***	(0.310)***	(0.047)***	(0.036)***	(0.048)***
R^2	0.95	0.91	0.31	0.28	0.29
N	37,779	103,558	102,726	96,718	103,598

Notes: Robust asymptotic standard errors reported in parentheses are clustered at the firm level. The sample period is from 1976 to 2004 at the frequency. All regressions control for profitability, leverage, market-to-book ratio, log R&D stock, firm age, the coefficient of variation of the firm's stock price, year dummies, and firm fixed effects. $***p < 0.01, **p < 0.05, *p < 0.1$.

Table B7: Firm Innovation and Relative Sales – Tradable Industries

Panel A

	avg. citations	tail innov. (10%)	avg. originality	avg. generality
relative sales	7.333	6.403	6.156	19.557
	(1.689)***	(2.019)***	(2.771)**	(2.816)***
relative sales sq.	-7.266	-6.581	-3.886	-16.519
	(2.051)***	(2.628)**	(3.350)	(3.365)***
R^2	0.16	0.10	0.24	0.23
N	62,621	62,621	62,621	62,621

Panel B

	log total patents	log total citations	log R&D spending	log R&D spending 2
relative sales	2.735	4.354	1.197	1.165
	(0.291)***	(0.447)***	(0.113)***	(0.120)***
relative sales sq.	-1.941	-3.290	-1.014	-1.105
	(0.355)***	(0.530)***	(0.141)***	(0.153)***
R^2	0.58	0.50	0.96	0.94
N	62,621	62,621	41,296	62,621

Panel C

	log(xad)	log(capx)	sales growth	employment growth	asset growth
relative sales	9.089	10.090	0.225	0.167	0.232
	(0.436)***	(0.289)***	(0.023)***	(0.019)***	(0.026)***
relative sales sq.	-8.300	-9.196	-0.189	-0.148	-0.191
	(0.566)***	(0.369)***	(0.028)***	(0.023)***	(0.030)***
R^2	0.79	0.74	0.13	0.12	0.15
N	21,140	61,913	61,247	58,266	61,859

Notes: Robust asymptotic standard errors reported in parentheses are clustered at the firm level. The sample is restricted to firms that operate in tradable industries which consist of agriculture, forestry, and fishing (01-09), mining (10-14), and manufacturing (20-39), where the numbers in parentheses refer to the two-digit SIC codes. All regressions control for profitability, leverage, market-to-book ratio, log R&D stock, firm age, the coefficient of variation of the firm's stock price, year dummies, and a full set of four-digit SIC industry dummies. $***p < 0.01, **p < 0.05, *p < 0.1$.

TABLE B8: FIRM INNOVATION AND RELATIVE SALES – NON-TRADABLE INDUSTRIES

Panel A

	avg. citations	tail innov. (10%)	avg. originality	avg. generality
relative sales	6.369	5.777	8.067	9.357
	(1.583)***	(1.738)***	(2.029)***	(2.398)***
relative sales sq.	-5.803	-4.941	-6.592	-6.439
	(1.775)***	(2.180)**	(2.361)***	(3.044)**
R^2	0.08	0.07	0.12	0.10
N	41,189	41,189	41,189	41,189

Panel B

	log total patents	log total citations	log R&D spending	log R&D spending 2
relative sales	0.603	1.330	0.848	0.347
	(0.206)***	(0.356)***	(0.153)***	(0.093)***
relative sales sq.	-0.147	-0.705	-0.745	-0.267
	(0.404)	(0.608)	(0.222)***	(0.135)**
R^2	0.34	0.27	0.95	0.92
N	41,189	41,189	19,293	41,189

Panel C

	log(xad)	log(capx)	sales growth	employment growth	asset growth
relative sales	12.398	13.662	0.244	0.182	0.226
	(0.500)***	(0.362)***	(0.034)***	(0.028)***	(0.035)***
relative sales sq.	-11.695	-12.556	-0.255	-0.184	-0.237
	(0.638)***	(0.477)***	(0.042)***	(0.033)***	(0.044)***
R^2	0.65	0.63	0.15	0.14	0.17
N	16,375	40,578	40,421	37,493	40,655

Notes: Robust asymptotic standard errors reported in parentheses are clustered at the firm level. The sample is restricted to firms that operate in nontradable industries which consist of construction (15-17), transportation and public utilities (40-49), wholesale trade (50-51), retail trade (52-59), finance, insurance, and real estate (60-67), services (70-89), and public administration (91-98), where the numbers in parentheses refer to the two-digit SIC codes. All regressions control for profitability, leverage, market-to-book ratio, log R&D stock, firm age, the coefficient of variation of the firm's stock price, year dummies, and a full set of four-digit SIC industry dummies. $***p < 0.01, **p < 0.05, *p < 0.1$.

Table B9: Firm Innovation and Relative Sales – Early Sample (1976-1994)

Panel A

	avg. citations	tail innov. (10%)	avg. originality	avg. generality
relative sales	5.508	5.163	7.746	16.394
	(1.605)***	(1.891)***	(1.895)***	(2.610)***
relative sales sq.	-6.255	-6.285	-6.581	-14.999
	(1.886)***	(2.417)***	(2.304)***	(3.166)***
R^2	0.18	0.13	0.23	0.29
N	59,236	59,236	59,236	59,236

Panel B

	log total patents	log total citations	log R&D spending	log R&D spending 2
relative sales	1.757	2.913	1.216	0.791
	(0.236)***	(0.376)***	(0.109)***	(0.092)***
relative sales sq.	-1.045	-2.046	-0.994	-0.691
	(0.334)***	(0.498)***	(0.145)***	(0.128)***
R^2	0.61	0.55	0.97	0.95
N	59,236	59,236	32,673	59,236

Panel C

	log(xad)	log(capx)	sales growth	employment growth	asset growth
relative sales	9.337	10.916	0.271	0.219	0.319
	(0.353)***	(0.262)***	(0.024)***	(0.020)***	(0.025)***
relative sales sq.	-8.501	-10.013	-0.268	-0.221	-0.309
	(0.482)***	(0.339)***	(0.030)***	(0.025)***	(0.032)***
R^2	0.78	0.72	0.13	0.12	0.12
N	25,308	58,521	57,742	54,622	58,132

Notes: Robust asymptotic standard errors reported in parentheses are clustered at the firm level. The sample period is from 1976 to 1994 at the annual frequency. All regressions control for profitability, leverage, market-to-book ratio, log R&D stock, firm age, the coefficient of variation of the firm's stock price, year dummies, and a full set of four-digit SIC industry dummies. $***p < 0.01, **p < 0.05, *p < 0.1$.

TABLE B10: FIRM INNOVATION AND RELATIVE SALES – LATE SAMPLE (1995-2005)

Panel A

	avg. citations	tail innov. (10%)	avg. originality	avg. generality
relative sales	7.747	5.361	14.412	9.819
	(1.292)***	(1.494)***	(2.546)***	(1.519)***
relative sales sq.	-7.483	-5.039	-10.516	-7.252
	(1.567)***	(1.865)***	(3.250)***	(1.978)***
R^2	0.14	0.10	0.28	0.21
N	49,522	49,522	49,522	49,522

Panel B

	log total patents	log total citations	log R&D spending	log R&D spending 2
relative sales	2.670	3.925	1.794	1.405
	(0.246)***	(0.339)***	(0.139)***	(0.122)***
relative sales sq.	-1.869	-2.961	-1.586	-1.329
	(0.318)***	(0.431)***	(0.179)***	(0.176)***
R^2	0.54	0.45	0.95	0.94
N	49,522	49,522	31,004	49,522

Panel C

	log(xad)	log(capx)	sales growth	employment growth	asset growth
relative sales	14.715	14.154	0.226	0.174	0.226
	(0.577)***	(0.299)***	(0.032)***	(0.027)***	(0.035)***
relative sales sq.	-14.015	-13.226	-0.172	-0.141	-0.187
	(0.776)***	(0.408)***	(0.042)***	(0.034)***	(0.046)***
R^2	0.72	0.68	0.14	0.13	0.15
N	14,063	48,881	44,984	42,096	45,466

Notes: Robust asymptotic standard errors reported in parentheses are clustered at the firm level. The sample period is from 1995 to 2005 at the annual frequency. All regressions control for profitability, leverage, market-to-book ratio, log R&D stock, firm age, the coefficient of variation of the firm's stock price, year dummies, and a full set of four-digit SIC industry dummies. $***p < 0.01, **p < 0.05, *p < 0.1$.

Table B11: Industry Innovation and Market Concentration (HHI) – Baseline Specification (Inverted-U Hypothesis Test)

Panel A: Total Innovation by Industry

	patent count	total citations	tail count	original count	general count
lower bound					
t-value	3.488	4.195	4.415	2.583	2.084
P>|t|	0.000	0.000	0.000	0.005	0.019
upper bound					
t-value	-2.472	-2.612	-2.377	-1.352	-1.470
P>|t|	0.007	0.005	0.009	0.088	0.071

Panel B: Industry Average of Total Innovation by Firms

	patent count	total citations	tail count	original count	general count
lower bound					
t-value	2.008	2.853	3.004	1.528	2.889
P>|t|	0.022	0.002	0.001	0.063	0.002
upper bound					
t-value	-2.617	-3.658	-3.605	-1.426	-2.989
P>|t|	0.004	0.000	0.000	0.077	0.001

Panel C: Industry Average of Average Innovation Quality by Firms

	avg. citations	tail innov	avg. orginality	avg. generality
lower bound				
t-value	4.172	3.219	3.340	3.158
P>|t|	0.000	0.001	0.000	0.001
upper bound				
t-value	-3.271	-1.944	-1.088	-2.492
P>|t|	0.001	0.026	0.138	0.006

Notes: To further check the robustness of the inverted-U relationship between industry innovation and market concentration, we test whether or not the slope of the fitted curve is positive at the start and negative at the end of the interval of the market concentration following Lind and Mehlum (2010). This table reports the hypothesis testing results.

Table B12: Firm Innovation and Relative Sales – Baseline Specification (Inverted-U Hypothesis Test)

Panel A

	avg. citations	tail innov	avg. orginality	avg. generality
lower bound				
t-value	6.614	4.952	4.625	9.277
P>|t|	0.000	0.000	0.000	0.000
upper bound				
t-value	-4.246	-2.779	-1.442	-4.020
P>|t|	0.000	0.003	0.075	0.000

Panel B

	log total patents	log total citations	log R&D Spending	log R&D spending 2
lower bound				
t-value	10.671	11.494	14.158	12.012
P>|t|	0.000	0.000	0.000	0.000
upper bound				
t-value	-2.048	-3.256	-6.363	-5.808
P>|t|	0.020	0.001	0.000	0.000

Panel C

	log(xad)	log(capx)	sales growth	employment growth	asset growth
lower bound					
t-value	31.841	52.415	12.784	11.965	12.938
P>|t|	0.000	0.000	0.000	0.000	0.000
upper bound					
t-value	-16.054	-25.941	-6.824	-6.877	-7.018
P>|t|	0.000	0.000	0.000	0.000	0.000

Notes: To further check the robustness of the inverted-U relationship between firm innovation and relative sales, we test whether or not the slope of the fitted curve is positive at the start and negative at the end of the interval of relative sales following Lind and Mehlum (2010). This table reports the hypothesis testing results.

TABLE B13: MODEL PARAMETERS AND TARGET MOMENTS — LATER SUBSAMPLE (2006-2016)

A. Parameter estimates

Parameter	*Description*	*Values*
λ	innovation step size	0.2607
η	elasticity within industry	6.5806
χ	superstar cost scale	34.3063
ν	small firm cost scale	1.8523
ζ	competitive fringe ratio	0.5150
ϕ	superstar cost convexity	3.1140
ϵ	small firm cost convexity	1.9713
τ	exit rate	0.0833
ψ	entry cost scale	0.0240

B. Moments

Target moments	Data	Model
growth rate	1.45%	1.67%
R&D intensity	2.70%	2.67%
average markup	1.4856	1.4855
std. dev. markup	0.508	0.454
labor share	0.613	0.611
firm entry rate	0.083	0.083
β(innovation, relative sales)	0.579	0.701
top point (intra-industry)	0.463	0.420
average profitability	0.163	0.230
average leader relative quality	0.765	0.649
std. dev. leader relative quality	0.213	0.160

Notes: The estimation is done with the simulated method of moments. Panel A reports the estimated parameters. Panel B reports the simulated and actual moments. We rely on patent citation data from UVA Darden Global Corporate Patent Dataset to construct the innovation-related data moments (Bena, Ferreira, Matos, and Pires (2017)). We exclude the years 2007-2009 when constructing the data moment for the output growth rate to exclude the effect of the Great Recession.

Table B14: Disentangling the Structural Transition — Later Subsample (2006-2016)

	Benchmark	Early η	% change	Early ζ	% change	Early ν, ϵ	% change
growth rate	1.67%	1.41%	-15.35%	1.17%	-30.01%	2.04%	22.09%
R&D intensity	2.67%	2.49%	-6.46%	1.44%	-45.92%	3.94%	47.98%
average markup	1.486	1.503	1.21%	1.327	-10.65%	1.488	0.19%
std. dev. markup	0.454	0.435	-4.12%	0.378	-16.75%	0.424	-6.63%
labor share	0.611	0.600	-1.76%	0.668	9.41%	0.603	-1.30%
entry rate	0.083	0.083	0.00%	0.083	0.00%	0.083	0.00%
β(innov, relative sales)	0.701	0.521	-25.62%	0.673	-3.95%	0.990	41.25%
top point (intra-industry)	0.420	0.384	-8.71%	0.413	-1.68%	0.436	3.84%
avg. profitability	0.230	0.245	6.29%	0.172	-25.22%	0.228	-1.17%
avg. leader relative quality	0.649	0.714	9.97%	0.726	11.91%	0.487	-25.00%
std. dev. leader rel. quality	0.160	0.189	18.05%	0.193	20.50%	0.106	-34.04%
superstar innovation	0.146	0.120	-18.13%	0.100	-31.95%	0.225	53.69%
small firm innovation	0.012	0.004	-61.91%	0.005	-56.55%	0.066	467.53%
output share of superstars	0.564	0.600	6.42%	0.444	-21.17%	0.618	9.60%
avg. superstars per industry	2.022	1.753	-13.28%	1.749	-13.52%	3.220	59.27%
mass of small firms	1.000	0.569	-43.12%	0.553	-44.71%	1.886	88.60%
initial output	0.808	0.744	-8.01%	0.829	2.58%	0.840	3.88%
CE Welfare change		-13.56%		-8.36%		12.38%	

	Benchmark	Early χ, ϕ	% change	Early ψ, τ	% change	All	% change
growth rate	1.67%	2.04%	22.19%	1.75%	4.41%	2.19%	30.87%
R&D intensity	2.67%	2.70%	1.38%	2.82%	5.94%	2.07%	-22.31%
average markup	1.486	1.482	-0.26%	1.487	0.12%	1.301	-12.40%
std. dev. markup	0.454	0.452	-0.39%	0.450	-0.90%	0.325	-28.31%
labor share	0.611	0.612	0.18%	0.609	-0.26%	0.628	2.79%
entry rate	0.083	0.083	0.00%	0.114	37.22%	0.114	37.22%
β(innov, relative sales)	0.701	0.677	-3.38%	0.705	0.64%	0.683	-2.52%
top point (intra-industry)	0.420	0.423	0.61%	0.423	0.67%	0.462	9.93%
avg. profitability	0.230	0.229	-0.53%	0.231	0.19%	0.162	-29.42%
avg. leader relative quality	0.649	0.630	-2.85%	0.628	-3.14%	0.607	-6.42%
std. dev. leader rel. quality	0.160	0.161	0.44%	0.149	-6.94%	0.140	-12.66%
superstar innovation	0.146	0.187	27.84%	0.155	5.94%	0.180	23.13%
small firm innovation	0.012	0.020	71.74%	0.014	19.93%	0.028	139.75%
output share of superstars	0.564	0.566	0.34%	0.572	1.39%	0.483	-14.34%
avg. superstars per industry	2.022	2.173	7.47%	2.109	4.32%	2.412	19.28%
mass of small firms	1.000	1.394	39.40%	1.327	32.68%	1.000	0.00%
initial output	0.808	0.808	-0.06%	0.813	0.59%	0.769	-4.88%
CE Welfare change		9.59%		2.29%		8.88%	

Notes: The table reports the change in model moments when setting the parameter of interest back to its estimated level in early sub-sample while keeping other parameters fixed at their estimated values in the later sub-sample.

TABLE B15: MODEL PARAMETERS AND TARGET MOMENTS — CRRA UTILITY

A. Parameter estimates

Parameter	*Description*	*Early sub-sample*	*Late sub-sample*
λ	innovation step size	0.2933	0.3140
η	elasticity within industry	20.4221	6.4435
χ	superstar cost scale	105.5469	72.8941
ν	small firm cost scale	1.3751	2.4345
ζ	competitive fringe ratio	0.5987	0.5354
ϕ	superstar cost convexity	3.8834	3.6608
ϵ	small firm cost convexity	2.7628	2.4430
τ	exit rate	0.1144	0.0964
ψ	entry cost scale	0.0102	0.0268

B. Moments

	Early sub-sample		Late sub-sample	
Target moments	Data	Model	Data	Model
growth rate	2.19%	2.21%	2.31%	2.31%
R&D intensity	2.40%	2.07%	2.50%	2.42%
average markup	1.3014	1.3143	1.4442	1.4391
std. dev. markup	0.306	0.332	0.421	0.444
labor share	0.656	0.623	0.644	0.611
firm entry rate	0.114	0.114	0.096	0.096
β(innovation, relative sales)	0.449	0.671	0.631	0.767
top point (intra-industry)	0.443	0.456	0.515	0.445
average profitability	0.136	0.169	0.152	0.211
average leader relative quality	0.751	0.608	0.746	0.652
std. dev. leader relative quality	0.224	0.142	0.222	0.160

Notes: We estimate the model with CRRA preferences using the simulated method of moments. Panel A reports the estimated parameters. Panel B reports the simulated and actual moments.

TABLE B16: DISENTANGLING THE STRUCTURAL TRANSITION — CRRA UTILITY

	Benchmark	Early η	% change	Early ζ	% change	Early ν,ϵ	% change
growth rate	2.31%	2.12%	-8.14%	2.04%	-11.39%	2.54%	10.28%
R&D intensity	2.42%	2.41%	-0.28%	1.84%	-23.77%	2.80%	15.86%
average markup	1.439	1.441	0.10%	1.336	-7.18%	1.443	0.29%
std. dev. markup	0.444	0.416	-6.47%	0.386	-13.06%	0.433	-2.53%
labor share	0.611	0.605	-0.96%	0.647	5.83%	0.607	-0.69%
entry rate	0.096	0.096	0.00%	0.096	0.00%	0.096	0.00%
β(innov, relative sales)	0.767	0.683	-11.02%	0.768	0.18%	0.809	5.53%
top point (intra-industry)	0.445	0.432	-2.79%	0.443	-0.29%	0.457	2.68%
avg. profitability	0.211	0.218	3.42%	0.171	-18.56%	0.212	0.73%
avg. leader relative quality	0.652	0.697	6.98%	0.682	4.64%	0.573	-12.15%
std. dev. leader rel. quality	0.160	0.171	6.94%	0.170	6.32%	0.135	-15.79%
superstar innovation	0.178	0.155	-13.06%	0.153	-13.82%	0.219	22.74%
small firm innovation	0.022	0.012	-43.83%	0.016	-26.91%	0.045	104.83%
output share of superstars	0.521	0.554	6.21%	0.449	-13.93%	0.545	4.56%
avg. superstars per industry	2.196	1.940	-11.66%	2.048	-6.75%	2.731	24.35%
mass of small firms	1.000	0.710	-28.98%	0.792	-20.81%	1.214	21.35%
initial output	0.805	0.739	-8.22%	0.827	2.75%	0.818	1.72%
CE Welfare change		-12.20%		-2.94%		6.88%	

	Benchmark	Early χ,ϕ	% change	Early ψ,τ	% change	All	% change
growth rate	2.31%	2.40%	3.97%	2.41%	4.46%	2.21%	-4.24%
R&D intensity	2.42%	2.35%	-2.83%	2.55%	5.58%	2.07%	-14.36%
average markup	1.439	1.439	-0.02%	1.441	0.13%	1.314	-8.67%
std. dev. markup	0.444	0.443	-0.19%	0.440	-0.95%	0.332	-25.22%
labor share	0.611	0.611	-0.01%	0.609	-0.29%	0.623	1.98%
entry rate	0.096	0.096	0.00%	0.114	18.65%	0.114	18.65%
β(innov, relative sales)	0.767	0.740	-3.55%	0.766	-0.09%	0.671	-12.59%
top point (intra-industry)	0.445	0.443	-0.40%	0.449	0.90%	0.456	2.50%
avg. profitability	0.211	0.211	0.40%	0.211	0.42%	0.169	-19.74%
avg. leader relative quality	0.652	0.646	-0.94%	0.623	-4.43%	0.608	-6.68%
std. dev. leader rel. quality	0.160	0.161	0.59%	0.149	-6.81%	0.142	-11.55%
superstar innovation	0.178	0.188	5.68%	0.192	7.81%	0.185	3.96%
small firm innovation	0.022	0.024	10.63%	0.028	26.47%	0.027	20.13%
output share of superstars	0.521	0.523	0.28%	0.531	1.75%	0.495	-5.05%
avg. superstars per industry	2.196	2.243	2.13%	2.358	7.39%	2.372	8.03%
mass of small firms	1.000	1.066	6.64%	1.452	45.19%	1.000	0.00%
initial output	0.805	0.805	0.08%	0.810	0.67%	0.767	-4.71%
CE Welfare change		2.28%		2.93%		-6.53%	

Notes: Using the model with CRRA preferences, we carry out the same experiments as in the baseline. The table reports the changes in model moments when setting parameters of interest back to their estimated levels in the early sub-sample while keeping other parameters fixed at their estimated values in the late sub-sample.

Table B17: Disentangling the Structural Transition — With Capital

	Benchmark	Early η	% change	Early ζ	% change	Early ν, ϵ	% change
growth rate	2.31%	2.07%	-10.39%	1.83%	-20.99%	2.74%	18.28%
R&D intensity	1.99%	1.89%	-5.07%	1.24%	-37.68%	2.65%	33.02%
average markup	1.444	1.448	0.24%	1.320	-8.56%	1.450	0.39%
std. dev. markup	0.452	0.425	-6.01%	0.381	-15.74%	0.438	-3.25%
labor share	0.610	0.604	-1.06%	0.653	6.98%	0.605	-0.92%
entry rate	0.096	0.096	0.00%	0.096	0.00%	0.096	0.00%
β(innov, relative sales)	0.783	0.706	-9.94%	0.795	1.42%	0.838	7.01%
top point (intra-industry)	0.448	0.435	-2.91%	0.443	-1.01%	0.466	4.16%
avg. profitability	0.210	0.219	4.35%	0.166	-21.04%	0.209	-0.49%
avg. leader relative quality	0.678	0.720	6.18%	0.728	7.42%	0.569	-16.02%
std. dev. leader rel. quality	0.165	0.176	6.78%	0.181	9.40%	0.130	-21.06%
superstar innovation	0.169	0.145	-13.86%	0.129	-23.48%	0.230	36.16%
small firm innovation	0.019	0.011	-41.25%	0.011	-41.58%	0.052	175.60%
output share of superstars	0.516	0.549	6.25%	0.429	-16.89%	0.547	6.04%
avg. superstars per industry	2.090	1.874	-10.33%	1.868	-10.59%	2.819	34.89%
mass of small firms	1.000	0.719	-28.13%	0.667	-33.28%	1.387	38.71%
initial output	0.883	0.821	-7.06%	0.921	4.30%	0.896	1.42%
CE Welfare change		-12.24%		-6.53%		11.57%	

	Benchmark	Early χ, ϕ	% change	Early ψ, τ	% change	All	% change
growth rate	2.31%	2.39%	3.14%	2.46%	6.37%	2.19%	-5.40%
R&D intensity	1.99%	1.97%	-0.97%	2.17%	9.05%	1.59%	-20.04%
average markup	1.444	1.444	0.02%	1.446	0.14%	1.301	-9.88%
std. dev. markup	0.452	0.451	-0.31%	0.448	-0.91%	0.325	-28.06%
labor share	0.610	0.610	-0.07%	0.609	-0.29%	0.628	2.87%
entry rate	0.096	0.096	0.00%	0.114	18.65%	0.114	18.65%
β(innov, relative sales)	0.783	0.748	-4.55%	0.775	-1.06%	0.683	-12.82%
top point (intra-industry)	0.448	0.447	-0.28%	0.452	0.98%	0.462	3.17%
avg. profitability	0.210	0.211	0.40%	0.210	0.01%	0.162	-22.74%
avg. leader relative quality	0.678	0.668	-1.48%	0.649	-4.31%	0.607	-10.44%
std. dev. leader rel. quality	0.165	0.164	-0.70%	0.154	-6.72%	0.140	-15.27%
superstar innovation	0.169	0.177	4.85%	0.184	9.24%	0.180	6.75%
small firm innovation	0.019	0.021	11.64%	0.024	27.35%	0.028	46.31%
output share of superstars	0.516	0.519	0.55%	0.525	1.74%	0.483	-6.46%
avg. superstars per industry	2.090	2.149	2.82%	2.239	7.16%	2.412	15.41%
mass of small firms	1.000	1.074	7.38%	1.438	43.84%	1.000	0.00%
initial output	0.883	0.883	0.05%	0.886	0.37%	0.913	3.35%
CE Welfare change		1.86%		3.82%		-1.69%	

Notes: Using the model with endogenous physical capital accumulation in Appendix A.7, we carry out the same experiments as in the baseline. The table reports the changes in model moments when setting parameters of interest back to their estimated levels in the early sub-sample while keeping other parameters fixed at their estimated values in the late sub-sample.

Table B18: Model Parameters and Target Moments — Cost-weighted Markups

A. Parameter estimates

Parameter	*Description*	*Early sub-sample*	*Late sub-sample*
λ	innovation step size	0.2854	0.3055
η	elasticity within industry	20.6825	6.8640
χ	superstar cost scale	105.9986	73.6554
ν	small firm cost scale	1.3957	2.5565
ζ	competitive fringe ratio	0.6130	0.5620
ϕ	superstar cost convexity	3.8583	3.6532
ϵ	small firm cost convexity	3.2012	2.6433
τ	exit rate	0.1144	0.0964
ψ	entry cost scale	0.0111	0.0246

B. Moments

	Early sub-sample		Late sub-sample	
Target moments	Data	Model	Data	Model
growth rate	2.19%	2.20%	2.31%	2.30%
R&D intensity	2.40%	2.17%	2.50%	2.49%
average markup (cost-weighted)	1.1793	1.2187	1.2470	1.2764
std. dev. markup	0.306	0.314	0.421	0.410
labor share	0.656	0.631	0.644	0.625
firm entry rate	0.114	0.114	0.096	0.096
β(innovation, relative sales)	0.449	0.702	0.631	0.765
top point (intra-industry)	0.443	0.471	0.515	0.451
average profitability	0.136	0.158	0.152	0.192
average leader relative quality	0.751	0.568	0.746	0.628
std. dev. leader relative quality	0.224	0.131	0.222	0.154

Notes: We re-estimate the baseline model using cost-weighted markups. The estimation is done with the simulated method of moments. Panel A reports the estimated parameters. Panel B reports the simulated and actual moments.

TABLE B19: DISENTANGLING THE STRUCTURAL TRANSITION — COST-WEIGHTED MARKUPS

	Benchmark	Early η	% change	Early ζ	% change	Early ν, ϵ	% change
growth rate	2.30%	2.07%	-9.86%	2.04%	-11.36%	2.57%	11.55%
R&D intensity	2.49%	2.28%	-8.57%	1.86%	-25.51%	3.10%	24.49%
average markup (cost-weighted)	1.276	1.278	0.16%	1.221	-4.31%	1.285	0.66%
average markup (sales-weighted)	1.392	1.382	-0.73%	1.314	-5.61%	1.396	0.31%
std. dev. markup (sales-weighted)	0.410	0.380	-7.39%	0.365	-11.02%	0.400	-2.41%
labor share	0.625	0.624	-0.16%	0.653	4.50%	0.621	-0.65%
entry rate	0.096	0.096	0.00%	0.096	0.00%	0.096	0.00%
β(innov, relative sales)	0.765	0.690	-9.79%	0.759	-0.83%	0.849	11.03%
top point (intra-industry)	0.451	0.439	-2.76%	0.447	-0.92%	0.465	3.05%
avg. profitability	0.192	0.195	1.65%	0.163	-15.13%	0.191	-0.45%
avg. leader relative quality	0.628	0.678	7.93%	0.657	4.66%	0.536	-14.70%
std. dev. leader rel. quality	0.154	0.166	8.09%	0.164	6.50%	0.124	-19.49%
superstar innovation	0.186	0.157	-15.62%	0.160	-14.15%	0.238	27.94%
small firm innovation	0.027	0.015	-46.26%	0.019	-29.04%	0.062	128.51%
output share of superstars	0.503	0.525	4.39%	0.443	-11.84%	0.527	4.84%
avg. superstars per industry	2.323	2.021	-12.99%	2.157	-7.15%	3.033	30.59%
mass of small firms	1.000	0.672	-32.81%	0.766	-23.42%	1.318	31.78%
initial output	0.812	0.748	-7.81%	0.830	2.23%	0.825	1.60%
CE Welfare change		-12.70%		-3.60%		7.88%	

	Benchmark	Early χ, ϕ	% change	Early ψ, τ	% change	All	% change
growth rate	2.30%	2.40%	4.13%	2.39%	3.77%	2.20%	-4.43%
R&D intensity	2.49%	2.49%	0.03%	2.65%	6.48%	2.17%	-12.83%
average markup (cost-weighted)	1.276	1.277	0.03%	1.279	0.20%	1.219	-4.52%
average markup (sales-weighted)	1.392	1.392	-5.58E-05	1.393	0.10%	1.292	-7.20%
std. dev. markup	0.410	0.409	-0.26%	0.407	-0.64%	0.314	-23.31%
labor share	0.625	0.625	-0.03%	0.624	-0.20%	0.631	0.91%
entry rate	0.096	0.096	0.00%	0.114	18.65%	0.114	18.65%
β(innov, relative sales)	0.765	0.736	-3.74%	0.772	0.89%	0.702	-8.21%
top point (intra-industry)	0.451	0.451	-0.15%	0.456	0.94%	0.471	4.31%
avg. profitability	0.192	0.192	0.17%	0.192	-0.02%	0.158	-17.77%
avg. leader relative quality	0.628	0.618	-1.54%	0.604	-3.79%	0.568	-9.48%
std. dev. leader rel. quality	0.154	0.154	0.06%	0.145	-5.81%	0.131	-14.71%
superstar innovation	0.186	0.198	6.48%	0.199	6.83%	0.199	6.64%
small firm innovation	0.027	0.031	12.95%	0.033	22.70%	0.039	44.38%
output share of superstars	0.503	0.505	0.45%	0.510	1.33%	0.485	-3.53%
avg. superstars per industry	2.323	2.396	3.15%	2.472	6.42%	2.646	13.90%
mass of small firms	1.000	1.090	9.01%	1.360	35.95%	1.000	0.00%
initial output	0.812	0.813	0.13%	0.815	0.44%	0.774	-4.61%
CE Welfare change		2.53%		2.48%		-6.70%	

Notes: Using the reestimated baseline model targeting cost-weighted markups, we carry out the same experiments as in the baseline. The table reports the changes in model moments when setting parameters of interest back to their estimated levels in the early sub-sample while keeping other parameters fixed at their estimated values in the late sub-sample.

TABLE B20: MODEL PARAMETERS AND TARGET MOMENTS — WITHOUT MARKUP-BASED TARGETS

A. Parameter estimates

Parameter	*Description*	*Early sub-sample*	*Late sub-sample*
λ	innovation step size	0.2869	0.3117
η	elasticity within industry	19.4859	6.6690
χ	superstar cost scale	104.8459	69.7572
ν	small firm cost scale	1.3992	2.3629
ζ	competitive fringe ratio	0.6111	0.5563
ϕ	superstar cost convexity	3.8765	3.6370
ϵ	small firm cost convexity	2.8704	2.4661
τ	exit rate	0.1144	0.0964
ψ	entry cost scale	0.0099	0.0222

B. Moments

	Early sub-sample		Late sub-sample	
Target moments	Data	Model	Data	Model
growth rate	2.19%	2.18%	2.31%	2.29%
R&D intensity	2.40%	2.08%	2.50%	2.43%
labor share	0.656	0.631	0.644	0.623
firm entry rate	0.114	0.114	0.096	0.096
β(innovation, relative sales)	0.449	0.681	0.631	0.770
top point (intra-industry)	0.443	0.462	0.515	0.448
average profitability	0.136	0.159	0.152	0.195
average leader relative quality	0.751	0.597	0.746	0.652
std. dev. leader relative quality	0.224	0.140	0.222	0.160

Notes: We re-estimate the model after dropping the average markup and the standard deviation of markups from the set of targeted moments. The estimation is done with the simulated method of moments. Panel A reports the estimated parameters. Panel B reports the simulated and actual moments.

TABLE B21: DISENTANGLING THE STRUCTURAL TRANSITION — WITHOUT MARKUP-BASED TARGETS

	Benchmark	Early η	% change	Early ζ	% change	Early ν, ϵ	% change
growth rate	2.29%	2.00%	-12.49%	1.98%	-13.54%	2.60%	13.38%
R&D intensity	2.43%	2.17%	-10.51%	1.75%	-27.82%	3.05%	25.47%
labor share	0.623	0.623	-0.10%	0.653	4.83%	0.619	-0.69%
entry rate	0.096	0.096	0.00%	0.096	0.00%	0.096	0.00%
β(innov, relative sales)	0.770	0.712	-7.62%	0.772	0.17%	0.828	7.44%
top point (intra-industry)	0.448	0.437	-2.53%	0.444	-0.93%	0.464	3.41%
avg. profitability	0.195	0.198	1.59%	0.164	-15.98%	0.194	-0.33%
avg. leader relative quality	0.652	0.706	8.31%	0.685	5.14%	0.562	-13.71%
std. dev. leader rel. quality	0.160	0.175	9.36%	0.171	7.28%	0.131	-17.99%
average markup	1.401	1.389	-0.81%	1.316	-6.04%	1.405	0.33%
std. dev. markup	0.421	0.390	-7.38%	0.372	-11.72%	0.411	-2.41%
superstar innovation	0.177	0.147	-17.23%	0.149	-16.04%	0.227	28.08%
small firm innovation	0.022	0.012	-46.42%	0.015	-31.71%	0.051	129.17%
output share of superstars	0.500	0.520	4.08%	0.437	-12.63%	0.525	4.89%
avg. superstars per industry	2.196	1.915	-12.81%	2.028	-7.65%	2.818	28.34%
mass of small firms	1.000	0.671	-32.91%	0.749	-25.06%	1.310	30.99%
initial output	0.808	0.744	-7.97%	0.828	2.44%	0.822	1.66%
CE Welfare change		-14.09%		-4.53%		9.04%	

	Benchmark	Early χ, ϕ	% change	Early ψ, τ	% change	All	% change
growth rate	2.29%	2.42%	5.62%	2.40%	4.65%	2.18%	-4.64%
R&D intensity	2.43%	2.44%	0.44%	2.60%	7.19%	2.08%	-14.34%
labor share	0.623	0.623	-0.05%	0.622	-0.22%	0.631	1.23%
entry rate	0.096	0.096	0.00%	0.114	18.65%	0.114	18.65%
β(innov, relative sales)	0.770	0.738	-4.19%	0.770	-0.09%	0.681	-11.66%
top point (intra-industry)	0.448	0.448	-0.17%	0.453	0.93%	0.462	2.99%
avg. profitability	0.195	0.195	0.18%	0.195	0.01%	0.159	-18.63%
avg. leader relative quality	0.652	0.641	-1.68%	0.627	-3.81%	0.597	-8.47%
std. dev. leader rel. quality	0.160	0.159	-0.25%	0.150	-5.98%	0.140	-12.42%
average markup	1.401	1.401	0.00%	1.402	0.12%	1.293	-7.70%
std. dev. markup	0.421	0.420	-0.29%	0.418	-0.69%	0.319	-24.34%
superstar innovation	0.177	0.192	8.02%	0.190	7.43%	0.189	6.71%
small firm innovation	0.022	0.026	16.60%	0.028	23.63%	0.030	34.88%
output share of superstars	0.500	0.503	0.52%	0.507	1.44%	0.480	-4.02%
avg. superstars per industry	2.196	2.272	3.46%	2.336	6.35%	2.446	11.36%
mass of small firms	1.000	1.109	10.94%	1.370	36.97%	1.000	0.00%
initial output	0.808	0.810	0.15%	0.812	0.49%	0.773	-4.38%
CE Welfare change		3.41%		3.01%		-6.55%	

Notes: Using the re-estimated model without markup-based targets, we carry out the same experiments as in the baseline. The table reports the changes in model moments when setting parameters of interest back to their estimated levels in the early sub-sample while keeping other parameters fixed at their estimated values in the late sub-sample.

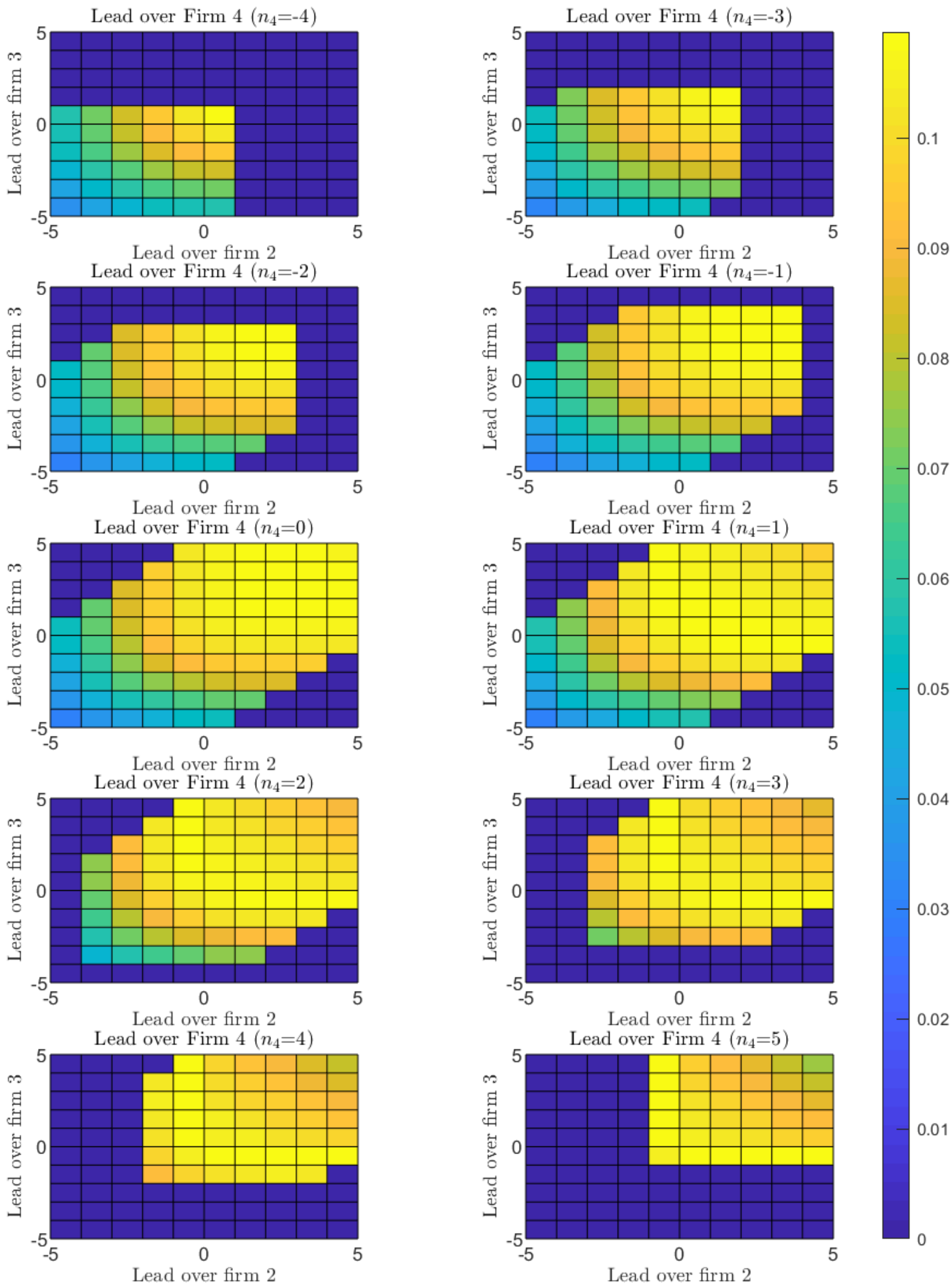
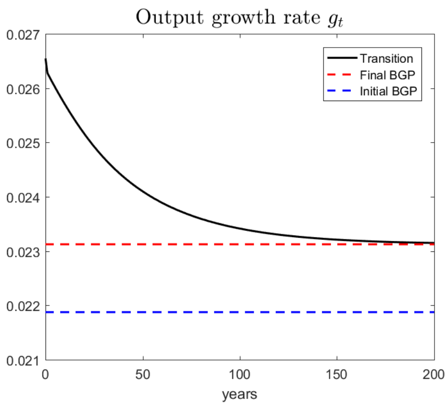
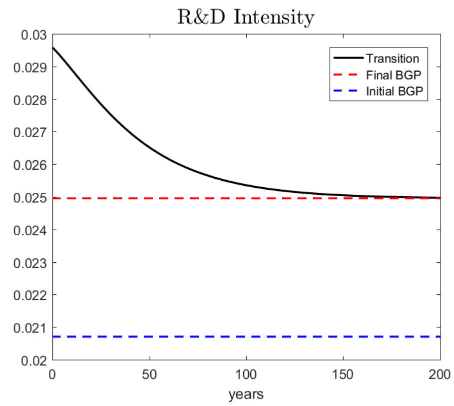


FIGURE B1: INNOVATION POLICY FUNCTION (N=4)

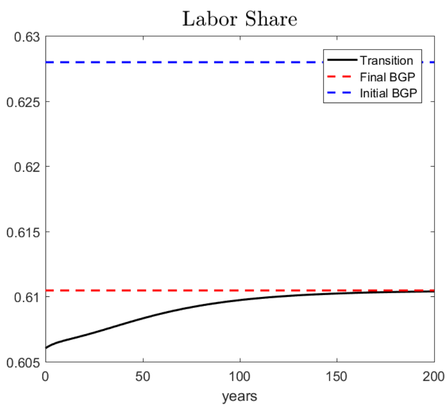
Notes: This figure displays the optimal innovation policy functions followed by the firms in an industry with four superstars. Each subfigure corresponds to the fourth competitor being a certain number of steps behind the current firm.



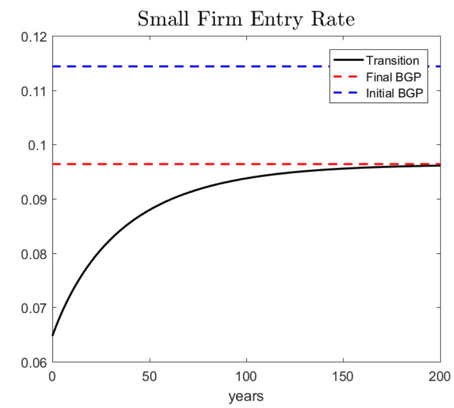
(A) Growth rate



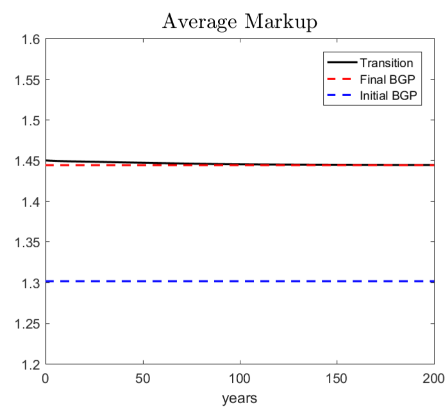
(B) R&D intensity



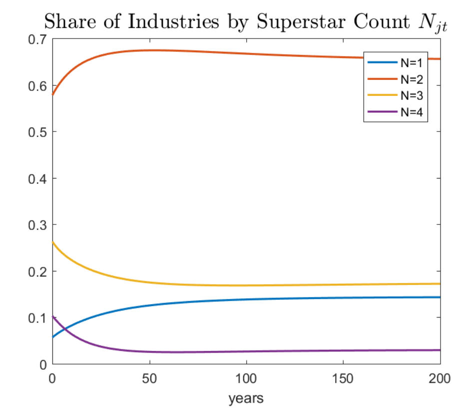
(C) Labor share



(D) Entry rate



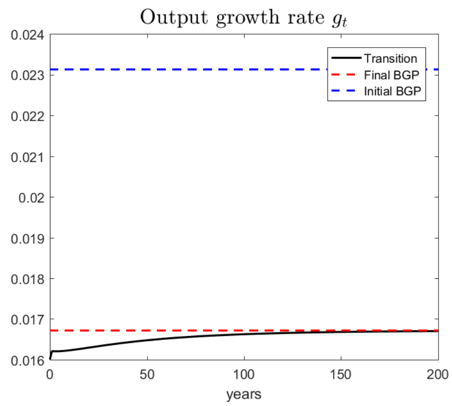
(E) Average markup



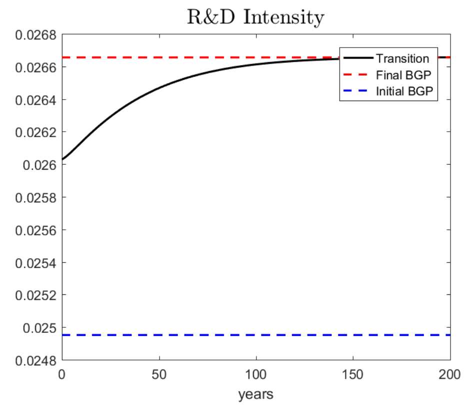
(F) Superstar count

Figure B2: Transitional Dynamics: Early to Late Period

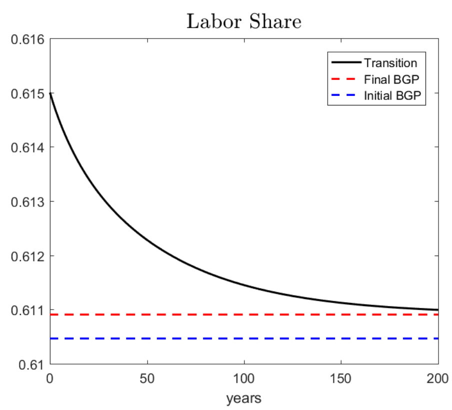
Notes: This figure illustrates the time paths of selected variables over time during the transition from the early to the late period stationary equilibrium.



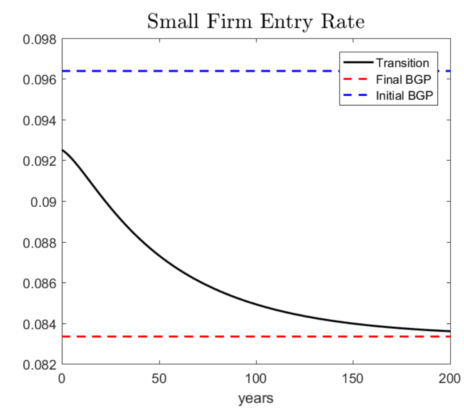
(A) GROWTH RATE



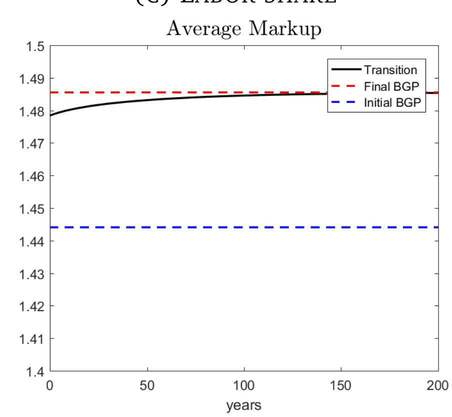
(B) R&D INTENSITY



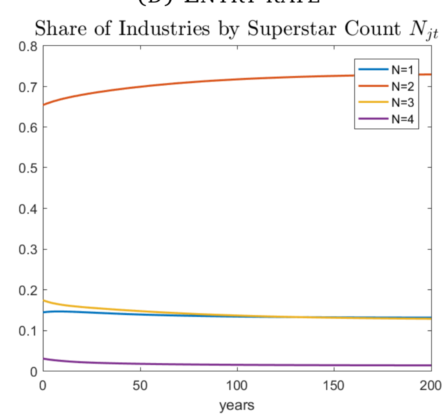
(C) LABOR SHARE



(D) ENTRY RATE



(E) AVERAGE MARKUP



(F) SUPERSTAR COUNT

FIGURE B3: TRANSITIONAL DYNAMICS: LATE TO LATER PERIOD

Notes: This figure illustrates the time paths of selected variables over time during the transition from the late to the later period stationary equilibrium.

Revision Appendices:

Are Markups Too High?
Competition, Strategic Innovation, and Industry Dynamics[†]

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C Model Extension: Letting Small Firms Have Positive Profits

This section derives the static equilibrium conditions and the level of output for two alternative models: (i) decreasing returns to scale in production technology (for both small firms and superstars), and (ii) letting the small firms in the competitive fringe collude, and thereby act as if they were a superstar with the productivity of the fringe ζq_{jt}^{leader}. In both cases, we depart from the baseline model in which small firms were making zero profits.

C.1 Decreasing Returns to Scale

In this section, we introduce decreasing returns to labor in the production function of superstar and small firms so that the production function for a superstar now reads $y_{ijt} = q_{ijt} l_{ijt}^{\alpha}$ and $y_{ckjt} = q_{cjt} l_{ckjt}^{\alpha}$ for a small firm, with $\alpha < 1$. The rest of the model is kept unchanged.

$\alpha < 1$ implies that small firms also make positive profits in equilibrium. We can rederive the first order conditions for both superstars and small firms. Starting with superstar firms, we obtain:

$$y_{ijt} = \alpha^{\alpha} \left(\frac{Y_t}{w_t}\right)^{\alpha} \left(\frac{\eta-1}{\eta}\right)^{\alpha} q_{ijt} \left[\frac{\sum_{k\neq i} \left(\frac{y_{kjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}} + \left(\frac{\tilde{y}_{cjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}}}{\left[\sum_{k=1}^{N_{jt}} \left(\frac{y_{kjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}} + \left(\frac{\tilde{y}_{cjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}}\right]^2} \right]^{\alpha} \tag{92}$$

The relative output between two superstar firms in the same industry is given by the solution to:

$$\left(\frac{y_{ijt}}{y_{kjt}}\right)^{\frac{\eta(1-\alpha)+\alpha}{\eta}} = \frac{q_{ijt}}{q_{kjt}} \left(\frac{\sum_{l\neq i} \left(\frac{y_{ljt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}} + \left(\frac{\tilde{y}_{cjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}}}{\sum_{l\neq k} \left(\frac{y_{ljt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}} + \left(\frac{\tilde{y}_{cjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}}} \right)^{\alpha} \tag{93}$$

Small firms in the fringe are price takers and charge a price equal to marginal cost so that each small firm in the fringe produces:

$$y_{ckjt} = \frac{Y_t^{\alpha} \alpha^{\alpha}}{w_t^{\alpha} m_{jt}^{\alpha} \left(\sum_{k=1}^{N_{jt}} \frac{y_{kjt}}{\tilde{y}_{cjt}}^{\frac{\eta-1}{\eta}} + 1\right)^{\alpha}} q_{cjt} \tag{94}$$

Total output of the competitive fringe is thus equal to:

$$\tilde{y}_{cjt} = m_{jt} y_{ckjt} = \frac{m_{jt}^{1-\alpha} Y_t^{\alpha} \alpha^{\alpha}}{w_t^{\alpha} \left(\sum_{k=1}^{N_{jt}} \frac{y_{kjt}}{\tilde{y}_{cjt}}^{\frac{\eta-1}{\eta}} + 1\right)^{\alpha}} q_{cjt} \tag{95}$$

This implies that the output ratio between any superstar firm and the total output of the fringe is equal to:

$$\left(\frac{y_{ijt}}{\tilde{y}_{cjt}}\right)^{\frac{\eta(1-\alpha)+\alpha}{\eta}} = \left(\frac{\eta-1}{\eta}\right)^{\alpha} \frac{q_{ijt}}{q_{cjt}} m_{jt}^{\alpha-1} \left(\frac{\sum_{l\neq i}\left(\frac{y_{ljt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}} + \left(\frac{\tilde{y}_{cjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}}}{\sum_{l=1}^{N_{jt}}\left(\frac{y_{ljt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}} + \left(\frac{\tilde{y}_{cjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}}}\right)^{\alpha} \tag{96}$$

Note that small firms charge no markups and that the markup of superstar firms as a function of relative output is unchanged compared to our baseline model:

$$M_{ijt} = \frac{\eta}{\eta-1} \frac{\sum_{k=1}^{N_{jt}}\left(\frac{y_{kjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}} + \left(\frac{\tilde{y}_{cjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}}}{\sum_{k\neq i}\left(\frac{y_{kjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}} + \left(\frac{\tilde{y}_{cjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}}} \tag{97}$$

Even though small firms do not charge a markup over marginal cost, both superstars and small firms make positive profits in equilibrium, which are respectively given by:

$$\pi_{ijt} = \frac{Y_t}{\left[\sum_{k=1}^{N_{jt}}\left(\frac{y_{kjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}} + \left(\frac{\tilde{y}_{cjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}}\right]^2} \frac{\eta + [(1-\alpha)\eta+\alpha)]\left[\sum_{k\neq i}\left(\frac{y_{kjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}} + \left(\frac{\tilde{y}_{cjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}}\right]}{\eta} \tag{98}$$

$$\pi_{ckjt} = (1-\alpha)\frac{Y_t}{m_{jt}\left(\sum_{k=1}^{N_{jt}} \frac{y_{kjt}}{\tilde{y}_{cjt}}^{\frac{\eta-1}{\eta}} + 1\right)} \tag{99}$$

Dynamically, the value functions for both entrepreneurs and superstar firms remain unchanged. For small firms in the competitive fringe, the value function now includes an additional term related to the static profit that small firms make with decreasing returns to scale:

$$\begin{aligned} rV^e(\Theta_j) &= \max_{X_{kj}} \pi_{ckjt} + X_{kj}V(\{\tilde{\mathbf{n}}_j - \bar{n}\} \cup \{-\bar{n}\}, N_j+1) - \tau V^e(\Theta_j) - \nu X_{kj}^{\epsilon} Y \\ &+ \sum_{\Theta'} p(\Theta_j, \Theta')(V^e(\Theta') - V^e(\Theta_j)) + \dot{V}^e(\Theta_j) \end{aligned} \tag{100}$$

Small firm innovation policy is still given by:

$$X_{kj} = \left(\frac{v(\{\tilde{\mathbf{n}}_j - \bar{n}\} \cup \{-\bar{n}\}, N_j+1)}{\nu\epsilon}\right)^{\frac{1}{\epsilon-1}}. \tag{101}$$

Finally, we can derive the level of aggregate output at any time t as:

$$
\begin{aligned}
\ln(Y_t) &= \int_0^1 \frac{\eta}{\eta-1} \ln\left[\sum_{i=1}^{N_{jt}} y_{ijt}^{\frac{\eta-1}{\eta}} + \tilde{y}_{cjt}^{\frac{\eta-1}{\eta}}\right] dj \\
&= \int_0^1 \frac{\eta}{\eta-1} \ln\left[\tilde{y}_{cjt}^{\frac{\eta-1}{\eta}} \left(\sum_{i=1}^{N_{jt}} \left(\frac{y_{ijt}}{\tilde{y}_{cjt}}\right)^{\frac{\eta-1}{\eta}} + 1\right)\right] dj \\
&= \int_0^1 \left[\ln\left(\tilde{y}_{cjt}\right) + \frac{\eta}{\eta-1} \ln\left(\sum_{i=1}^{N_{jt}} \left(\frac{y_{ijt}}{\tilde{y}_{cjt}}\right)^{\frac{\eta-1}{\eta}} + 1\right)\right] dj \\
&= \int_0^1 \left[\ln\left(m_{jt}^{1-\alpha}\alpha^{\alpha}\omega_t^{-\alpha} \frac{q_{cjt}}{\left[\sum_{i=1}^{N_{jt}} \left(\frac{y_{ijt}}{\tilde{y}_{cjt}}\right)^{\frac{\eta-1}{\eta}} + 1\right]^{\alpha}}\right) + \frac{\eta}{\eta-1} \ln\left(\sum_{i=1}^{N_{jt}} \left(\frac{y_{ijt}}{\tilde{y}_{cjt}}\right)^{\frac{\eta-1}{\eta}} + 1\right)\right] dj \\
&= (1-\alpha)\ln(m_{jt}) + \alpha\ln(\alpha) - \alpha\ln(\omega_t) + \int_0^1 \left[\ln\left(q_{cjt}\right) + \frac{\eta+\alpha-\alpha\eta}{\eta-1} \ln\left(\sum_{i=1}^{N_{jt}} \left(\frac{y_{ijt}}{\tilde{y}_{cjt}}\right)^{\frac{\eta-1}{\eta}} + 1\right)\right] dj
\end{aligned}
$$

We estimate this model with decreasing returns to scale for both the early and late periods, and repeat our counterfactual experiments to assess whether any of our results crucially depends on the zero-profit assumption for the competitive fringe. The details of the estimation and the counterfactual experiments can be found in Tables L16 and L7, respectively.

Our baseline results are maintained even when small firms earn a profit. A decline in competition among superstars and from small firms (declines in η and ζ) still improves growth and welfare, and the change in the relative productivity of the competitive fringe ζ continues to be responsible for the increase in the average markup despite the fact that small firms are now making positive profits. Ideas are also still getting harder to find for both small and large firms, and the effect is once again larger for the small firms, as it was in the baseline.

In our baseline model in which small firms do not make profits, these firms have an incentive to perform innovation in order to reap future expected profits as a superstar firm. In the alternative model with decreasing returns to scale, small firms now make positive profits even before becoming a superstar. Adding positive profits for small firms therefore increases their value compared to the baseline model. Consequently, the new business creation decision of the entrepreneurs is affected by the positive profits of the fringe firms, which in turn affects the stationary mass of small firms in the counterfactual experiments.

The most important difference between the baseline model and the alternative model with decreasing returns to scale is how much total innovation by the fringe firms responds to changes in parameter values in the counterfactual experiments. Once we let fringe firms have positive current profits, expected future profits from becoming a superstar no longer totally dominate the value of small firms. Consequently, the value of small firms are now less elastic to changes in the value of the superstar firm they try to become. And when the value of small firms is less elastic, so is the new business creation decision of the entrepreneurs, and the consequent stationary mass of small firms. Comparing Table L7 in the Revision Appendix against Table 2 in the main text, we see that the mass of small firms responds much less to parameter value changes in absolute terms, and therefore, the response of small firm innovation is also more muted. However, the direction of the change in small firm innovation remains the same in the experiments of interest, and none of our conclusions regarding the total effect on aggregate growth or welfare change. In conclusion, all our main results go through regardless of whether we let the fringe firms make positive or zero profits, although the exact quantitative magnitudes change.

C.2 Colluding Fringe

We assume that the firms in the competitive fringe collude and behave as a single superstar firm statically. Dynamically, the model assumptions remain unchanged as small firms invest in innovation in order to create a new differentiated variety and become a superstar firm on their own.

Statically, this is equivalent to assuming that there is one additional superstar firm in each industry (and no competitive fringe). The system of equations that needs to be solved is unchanged regarding superstar relative output. The only difference is the equilibrium output of the fringe, as the fringe now solves the same problem as a superstar firm, i.e., it interacts strategically with other firms. The total output of the fringe in equilibrium is given by:

$$\tilde{y}_{cjt} = \frac{\eta-1}{\eta} q_{cjt} \frac{\sum_{k=1}^{N_{jt}} \left(\frac{y_{kjt}}{\tilde{y}_{cjt}}\right)^{\frac{\eta-1}{\eta}}}{\left[\sum_{k=1}^{N_{jt}} \left(\frac{y_{kjt}}{\tilde{y}_{cjt}}\right)^{\frac{\eta-1}{\eta}} + 1\right]^2} \frac{Y_t}{w_t} \tag{102}$$

The relative output between the colluding fringe and any superstar i is thus given by:

$$\left(\frac{y_{ijt}}{\tilde{y}_{cjt}}\right)^{\frac{1}{\eta}} = \frac{q_{ijt}}{q_{cjt}} \frac{\sum_{l\neq i} \left(\frac{y_{ljt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}} + \left(\frac{\tilde{y}_{cjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}}}{\sum_{l=1}^{N_{jt}} \left(\frac{y_{ljt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}}}. \tag{103}$$

We can further derive the level of aggregate output Y_t at any time using:

$$\begin{aligned}
\ln(Y_t) &= \int_0^1 \frac{\eta}{\eta-1} ln \left[\sum_{i=1}^{N_{jt}} y_{ijt}^{\frac{\eta-1}{\eta}} + \tilde{y}_{cjt}^{\frac{\eta-1}{\eta}}\right] dj \\
&= \int_0^1 \left[\ln\left(\tilde{y}_{cjt}\right) + \frac{\eta}{\eta-1} \ln\left(\sum_{i=1}^{N_{jt}} \left(\frac{y_{ijt}}{\tilde{y}_{cjt}}\right)^{\frac{\eta-1}{\eta}} + 1\right)\right] dj \\
&= \int_0^1 \left[\ln\left(\frac{(\eta-1) q_{cjt} \sum_{k=1}^{N_{jt}} \left(\frac{y_{ijt}}{\tilde{y}_{cjt}}\right)^{\frac{\eta-1}{\eta}}}{\eta \omega_t \left[\sum_{i=1}^{N_{jt}} \left(\frac{y_{ijt}}{\tilde{y}_{cjt}}\right)^{\frac{\eta-1}{\eta}} + 1\right]^2}\right) + \frac{\eta}{\eta-1} \ln\left(\sum_{i=1}^{N_{jt}} \left(\frac{y_{ijt}}{\tilde{y}_{cjt}}\right)^{\frac{\eta-1}{\eta}} + 1\right)\right] dj \\
&= \int_0^1 \left[\ln\left(\frac{(\eta-1) q_{cjt}}{\eta \omega_t}\right) + \ln\left(\sum_{k=1}^{N_{jt}} \left(\frac{y_{ijt}}{\tilde{y}_{cjt}}\right)^{\frac{\eta-1}{\eta}}\right) + \frac{2-\eta}{\eta-1} \ln\left(\sum_{i=1}^{N_{jt}} \left(\frac{y_{ijt}}{\tilde{y}_{cjt}}\right)^{\frac{\eta-1}{\eta}} + 1\right)\right] dj
\end{aligned}$$

We use this alternative model for some quantitative experiments which demonstrate that moving from the baseline model to this alternative model with positive markups and profits for the competitive fringe lowers allocative efficiency across all industry states. Furthermore, repeating the welfare decomposition for the static impact of the ζ counterfactual experiment found in the first two columns of Table 3 in the main text, we find that this strengthens our baseline result that the dynamic losses from lower growth dominate the static gains from reverting ζ to its early period value. To obtain the first result, we did the following: we calculated log industry output in each industry state using the baseline model that was estimated using the whole sample, as well as the early and late period subsamples. Then, using the same parameter values, we calculated the log industry output in each industry state using the alternative model with the colluding fringe. The three subfigures of Figure L3 in the Revision Appendix plot the log industry output in the baseline model minus that in the alternative model for each industry state for the three estimations, where the size of a dot indicates the fraction of the industry state $\mu(\Theta)$ in the stationary equilibrium. We find that the allocative efficiency is higher in the baseline model compared to the alternative with the colluding fringe.

Why is this the case? While the fringe in the baseline uses up more inputs, shifting sales away from more productive superstars towards themselves, they also charge no markups. In the alternative model with the colluding fringe, the fringe produces less, but now they are charging positive markups, the effect of which on industry output is negative. Which effect dominates is a quantitative question, and our experiment reveals that the latter dominates. It is also worth noting that this effect holds across all industry states. As can be seen in Figure L3, log industry output is always higher in all industry states using the baseline model, regardless of which estimation we use.

To obtain the second result, we use the alternative model with the colluding fringe. We repeat the counterfactual experiment in which we start from the late period estimation, and set the productivity of the competitive fringe ζ back to its value in the early period. We reproduce the first two columns of Table 3 in the main text. The new results can be found in Table L12 in the Revision Appendix. While the quantitative magnitudes change, it is still true that reverting ζ back to its early value increases static allocative efficiency. The total consumption-equivalent welfare change is still positive at 3.55% as opposed to 4.13% in the baseline model. The directions of all the individual components are also the same. This reduction in the static positive impact of increasing ζ strengthens our baseline results in the sense that the dynamic loss would dominate even more if small firms made positive profits and charged markups. Total welfare losses from an increase in the relative productivity of the fringe are thus magnified.

D Model Extension: Multi-Product Firms

This section derives the dynamic problem of superstar firms when we allow them to own multiple products (instead of single-product firms in the baseline model). To do so, we allow superstar firms to invest in so-called "expansion" innovation which, if successful, allows the firm to enter a new market. As for successful small firms, we assume that a successful small firm enters the market as the smallest superstar (i.e., $\bar{n}$ below the industry leader). Unlike small firms, superstar expansion innovation is not directed. That is, a superstar is randomly allocated to one industry upon successful expansion innovation as in Klette and Kortum (2004) or Akcigit and Kerr (2018) among many others. The rest of the model is unchanged. This implies that the within-industry static equilibrium as well as the dynamic problems of entrepreneurs and small firms are unaffected by the extension to multi-product superstar firms. The main changes apply to the dynamic problem of superstar firms which now have to decide how much to invest in expansion innovation.

D.1 Dynamic problem of the superstar firms

We now assume that superstar firms can invest in expansion innovation as well. The cost for superstar i of generating a Poisson rate $X_{i,EXP}$ of success in expansion innovation is given by $\chi^{EXP} X_{i,EXP}^{\phi^{EXP}} H_i^{1-\phi^{EXP}} Y$, where $\chi^{EXP} > 0$, $\phi^{EXP} > 1$ and H_i is the number of products in firm i's (active) portfolio. Upon successful expansion innovation, a superstar firm is randomly allocated to an industry and starts as the smallest superstar firm (i.e., $\bar{n}$ steps below the leader).[67]

The relevant state variables for a firm i can be summarized by the vectors of the number of productivity steps between superstar firm i and every other superstar firm $k \in \{(1, 2, ..., N_{jt}) \backslash \{i\}\}$ in each of the industries in which firm i is operating. Letting $n_{ij}^k \in \{-\bar{n}, -\bar{n}+1, ..., \bar{n}-1, \bar{n}\}$ be the number of steps by which firm i in industry j leads firm k, the relevant state variables for firm i are given by the collection of vectors $\mathbf{n}_{ij} = \{n_{ij}^k\}_{k \neq i}$ and $N_j = |\mathbf{n}_{ij}| + 1$ for all industries j in which the firm is operating.[68] Recalling that H_i is the number of industries in which the firm is operating and letting $\mathcal{H}_i$ be the set of such industries, a superstar firm i chooses an innovation rate (z_{ij}) for all industries $j \in \mathcal{H}_i$ and a rate of expansion innovation $X_{i,EXP}$ to maximize the value of the firm given by:

$$
\begin{aligned}
& rV(\{\mathbf{n}_{ij}\}_{j\in\mathcal{H}_i}, \{N_j\}_{j\in\mathcal{H}_i}, H_i) = \max_{\{z_{ih}\}_{h\in\mathcal{H}_i}, X_{i,EXP}} \sum_{h\in\mathcal{H}_i} \left[\pi(\mathbf{n}_{ih}, N_h) - \chi z_{ih}^{\phi} Y\right] - \chi^{EXP} X_{i,EXP}^{\phi^{EXP}} H_i^{1-\phi^{EXP}} Y \\
& + \sum_{h\in\mathcal{H}_i} z_{ih} \left[V(\{\mathbf{n}_{ij}\}_{j\in\mathcal{H}_i} \backslash \{n_{ih}^k = \bar{n}\} + \mathbf{1}_h, \{N_j\}_{j\in\mathcal{H}_i} - |\{n_{ih}^k = \bar{n}\}|, H_i) - V(\{\mathbf{n}_{ij}\}_{j\in\mathcal{H}_i}, \{N_j\}_{j\in\mathcal{H}_i}, H_i)\right] \\
& + \sum_{h\in\mathcal{H}_i} \sum_{k: n_{ih}^k = -\bar{n}} z_{kh} \left[V(\{\mathbf{n}_{ij}\}_{j\in\mathcal{H}_i} \backslash \{\mathbf{n}_{ih}\}, \{N_j\}_{j\in\mathcal{H}_i} \backslash \{N_h\}, H_i - 1) - V(\{\mathbf{n}_{ij}\}_{j\in\mathcal{H}_i}, \{N_j\}_{j\in\mathcal{H}_i}, H_i)\right] \\
& + \sum_{h\in\mathcal{H}_i} \sum_{k: n_{ih}^k \neq -\bar{n}} z_{kh} \left[V(\{\mathbf{n}_{ij}\}_{j\in\mathcal{H}_i} \backslash \{n_{ih}^k\} \cup \{n_{ih}^k - 1\} \backslash \{n_{ih}^l = \bar{n} + n_{ih}^k\}, \{N_j\}_{j\in\mathcal{H}_i} - |\{n_{ih}^l = \bar{n} + n_{ih}^k\}|, H_i) - V(\{\mathbf{n}_{ij}\}_{j\in\mathcal{H}_i}, \{N_j\}_{j\in\mathcal{H}_i}, H_i)\right] \\
& + X_{i,EXP} \mathcal{K}_i \\
& + \sum_{h\in\mathcal{H}_i} X_h \left[V(\{\mathbf{n}_{ij}\}_{j\in\mathcal{H}_i} \cup \{\min\{\bar{n}, \bar{n} + \min(\mathbf{n}_{ih})\}\}, \{N_j\}_{j\in\mathcal{H}_i} \backslash \{N_h\} \cup \min(N_h + 1, \bar{N}), H_i) - V(\{\mathbf{n}_{ij}\}_{j\in\mathcal{H}_i}, \{N_j\}_{j\in\mathcal{H}_i}, H_i)\right] \\
& + \dot{V}(\{\mathbf{n}_{ij}\}_{j\in\mathcal{H}_i}, \{N_j\}_{j\in\mathcal{H}_i}, H_i)
\end{aligned}
$$

[67] We assume that if the superstar is allocated to an industry which already has $\bar{N}$ firms, then the superstar does not enter.

[68] We can rewrite the relative productivity of firm i and k as $\frac{q_{ij}}{q_{kj}} = (1+\lambda)^{n_{ij}^k}$.

where $\mathbf{1}_h$ is a collection of vectors (of appropriate dimension) whose elements are equal to one if the industry is h and zero otherwise. The rate of creation of a new superstar firm in any industry now depends on the innovation of small firms in the fringe and of superstar firms' expansion innovation i.e. $X_j = X_{cj} + \int_{\mathcal{F}} X_{i,EXP} di$ where $\mathcal{F}$ is the set of active superstar firms in the economy. $\mathcal{K}_i$ is the expected gain from being successful at expansion innovation for firm i and is given by:[69]

$$\mathcal{K}_i = \int_0^1 V(\{\mathbf{n}_{ij}\}_{j\in\mathcal{H}_i} \cup \{\{\tilde{\mathbf{n}}_l - \bar{n}\} \cup \{-\bar{n}\}\}, \{N_j\}_{j\in\mathcal{H}_i} \cup \{N_l+1\}, H_i + \mathbf{1}_{N_l<\bar{N}})dl - V(\{\mathbf{n}_{ij}\}_{j\in\mathcal{H}_i}, \{N_j\}_{j\in\mathcal{H}_i}, H_i)$$

where $\tilde{\mathbf{n}}_l = \mathbf{n}_{kl}$ where k denotes a productivity leader in industry l.

We can guess and verify that $V(\{\mathbf{n}_{ij}\}_{j\in\mathcal{H}_i}, \{N_j\}_{j\in\mathcal{H}_i}, H_i) = v(\{\mathbf{n}_{ij}\}_{j\in\mathcal{H}_i}, \{N_j\}_{j\in\mathcal{H}_i}, H_i)Y$ and rewrite:

$$\begin{aligned}
&\rho v(\{\mathbf{n}_{ij}\}_{j\in\mathcal{H}_i}, \{N_j\}_{j\in\mathcal{H}_i}, H_i) = \max_{\{z_{ih}\}_{h\in\mathcal{H}_i}, X_{i,EXP}} \sum_{h\in\mathcal{H}_i} \left[\frac{\pi(\mathbf{n}_{ih}, N_h)}{Y} - \chi z_{ih}^{\phi}\right] - \chi^{EXP} X_{i,EXP}^{\phi^{EXP}} H_i^{1-\phi^{EXP}} \\
&+ \sum_{h\in\mathcal{H}_i} z_{ih}\left[v(\{\mathbf{n}_{ij}\}_{j\in\mathcal{H}_i} \backslash \{n_{ih}^k = \bar{n}\} + \mathbf{1}_h, \{N_j\}_{j\in\mathcal{H}_i} - |\{n_{ih}^k = \bar{n}\}|, H_i) - v(\{\mathbf{n}_{ij}\}_{j\in\mathcal{H}_i}, \{N_j\}_{j\in\mathcal{H}_i}, H_i)\right] \\
&+ \sum_{h\in\mathcal{H}_i} \sum_{k:n_{ih}^k=-\bar{n}} z_{kh}\left[v(\{\mathbf{n}_{ij}\}_{j\in\mathcal{H}_i} \backslash \{\mathbf{n}_{ih}\}, \{N_j\}_{j\in\mathcal{H}_i} \backslash \{N_h\}, H_i - 1) - v(\{\mathbf{n}_{ij}\}_{j\in\mathcal{H}_i}, \{N_j\}_{j\in\mathcal{H}_i}, H_i)\right] \\
&+ \sum_{h\in\mathcal{H}_i} \sum_{k:n_{ih}^k\neq-\bar{n}} z_{kh}\left[v(\{\mathbf{n}_{ij}\}_{j\in\mathcal{H}_i} \backslash \{n_{ih}^k\} \cup \{n_{ih}^k - 1\} \backslash \{n_{ih}^l = \bar{n} + n_{ih}^k\}, \{N_j\}_{j\in\mathcal{H}_i} - |\{n_{ih}^l = \bar{n} + n_{ih}^k\}|, H_i) - v(\{\mathbf{n}_{ij}\}_{j\in\mathcal{H}_i}, \{N_j\}_{j\in\mathcal{H}_i}, H_i)\right] \\
&+ X_{i,EXP}\frac{\mathcal{K}_i}{Y} \\
&+ \sum_{h\in\mathcal{H}_i} X_h\left[v(\{\mathbf{n}_{ij}\}_{j\in\mathcal{H}_i} \cup \{\min\{\bar{n}, \bar{n} + \min(\mathbf{n}_{ih})\}\}, \{N_j\}_{j\in\mathcal{H}_i} \backslash \{N_h\} \cup \min(N_h+1, \bar{N}), H_i) - v(\{\mathbf{n}_{ij}\}_{j\in\mathcal{H}_i}, \{N_j\}_{j\in\mathcal{H}_i}, H_i)\right]
\end{aligned}$$

We can derive the optimal innovation policy for superstar firms as:

$$\begin{aligned}
z_{ih} &= \left[\frac{v(\{\mathbf{n}_{ij}\}_{j\in\mathcal{H}_i} \backslash \{n_{ih}^k = \bar{n}\} + \mathbf{1}_h, \{N_j\}_{j\in\mathcal{H}_i} - |\{n_{ih}^k = \bar{n}\}|, H_i) - v(\{\mathbf{n}_{ij}\}_{j\in\mathcal{H}_i}, \{N_j\}_{j\in\mathcal{H}_i}, H_i)}{\chi\phi}\right]^{\frac{1}{\phi-1}} \\
\frac{X_{i,EXP}}{H_i} &= \left[\frac{\mathcal{K}_i}{\chi^{EXP}\phi^{EXP}Y}\right]^{\frac{1}{\phi^{EXP}-1}}
\end{aligned}$$

We can notice that expansion R&D effort scales linearly with firm size as measured by the number of products.

We can further guess and verify that we can write

$$v(\{\mathbf{n}_{ij}\}_{j\in\mathcal{H}_i}, \{N_j\}_{j\in\mathcal{H}_i}, H_i) = \sum_{h\in\mathcal{H}_i} \tilde{v}(\mathbf{n}_{ih}, N_h)$$

i.e., the value of the firm can be written as the sum across all the products in the firm's portfolio of the value of owning that specific product, which is itself only a function of the number of firms and the relative productivities in that industry.

Optimal innovation policies become:

[69]We must keep in mind that this takes into account the fact that new firms cannot enter industries in which $N = \bar{N}$.

$$z_{ih} = \left[\frac{\tilde{v}(\mathbf{n}_{ih}\backslash\{n_{ih}^k = \bar{n}\} + \mathbf{1}, N_h - |\{n_{ih}^k = \bar{n}\}|) - \tilde{v}(\mathbf{n}_{ih}, N_h)}{\chi\phi}\right]^{\frac{1}{\phi-1}}$$

$$\frac{X_{i,EXP}}{H_i} = \left[\frac{\int_0^1 \tilde{v}(\{\tilde{\mathbf{n}}_l - \bar{n}\} \cup \{-\bar{n}\}, N_l + 1))dl}{\chi^{EXP}\phi^{EXP}}\right]^{\frac{1}{\phi^{EXP}-1}}$$

The value function can then be rewritten as:

$$\begin{aligned}
&\rho \sum_{h\in\mathcal{H}_i} \tilde{v}(\mathbf{n}_{ih}, N_h) = \sum_{h\in\mathcal{H}_i} \frac{\pi(\mathbf{n}_{ih}, N_h)}{Y} \\
&- \sum_{h\in\mathcal{H}_i} \chi \left[\frac{\tilde{v}(\mathbf{n}_{ih}\backslash\{n_{ih}^k = \bar{n}\} + \mathbf{1}, N_h - |\{n_{ih}^k = \bar{n}\}|) - \tilde{v}(\mathbf{n}_{ih}, N_h)}{\chi\phi}\right]^{\frac{\phi}{\phi-1}} \\
&- \sum_{h\in\mathcal{H}_i} \chi^{EXP} \left[\frac{\int_0^1 \tilde{v}(\{\tilde{\mathbf{n}}_l - \bar{n}\} \cup \{-\bar{n}\}, N_l + 1))dl}{\chi^{EXP}\phi^{EXP}}\right]^{\frac{\phi^{EXP}}{\phi^{EXP}-1}} \\
&+ \sum_{h\in\mathcal{H}_i} \left[\frac{\tilde{v}(\mathbf{n}_{ih}\backslash\{n_{ih}^k = \bar{n}\} + \mathbf{1}, N_h - |\{n_{ih}^k = \bar{n}\}|) - \tilde{v}(\mathbf{n}_{ih}, N_h)}{\chi\phi}\right]^{\frac{1}{\phi-1}} \left[\tilde{v}(\mathbf{n}_{ih}\backslash\{n_{ih}^k = \bar{n}\} + \mathbf{1}, N_h - |\{n_{ih}^k = \bar{n}\}|) - \tilde{v}(\mathbf{n}_{ih}, N_h)\right] \\
&+ \sum_{h\in\mathcal{H}_i} \sum_{k:n_{ih}^k = -\bar{n}} z_{kh}(-\tilde{v}(\mathbf{n}_{ih}, N_h)) \\
&+ \sum_{h\in\mathcal{H}_i} \sum_{k:n_{ih}^k \neq -\bar{n}} z_{kh} \left[\tilde{v}(\mathbf{n}_{ij}\backslash\{n_{ih}^k\} \cup \{n_{ih}^k - 1\}\backslash\{n_{ih}^l = \bar{n} + n_{ih}^k\}, N_h - |\{n_{ih}^l = \bar{n} + n_{ih}^k\}|) - \tilde{v}(\mathbf{n}_{ih}, N_h)\right] \\
&+ \sum_{h\in\mathcal{H}_i} \left[\frac{\int_0^1 \tilde{v}(\{\tilde{\mathbf{n}}_l - \bar{n}\} \cup \{-\bar{n}\}, N_l + 1))dl}{\chi^{EXP}\phi^{EXP}}\right]^{\frac{1}{\phi^{EXP}-1}} \left[\int_0^1 \tilde{v}(\{\tilde{\mathbf{n}}_l - \bar{n}\} \cup \{-\bar{n}\}, N_l + 1))dl\right] \\
&+ \sum_{h\in\mathcal{H}_i} X_h \left[\tilde{v}(\mathbf{n}_{ih} \cup \{\min\{\bar{n}, \bar{n} + \min(\mathbf{n}_{ih})\}\}, \min(N_h + 1, \bar{N})) - \tilde{v}(\mathbf{n}_{ih}, N_h)\right]
\end{aligned}$$

imposing $\tilde{v}(\cdot, x) = 0$ for all $x > \bar{N}$.

This verifies our guess. As a result, value functions $\tilde{v}(\mathbf{n}_{ih}, N_h)$ can be solved industry by industry.

E Model Extension: Bertrand Competition

Without differentiated goods, which is the most common assumption in the literature, Cournot and Bertrand competition differ substantially, since Bertrand competition with homogeneous goods implies that the industry leader captures the whole market, which leads to degenerate distributions of sales, profits, employment, and so on. With differentiated goods, both Cournot and Bertrand competition could deliver non-degenerate distributions. The difference lies in how skewed these distributions are towards the highest productivity firms.

Motivated by this, we built an alternative version of the model with Bertrand competition in each industry instead of Cournot. This changes the calculation of the static industry equilibria and the associated profit flows, but given these, the dynamics remain the same. Superstar firms now choose prices to maximize profit subject to demand from the final good producers for their product:[70]

$$\max_{p_{ijt}} \; p_{ijt} y_{ijt} - w_t l_{ijt} = \max_{p_{ijt}} \; \frac{Y_t p_{ijt}^{1-\eta}}{\sum_{k=1}^{N_{jt}} p_{kjt}^{1-\eta} + p_{cjt}^{1-\eta}} - \frac{w_t}{q_{ijt}} \frac{p_{ijt}^{-\eta}}{\sum_{k=1}^{N_{jt}} p_{kjt}^{1-\eta} + p_{cjt}^{1-\eta}}, \tag{104}$$

which delivers the following best response functions:

$$p_{ijt} = \frac{w_t}{(\eta-1) q_{ijt}} \frac{\eta \sum_{k \neq i} p_{kjt}^{1-\eta} + \eta p_{cjt}^{1-\eta} + p_{ijt}^{1-\eta}}{\sum_{k \neq i} p_{kjt}^{1-\eta} + p_{cjt}^{1-\eta}}.$$

We assume Cournot competition in our baseline analysis due to its ability to generate more variation in markups and more realistic market share distributions consistent with what is observed for large firms in the United States, but most of our results hold regardless of the specific assumption on whether firms compete in prices or quantities. To show this, we have re-estimated the model and performed a robustness check on our results with differentiated Bertrand competition instead. The estimation and counterfactual results are reported in Table L13 and Table L1, respectively.

Our quantitative results are very similar to what we have in the baseline analysis. The decline in competition among superstars and from small firms (decline in η and ζ) once again implies an increase in innovation, growth, and welfare. Ideas are getting harder to find for both small firms and superstars, hurting growth and welfare. The rise in markups is almost completely explained by the decline in the relative productivity of the competitive fringe ζ. While the exact quantitative magnitudes change, the relative quantitative magnitudes stay similar to that in the baseline.

[70] Small firms in the fringe still set price equal to marginal cost.

F Robustness Check: Intangible Investment

In the baseline estimation, we map firm-year-level innovation to firm-year-level average patent citations, and aggregate spending on innovation to aggregate spending on R&D in the data. We examine the robustness of our quantitative results by conducting an alternative estimation for the early and late subsamples in which we target (1) a measure of aggregate intangible investment to GDP rather than R&D to GDP alone, and (2) use a firm-year-level measure of total intangible investment to measure innovation, as opposed to relying on patent data, when obtaining the data moments that let us replicate the inverted-U relationship between innovation and relative sales.

To construct a firm-level measure of total intangible investment, we follow the methodology in Peters and Taylor (2017) that uses data on the observed R&D expenses (*xrd* in Compustat) and Selling, General and Administrative expenses (*xsga* in Compustat) of the firms in our sample.[71] To obtain a target for aggregate intangible investment to GDP, we start with the aggregate business R&D spending to GDP ratio, and then multiply it with an adjustment ratio. This adjustment ratio is calculated by summing up the total intangible investment by all firms in our Compustat sample, divided by the sum of all R&D spending by all firms in our Compustat sample. To obtain the targets for the inverted-U relationship, we repeat our regressions by using (standardized) log total intangible investment at the firm-year level instead of (standardized) average patent citations for the early and late subsamples. The results of the new estimation using these alternative targets and the associated counterfactual experiments are presented in Tables L15 and L4 in the Revision Appendix, respectively. As can be seen, the results of the counterfactual experiments are quite similar to the baseline analysis. Although the exact magnitudes change, it is still the case that (1) the decline in the relative productivity of the competitive fringe explains almost all the increase in the average markup, (2) decreasing competition among superstars and from small firms serves to increase innovation, growth, and welfare, and (3) ideas are getting harder to find for both small firms and superstars, and the decline in R&D efficiency once again explains why productivity growth did not increase as much in response to declining competition. The signs of all CEWC effects are the same.

[71] Other papers that use this methodology include Hulten and Hao (2008), Eisfeldt and Papanikolaou (2014), and Xiaolan (2014). Selling, General and Administrative expenses include, among other things, R&D, advertising, marketing and sales related expenses.

G Robustness Check: Increasing $\bar{n}$ and $\bar{N}$

In setting up the model, we had to make a choice on the upper bound for the maximum number of productivity steps ($\bar{n}$) and the maximum number of superstar firms within an industry ($\bar{N}$) so that the equilibria are computable. While in theory these numbers could be arbitrarily large, increasing any of them rapidly increases the number of potential firm and industry states, which slows down the numerical solution of the model. We have chosen the baseline values as a good compromise between allowing the model to have rich enough heterogeneity within and across industries and the time needed for estimation and computing counterfactual equilibria. This section proposes two robustness checks by increasing respectively for higher $\bar{n}$ and $\bar{N}$.

Before we move onto the details of our robustness checks, we would like to discuss how $\bar{N}$ and $\bar{n}$ affect the burden of computation. The dimensionalities of the two state variables are crucial: the firm state variable for superstars, $\mathbf{n}_{ijt} = \{n^k_{ijt}\}_{k \neq i}$, and the industry state variable $\Theta = (N, \vec{n})$. The dimensionality of the firm state variable governs how many different static industry equilibria are to be solved, and the dimensionalities of the superstar value function and the associated innovation policy function to be evaluated in the value function iteration. The dimensionality of the industry state variable governs the size of the instantaneous flow matrix $p_t(\Theta, \Theta')$ which is needed to compute the stationary industry state distribution $\mu(\Theta)$ in BGP equilibria. In non-stationary equilibria, it directly increases the number of continuous state variables for which the time paths must be calculated during the transition. In our baseline estimation, $\bar{N} = 4$ and $\bar{n} = 5$ imply that the firm state variable has 774 dimensions, whereas the industry state variable has 84 dimensions. In our robustness checks which will be discussed below, $\bar{N} = 5$ and $\bar{n} = 5$ implies 5,425 and 210, respectively, and $\bar{N} = 4$ and $\bar{n} = 6$ implies 1,246 and 120, respectively. Tables L18 and L19 in the Revision Appendix report the dimensionalities of the firm and industry state variables under different values of $\bar{N}$ and $\bar{n}$, respectively. The curse of dimensionality is apparent. For instance, picking $\bar{N} = 8$ and $\bar{n} = 8$, which might sound reasonable at first, requires us to solve 29 million systems of 8 nonlinear equations in each iteration given particular values for general-equilibrium variables such as m_t and Θ_t. The matrix $p_t(\Theta, \Theta')$ has $(289{,}795)^2 \approx 84 \times 10^9$ entries, and we would need to keep track of 290 thousand continuous state variables during the transition. As the tables demonstrate, the problem becomes quite intractable for large values.

Increasing $\bar{N}$: In this robustness check, we increase $\bar{N}$ from 4 to 5. The results show that increasing $\bar{N}$ to 5 changes virtually nothing. Despite increasing $\bar{N}$ from 4 to 5, the parameter values we obtained using the baseline version of the model still generates almost exactly the same model moments, so much so that no re-estimation is needed, as the model fit is just as good. Table L3 in the Revision Appendix shows the model moments in the late and early subsamples in the first and second to last columns, respectively, and they are indistinguishable from the same in Table 2 in the main text. All the effects of the counterfactual experiments found in the same table are also almost identical.

The underlying reason is that the fraction of industries with the maximum allowed number of superstars $N_{jt} = 4$ in the estimated equilibria is quite small. Figure L2 in the Revision Appendix shows the fractions of industry states by the number of superstars under $\bar{N} = 4$ and $\bar{N} = 5$ for the early and late subsample estimations, visually demonstrating this situation. Therefore, relaxing the maximum number of superstars per industry does not lead to any meaningful changes, as the constraint was not binding much in the first place.

Increasing $\bar{n}$: In this robustness check, we increase $\bar{n}$ from 5 to 6. The upper bound $\bar{n}$ is a grid size for the quality ladder. Increasing $\bar{n}$ and re-estimating the model is then expected to reduce the inferred value of λ down. Since each step of the ladder represents less of a productivity increase, by extension, the estimated parameter values for the superstar innovation cost function must also adjust to account for the drop in the private value of the innovation to the firm so that the incentives remain the same. The overall frequency of successful innovation mechanically increases, but the value of each innovation is now lower.

We increase $\bar{n}$ from 5 to 6, re-estimate the model for the early and late subsamples, and repeat the counterfactuals, the results of which are shown in Tables L14 and L2 in the Revision Appendix, respectively. As expected, λ goes down in both re-estimations. Repeating the counterfactual experiments, we see that the exact quantitative magnitudes vary, but the message remains the same. Movements in growth, the average markup, and CEWC retain their directions and relative magnitudes, and so do almost all statistics of interest.

H Robustness Check: Quadratic R&D Cost Functions

While part of the existing literature uses quadratic R&D cost functions (see for instance Akcigit and Kerr (2018)), we have decided to estimate the convexity of the R&D cost functions for both small and superstar firms in our baseline analysis. As a robustness check, we re-estimate the model with quadratic R&D cost functions. In particular, we re-estimate the model for both early and late subsamples and repeat our counterfactual experiments. The details of the estimation and the experiments are reported in Tables L17 and L9, respectively.

We should note that, when we exogenously impose both convexity parameters (ϵ and ϕ) to be equal to 2, the model has a hard time matching the level of R&D intensity and the growth rate of the economy simultaneously. This is not very surprising, since this leaves us with 6 parameters to hit 11 targets, an acute case of over-identification.

Despite the fact that the fit of the model is significantly worse than under our baseline estimation, it is worth noting that setting the R&D convexities to 2 does not significantly change the results obtained from our counterfactual experiments in all but one case. The overall effect of setting the convexities equal to 2 is to increase the elasticity of the innovation decisions to changes in parameter values. As a consequence, innovation, growth, and welfare effects are magnified. For instance, reverting η to its early period value now delivers a consumption-equivalent welfare change of -18.32% as opposed to -12.76%. The effect for reverting η and the small firm R&D cost function are very similar to the baseline figures. The only difference is that the effect for superstar innovation now flips which can be attributed to the failure of the estimation in matching the aggregate R&D to GDP ratio.

Overall, the robustness check shows that our results are robust to setting R&D cost to the commonly used value of 2, but doing so reduces the model fit and further magnifies the dynamic gains from rising markups.

I Robustness Check: Non-Quadratic Entrepreneur Cost Function

In our baseline estimation, we assume a quadratic entrepreneur cost function due to the lack of available data that we could use to separately identify the scale (ψ) and convexity parameters. Unlike what is the case for incumbent firms, for which we can empirically observe their innovation inputs (R&D expenditures) and outputs (patents, citations, sales growth, productivity growth, ...), we do not have access to representative micro-data on business creation costs of entrepreneurs, which would ideally include not only the material costs of founding a new business, but also the opportunity cost of the entrepreneur(s).

Using our model, the expected value of a small firm, combined with the normalization $m_t = 1$, allows us to infer only the value of one of the parameters. Therefore, we have decided to assume a quadratic cost, as is often done in the endogenous growth literature (see, for instance, Akcigit and Kerr (2018), Akcigit and Ates (2021), and Liu, Mian, and Sufi (2022)).

How would assuming a different value for the convexity parameter alter our results? First of all, the value of this parameter does not affect estimation at all. Due to the normalization $m_t = 1$, the estimation algorithm never loops over it. Rather, we first estimate all the parameters except for ψ, and then recover its implied value given the entrepreneurs' optimality condition. What it affects is (1) the estimated value of ψ, and (2) how much the mass of small firms m_t changes in response to the changes in other parameter values in the counterfactual experiments. Higher values of the convexity parameter imply smaller changes in m_t, and lower values imply larger changes. If we go all the way down to 1, we reach the linear business creation cost/free entry case, where the response of m_t is the most inflated.

We conduct two separate robustness checks. In the first one, we assume a higher convexity value of 3 instead of 2 for the entrepreneurs. In the second one, we assume a linear technology instead.

Table L6 in the Revision Appendix repeats our counterfactual experiments while assuming a convexity of 3. As can be seen, the change in the stationary mass of small firms is always in the same direction, but the movement is smaller in absolute terms. Consequently, the changes in innovation and growth are all slightly muted, as the mass of small firms directly affects how much small firm innovation there is. This reduces the dynamic welfare changes coming from changes in the growth rate of the economy, but quantitatively, the differences are minor. For instance, the CEWC in the experiment for η goes from -12.76% to -11.12%, and that for ζ goes from -7.60% to -5.50%.

We also implement a robustness check in which we assume a linear cost and free entry. Once again, there is no need for re-estimation beyond recovering the new implied value of ψ. The moment fit is identical to that in the baseline estimation.

Table L5 in the Revision Appendix displays the results of repeating our counterfactual experiments. As discussed earlier, assuming a linear technology for the creation of new businesses magnifies the responses of m_t in all counterfactual experiments, and by extension, the magnitude of the changes in innovation and growth, and the associated dynamic welfare effects. All signs are maintained, and all the CEWC numbers are now inflated. Our results still hold, but all the implied changes are more dramatic.

J Discussion of the Competitive Fringe Assumption

In our baseline model, each industry is populated by an endogenous number of superstar firms ($N_{jt} \in \{1, ..., \bar{N}\}$), each producing a differentiated variety, as well as by a competitive fringe composed of a mass m_{jt} of small firms producing a homogeneous good. The productivity of superstar firms are heterogeneous, and denoted by q_{ijt}, whereas the small firms in the competitive fringe all share the same productivity $q_{cjt} = \zeta q_{jt}^{leader}$, where $\zeta > 0$ is a parameter.

In this section, we aim to elucidate the reasoning behind our modeling choices, and at the same time, demonstrate that the competitive fringe productivity $q_{cjt} = \zeta q_{jt}^{leader}$ corresponds to the productivity of the whole fringe, and therefore, that the parameter ζ can take any positive value, including values above unity, with no perverse implications (that is, the competitive fringe can produce more output than even the most productive superstar, even though the firms in the fringe themselves produce much smaller amounts individually). To do so, we will solve a static equilibrium in which a large number of small firms ($M_{jt} \in \mathbb{Z}_+$) will be treated exactly the same as the superstar firms, and provide a mapping between the productivities in this alternative model and q_{cjt} in our baseline.

J.1 Why do we need a competitive fringe?

Before laying out the details of this alternative model, it is useful to discuss why we need a competitive fringe in the first place. Industries in the United States are populated by thousands of firms on average, but in most industries, a handful of large firms ("superstars") account for a large fraction of the total industry sales. Data from the Business Dynamics Statistics database for the years 1980, 1990 and 2000 shows that, on average, the median number of firms in NAICS 4-level industries was 4386 rising from 3724 in 1980 to 5022 in 2000. Table J1 provides additional summary statistics for the number of firms in each industry, showing that these large numbers are not driven by some outlier industries. However, Autor, Dorn, Katz, Patterson, and Reenen (2017) show that in many industries the top 4 firms account for a significant share of both sales and employment. Focusing on four-digit SIC industries, they find that across six broadly defined sectors, top 4 firms account for between 15 and 43 percent of total sales and between 12 and 34 percent of total employment, on average in 2012. The firm size distribution is right skewed, as can be seen in Figure J1 which is borrowed from Axtell (2001). Using BDS data for the years 1980, 1990 and 2000, we find that firms with more than 500 employees represent on average 5.8 percent of firms across four-digit NAICS industries and between around 43 and 48 percent of employment.

When modeling an industry, theoretically speaking, we could treat all the firms in the industry the same, and allow them to have their own differentiated varieties. This would imply having an endogenous number of firms N_{jt}, where it can range from just a few hundred firms to tens of thousands. To be able to match the actual market share distribution observed in the real world, the firm productivity distribution would contain a handful of very productive firms (superstars), and a huge number of firms with very low productivities (the rest).

In practice, solving such a model computationally is impossible without further simplifications. Suppose that we maintain our baseline grid size $\bar{n} = 5$ for the quality ladder. Even with this rather coarse grid, for an industry with $N_{jt} = 10,000$ firms, we would have $5^{10000} \approx 5.01 \times 10^{6989}$ potential configurations, which is substantially larger than the number of atoms in the universe (estimated to be between 10^{78} and 10^{82}).

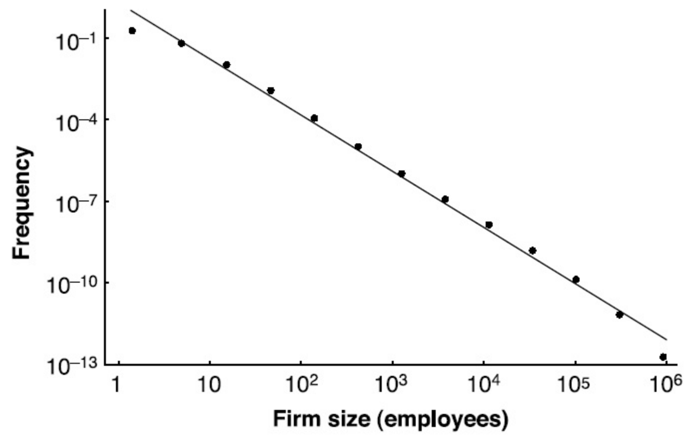

FIGURE J1: US FIRM SIZE DISTRIBUTION BORROWED FROM AXTELL (2001) (FIGURE 1)

Notes: Data are for 1997 from the US Census Bureau, tabulated in bins having width increasing in powers of three. The solid line is the OLS regression fit through the data.

TABLE J1: DISTRIBUTION OF THE NUMBER OF FIRMS PER NAICS 4 INDUSTRIES: BDS DATA

	Year		
Percentile	1980	1990	2000
10^{th}	396	391	425
25^{th}	1116	1309	1437
50^{th}	3724	4413	5022
75^{th}	12406	15731	16554
90^{th}	30906	39510	44732

Notes: This table shows the distribution of the number of firms per industry. It reports the 10^{th}, 25^{th}, 75^{th}, and 90^{th} percentile of the distribution as well as the median number of firms per NAICS 4 industry from BDS data for the years 1980, 1990 and 2000.

One could exploit the symmetry and just keep track of how many firms have a distinct quality instead, but even that would result in a cardinality slightly lower than $10000^5 = 10^{20}$ for industries with 10,000 firms, nevermind the fact that the model with an endogenous number of firms must calculate static equilibria for all possible values of $N_{jt} \in \{1, ..., \bar{N}\}$ rather than just $N_{jt} = 10,000$.

As demonstrated, even computing all possible static equilibria is troublesome when the number of firms is large. Since our model is a step-by-step innovation model, the complexity is even greater, as we have to compute the dynamic Markov-Perfect Equilibrium where innovation choices take as given the profits from the static industry equilibria in the value function iteration, which themselves would suffer from the curse of dimensionality.

Previous models in the endogenous growth literature have chosen to simplify the static industry equilibria substantially. For instance, Klette and Kortum (2004), Akcigit and Kerr (2018), Akcigit and Ates (2023), Aghion, Bergeaud, Boppart, Klenow, and Li (2023) or De Ridder (2023) assume that there is only a single active firm in each industry thanks to perfect substitution and Bertrand competition. The step-by-step innovation models such as Olmstead-Rumsey (2022) and Liu, Mian, and Sufi (2022) assume that all industries are (at most) duopolies, that is $N_{jt} \leq 2$ in all industries. Our framework, in comparison, allows for an endogenous number of superstar firms $N_{jt} = \{1, ..., \bar{N}\}$ with $\bar{N} = 4$ in the baseline, and $\bar{N} = 5$ as a robustness check. However, it remains a fact that there are thousands of firms other than superstars in each industry in the real world. These firms command very insignificant market shares individually, but collectively, they constitute a significant chunk of the total industry sales. There are industries in which the share of the top 4 firms (CR4) is less than 50%, which means the small firms collectively produce more than all the superstars.

Our baseline model captures this feature of the data by introducing a competitive fringe composed of a continuum of small firms in each industry. These firms individually produce infinitesimally small amounts (the Lebesgue measure of any single firm is zero, and so is its market share compared to that of any superstar), but their collective output and market share are determined according to the parameter $\zeta > 0$. In the next subsection, we offer a discretized version of the same idea to elucidate its workings, and show how ζ maps to productivities of small firms when they are countable and treated the same as superstars.

J.2 Discretized competitive fringe

Consider an alternative version of the industry-level model with a discretized competitive fringe. Industry j is populated by N_{jt} superstar firms ($N_{jt} \in \{1, ..., \bar{N}\}$) and $M_{jt} \in \mathbb{Z}_+$ small firms. Unlike in the baseline model, all firms are treated the same. That is, each firm has its own individual variety that is aggregated using a CES technology, and they all strategically compete in quantities. The industry output is therefore given by:

$$y_{jt} = \left(\sum_{i=1}^{N_{jt}} y_{ijt}^{\frac{\eta-1}{\eta}} + \sum_{i=N_{jt}+1}^{N_{jt}+M_{jt}} y_{ijt}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} = \left(\sum_{i=1}^{N_{jt}+M_{jt}} y_{ijt}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}} \tag{105}$$

The productivity of superstar firms $i \in \{1, ..., N_{jt}\}$ are heterogeneous, and denoted by q_{ijt} as in the baseline model, whereas the small firms $i \in \{N_{jt} + 1, N_{jt} + M_{jt}\}$ all share the same productivity $q_{ijt} = q_{cjt} = \alpha q_{jt}^{leader}, \forall i > N_{jt}$, where $\alpha > 0$ is a parameter. We will later on show how the parameter α maps to the parameter ζ in the baseline model. All firms produce their own variety using a linear production technology in labor:

$$y_{ijt} = q_{ijt} l_{ijt} \tag{106}$$

where q_{ijt} is the productivity of firm i in industry j at time t and l_{ijt} is labor. Using the same final good production technology as in the baseline model, the inverse demand function for firm i becomes:

$$p_{ijt} = \frac{y_{ijt}^{-\frac{1}{\eta}} Y_t}{\sum_{k=1}^{N_{jt}+M_{jt}} y_{kjt}^{\frac{\eta-1}{\eta}}} \tag{107}$$

implying

$$\frac{y_{ijt}}{y_{kjt}} = \left(\frac{p_{kjt}}{p_{ijt}} \right)^{\eta} \tag{108}$$

We assume now that all firms within the same industry, and not just the superstars, compete *à la* Cournot. Each firm maximizes profit:

$$\max_{y_{ijt}} \; p_{ijt} y_{ijt} - w_t l_{ijt} = \max_{y_{ijt}} \; \frac{y_{ijt}^{\frac{\eta-1}{\eta}} Y_t}{\sum_{k=1}^{N_{jt}+M_{jt}} y_{kjt}^{\frac{\eta-1}{\eta}}} - \frac{w_t y_{ijt}}{q_{ijt}}. \tag{109}$$

This delivers the following best response function for all firms:

$$y_{ijt} = \left[\frac{\eta-1}{\eta} q_{ijt} \frac{\sum_{k \neq i} y_{kjt}^{\frac{\eta-1}{\eta}}}{\left[\sum_{k=1}^{N_{jt}+M_{jt}} y_{kjt}^{\frac{\eta-1}{\eta}} \right]^2} \frac{Y_t}{w_t} \right]^{\eta} = \frac{\eta-1}{\eta} q_{ijt} \frac{\sum_{k \neq i} \left(\frac{y_{kjt}}{y_{ijt}} \right)^{\frac{\eta-1}{\eta}}}{\left[\sum_{k=1}^{N_{jt}+M_{jt}} \left(\frac{y_{kjt}}{y_{ijt}} \right)^{\frac{\eta-1}{\eta}} \right]^2} \frac{Y_t}{w_t} \tag{110}$$

Relative production between the varieties of firm i and k in industry j can then be written as:

$$\left(\frac{y_{ijt}}{y_{kjt}} \right)^{\frac{1}{\eta}} = \frac{q_{ijt}}{q_{kjt}} \frac{\sum_{l \neq i} \left(\frac{y_{ljt}}{y_{ijt}} \right)^{\frac{\eta-1}{\eta}}}{\sum_{l \neq k} \left(\frac{y_{ljt}}{y_{ijt}} \right)^{\frac{\eta-1}{\eta}}} \tag{111}$$

Since all small firms share the same productivity $q_{cjt} = \alpha q_{jt}^{leader}, \forall i > N_{jt}$, equation (111) implies their output, denoted y_{cjt} from now on, must also be the same; that is, $y_{ijt} = y_{cjt}, \forall i > N_{jt}$. Consequently, solving for the static equilibrium of the industry, $\{y_{ijt}\}_{i=1}^{N_{jt}+M_{jt}}$, only requires solving for N_{jt} ratios, not $N_{jt} + M_{jt}$. In fact, we can rewrite equation (111) as

$$\left(\frac{y_{ijt}}{y_{kjt}} \right)^{\frac{1}{\eta}} = \frac{q_{ijt}}{q_{kjt}} \frac{\sum_{l \neq i, l \leq N_{jt}} \left(\frac{y_{ljt}}{y_{ijt}} \right)^{\frac{\eta-1}{\eta}} + M_{jt} \left(\frac{y_{cjt}}{y_{ijt}} \right)^{\frac{\eta-1}{\eta}}}{\sum_{l \neq k, l \leq N_{jt}} \left(\frac{y_{ljt}}{y_{ijt}} \right)^{\frac{\eta-1}{\eta}} + M_{jt} \left(\frac{y_{cjt}}{y_{ijt}} \right)^{\frac{\eta-1}{\eta}}}, \forall i, k \leq N_{jt} \tag{112}$$

and

$$\left(\frac{y_{ijt}}{y_{cjt}} \right)^{\frac{1}{\eta}} = \frac{q_{ijt}}{q_{cjt}} \frac{\sum_{l \neq i, l \leq N_{jt}} \left(\frac{y_{ljt}}{y_{ijt}} \right)^{\frac{\eta-1}{\eta}} + M_{jt} \left(\frac{y_{cjt}}{y_{ijt}} \right)^{\frac{\eta-1}{\eta}}}{\sum_{l \leq N_{jt}} \left(\frac{y_{ljt}}{y_{ijt}} \right)^{\frac{\eta-1}{\eta}} + (M_{jt} - 1) \left(\frac{y_{cjt}}{y_{ijt}} \right)^{\frac{\eta-1}{\eta}}}, \forall i \leq N_{jt} \tag{113}$$

We can further derive variety prices (p_{ijt}) and profits before R&D expenditures (π_{ijt}) which only depend on relative productivities within the industry:

$$p_{ijt} = \frac{\eta}{\eta - 1} \frac{\sum_{k=1}^{N_{jt}} \left(\frac{y_{kjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}} + M_{jt} \left(\frac{y_{cjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}}}{\sum_{k=1}^{N_{jt}} \left(\frac{y_{kjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}} + M_{jt} \left(\frac{y_{cjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}} - 1} \frac{w_t}{q_{ijt}} \tag{114}$$

$$\pi_{ijt} = \frac{Y_t}{\left[\sum_{k=1}^{N_{jt}} \left(\frac{y_{kjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}} + M_{jt} \left(\frac{y_{cjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}}\right]^2} \frac{\eta + \sum_{k=1}^{N_{jt}} \left(\frac{y_{kjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}} + M_{jt} \left(\frac{y_{cjt}}{y_{ijt}}\right)^{\frac{\eta-1}{\eta}} - 1}{\eta} \tag{115}$$

J.3 Quantitative demonstration and mapping to the baseline model

Given the derivations, we are ready to calculate equilibria using the discretized competitive fringe. For the quantitative demonstration, we are going to rely on estimated parameter values from our baseline model, and focus on a particular industry state. For instance, consider an industry with $N_{jt} = 4$ firms, where firm 1 is the industry leader, firm 2 is one step behind the leader, firm 3 is two steps behind the leader, and firm 4 is three steps behind the leader (i.e., the state of the leading firm is $(1, 2, 3)$).

Our goal is to find the value of $\alpha > 0$ such that the total market share of the small firms (fringe), is the same as the industry equilibrium in the baseline model with the estimated value of $\zeta = 0.5912$ (whole sample). Naturally, the value of α that satisfies this requirement is a function of the number of small firms M_{jt}. Therefore, we shall repeat the same exercise for various values of M_{jt}, motivated by the empirically-observed firm counts per industry we previously provided in Table J1.

Table J2 displays the values of α that replicate the same competitive fringe market share distribution as in the industry equilibrium in the baseline model. As can be seen in the table, as M_{jt} increases, α, the relative productivity of a single small firm decreases, so that the total market share of all small firms (the competitive fringe) is the same as it is in the baseline model. We should note that, in this exercise, we have only targeted the market share of the competitive fringe. We can nevertheless notice that it directly follows that the entire market share distribution in the industry is preserved (compared to the baseline as well as across different numbers of small firms). This further shows that our simplifying assumption of a continuum of small firms does not affect our results regarding market share distribution across different firms. The small firms that are tiny compared to the superstars do not have meaningful strategic interactions with the superstars or the other small firms. However, thanks to their large number, the small firms as a whole still have a meaningful impact on the production choices of the superstars, as well as their markups and profits.

Overall, this exercise demonstrates that $\zeta \neq \alpha$. The parameter ζ in the baseline model captures the productivity of the competitive fringe as a whole, and not of individual small firms α. The value of α is negatively related to the number of small firms M_{jt}. When $\zeta > 1$, this does not mean the small firms are more productive than the superstars. It means that the competitive fringe as a collection of all small firms is more productive than individual superstars. But individually, a single small firm is much less productive than any superstar, i.e., $\alpha q_{jt}^{leader} << (1+\lambda)^{-\bar{n}} q_{jt}^{leader} \leq q_{ijt}, \forall i \leq N_{jt}$ for realistic values of M_{jt}.[72] We have repeated the same exercise as in Table J2 for every industry state in the model. In all cases, our baseline

[72] in our baseline calibration, $(1+\lambda)^{-\bar{n}} = 0.257$.

TABLE J2: α AS A FUNCTION OF THE NUMBER OF SMALL FIRMS IN THE FRINGE FOR A GIVEN FRINGE MARKET SHARE

Number of small firms	α	Market share fringe	Market share superstar 1	Market share superstar 2	Market share superstar 3	Market share superstar 4	Market share one small firm
391	0.243	0.378	0.323	0.191	0.083	0.025	8.273×10^{-4}
1309	0.197	0.378	0.323	0.191	0.083	0.025	2.471×10^{-4}
4413	0.159	0.378	0.323	0.191	0.083	0.025	7.330×10^{-5}
15731	0.127	0.378	0.323	0.191	0.083	0.025	2.056×10^{-5}
39510	0.108	0.378	0.323	0.191	0.083	0.025	8.187×10^{-6}
Baseline		0.378	0.323	0.191	0.083	0.025	0

Notes: This table shows the value of α required to maintain the same market share for the whole competitive fringe as in our baseline economy (whole sample) as a function of the number of firms in the fringe. The reported results relate to an industry with 4 superstar firms, in which the largest superstar firm leads its competitors by respectively one, two and three productivity steps. It also reports the market shares of all superstar firms, of the competitive fringe as a whole and of each small firm in the fringe. The last row shows the market share distribution in our baseline estimation.

calibration implies that small firms are less productive than the least productive superstar in the industry for realistic values of M_{jt}. In particular, this is true for every industry as long as the number of firms is larger than 291.

K Additional Quantitative Analysis

K.1 Distributional Changes between Early and Late Periods

Figure K1 shows that there are considerable changes in the industrial structure between the two periods. Panel (a) of Figure K1 shows the distribution of industries over states with a different number of superstar firms. There is an increase in the share of industries with one or two superstar firms, whereas the share of industries with more than two superstar firms decreases. There is an increase in market concentration. This is also seen in Panels (b) and (g) of Figure K1: the distribution of average markup within industries and of and HHI shifts strongly to the right. Panel (c) of Figure K1 depicts superstar innovation at the industry level, and the distribution moves to the left towards less innovation. The distribution of the superstar entry rate – or, equivalently, small firm innovation – across industries is seen in Panel (d) of Figure K1. A large overall decrease is observed, and the heterogeneity of superstar entry rates across industries also goes down. Despite the decrease in innovation by both small and large firms, the distribution of R&D costs remains largely the same with slight increases, as seen in Panels (e) and (f) of Figure K1. This owes to the overall increase in the costs to innovate. Panel (h) of Figure K1 shows that the distribution of within-industry standard deviation of markups shifts to the left between the early and the late period.

K.2 Decomposing the Source of the Decrease in the Labor Share

Using our model, we can write the change in the labor share (LS) between the early (subscript e) and late (subscript l) sub-samples as follows:

$$\begin{aligned}\Delta LS &= \sum_{\Theta}\mu_l(\Theta)\left[\sum_{k=1}^{N(\Theta)} l_{k,l}(\Theta) + m_l l_{c,l}(\Theta)\right]\omega_l - \sum_{\Theta}\mu_e(\Theta)\left[\sum_{k=1}^{N(\Theta)} l_{k,e}(\Theta) + m_e l_{c,e}(\Theta)\right]\omega_e \\ &= \sum_{\Theta}\mu_l(\Theta)\left[\sum_{k=1}^{N(\Theta)} LS_{k,l}(\Theta)\Upsilon_{k,l}(\Theta) + LS_{c,l}(\Theta)\Upsilon_{c,l}(\Theta)\right] \\ &\quad - \sum_{\Theta}\mu_e(\Theta)\left[\sum_{k=1}^{N(\Theta)} LS_{k,e}(\Theta)\Upsilon_{k,e}(\Theta) + LS_{c,e}(\Theta)\Upsilon_{c,e}(\Theta)\right] \qquad (116)\end{aligned}$$

where $\Upsilon_{k,t}(\Theta)$ is the market share of firm k in industry Θ at time t and $\Upsilon_{c,t}(\Theta)$ is the market share of the fringe. We can further decompose the change in the labor share as:

$$\begin{aligned}\Delta LS &= \left(\sum_{\Theta}\mu_l(\Theta) - \sum_{\Theta}\mu_e(\Theta)\right)\left[\sum_{k=1}^{N(\Theta)} LS_{k,l}(\Theta)\Upsilon_{k,l}(\Theta) + LS_{c,l}(\Theta)\Upsilon_{c,l}(\Theta)\right] \\ &+ \sum_{\Theta}\mu_e(\Theta)\left\{\left[\sum_{k=1}^{N(\Theta)} LS_{k,l}(\Theta)\Upsilon_{k,l}(\Theta) + LS_{c,l}(\Theta)\Upsilon_{c,l}(\Theta)\right] - \left[\sum_{k=1}^{N(\Theta)} LS_{k,l}(\Theta)\Upsilon_{k,e}(\Theta) + LS_{c,l}(\Theta)\Upsilon_{c,e}(\Theta)\right]\right\} \\ &+ \sum_{\Theta}\mu_e(\Theta)\left\{\left[\sum_{k=1}^{N(\Theta)} LS_{k,l}(\Theta)\Upsilon_{k,e}(\Theta) + LS_{c,l}(\Theta)\Upsilon_{c,e}(\Theta)\right] - \left[\sum_{k=1}^{N(\Theta)} LS_{k,e}(\Theta)\Upsilon_{k,e}(\Theta) + LS_{c,e}(\Theta)\Upsilon_{c,e}(\Theta)\right]\right\}\end{aligned}$$

where the first term captures the change in the aggregate labor share due to the change in the distribution of industry states $\mu(\Theta)$, the second term captures the change in the aggregate labor share due to within-industry reallocation of sales, and the third term captures the change in the aggregate labor share due to changes in firm-level labor shares.

Our model predicts that almost all of the 5.18% decrease in the aggregate labor share between both sub-samples is due to changes in market share reallocation (1.79%) and changes in firm-level labor shares (3.95%). Even though Autor, Dorn, Katz, Patterson, and Van Reenen (2020) use a different decomposition, our results are comparable, in that they also find within-industry market share reallocation to be responsible for a large part of the decrease in the aggregate labor share.

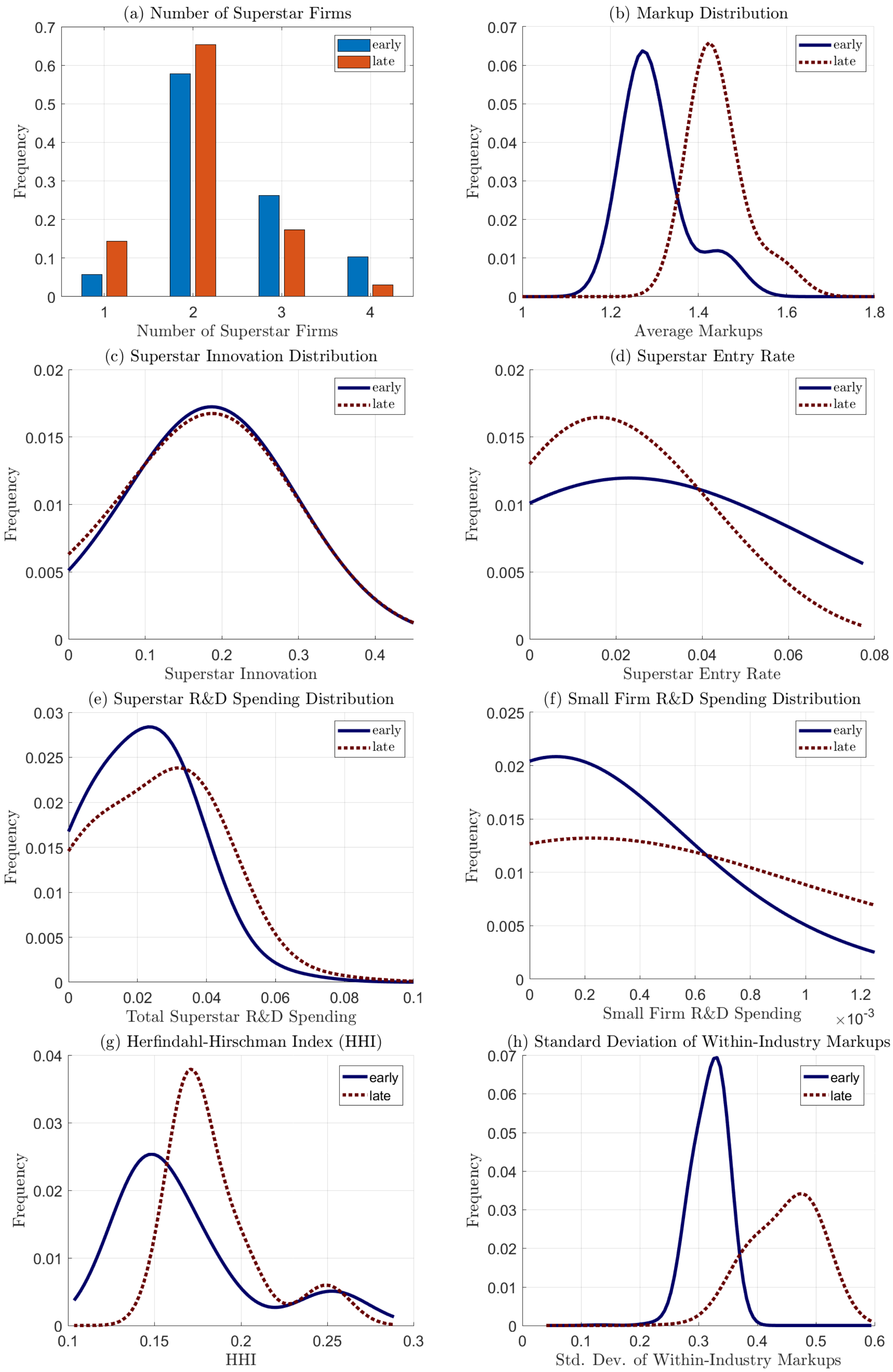

FIGURE K1: CHANGES IN DISTRIBUTION: EARLY VS. LATE

Notes: This figure compares the stationary distribution of the key variables of interest in the early vs. late subsamples. The stationary distributions for both subsamples were computed using the computational algorithms detailed in Section A.8 with parameter estimates reported in Table 1. The blue line depicts the distribution in early subsample, while the red dotted line depicts that in late subsample. Panel (a) shows the distribution of industries over states with a different number of superstar firms. Panel (b) illustrates the distribution of average markup within industries. Panel (c) depicts the distribution of superstar innovation at the industry level. Panel (d) presents the distribution of the superstar entry rate, or equivalently, small firm innovation, across industries. Panels (e) and (f) illustrate the distribution of R&D spending for superstars and small firms, respectively. Panel (g) and (h) show the distribution of the Herfindahl-Hirschman Index (HHI) and the standard deviation of within-industry markups.

L Additional Tables and Figures

Table L1: Disentangling the Structural Transition — Bertrand Competition

	Benchmark	Early η	% change	Early ζ	% change	Early ν, ϵ	% change
growth rate	2.30%	1.85%	-19.65%	2.05%	-11.15%	2.70%	17.12%
R&D intensity	2.59%	1.94%	-25.29%	1.91%	-26.17%	3.37%	30.11%
average markup	1.404	1.408	0.23%	1.318	-6.15%	1.397	-0.55%
std. dev. markup	0.303	0.295	-2.86%	0.275	-9.51%	0.285	-5.94%
labor share	0.599	0.596	-0.47%	0.634	5.84%	0.599	-0.01%
entry rate	0.096	0.096	0.00%	0.096	0.00%	0.096	0.00%
β(innov, relative sales)	1.053	1.250	18.65%	0.993	-5.69%	1.024	-2.75%
top point (intra-industry)	0.489	0.494	1.03%	0.480	-1.92%	0.493	0.90%
avg. profitability	0.224	0.233	4.29%	0.186	-16.74%	0.216	-3.33%
avg. leader relative quality	0.678	0.762	12.29%	0.694	2.26%	0.562	-17.06%
std. dev. leader rel. quality	0.173	0.180	4.05%	0.176	2.02%	0.138	-19.79%
superstar innovation	0.174	0.129	-25.98%	0.153	-12.36%	0.237	36.12%
small firm innovation	0.039	0.025	-36.87%	0.029	-27.10%	0.079	102.45%
output share of superstars	0.665	0.680	2.13%	0.587	-11.72%	0.691	3.81%
avg. superstars per industry	2.321	1.918	-17.33%	2.189	-5.65%	3.150	35.73%
mass of small firms	1.000	0.696	-30.44%	0.783	-21.70%	1.356	35.58%
initial output	0.966	0.934	-3.35%	0.990	2.47%	0.990	2.48%
CE Welfare change		-13.10%		-3.22%		12.15%	

	Benchmark	Early χ, ϕ	% change	Early ψ, τ	% change	All	% change
growth rate	2.30%	2.37%	2.82%	2.30%	-0.26%	2.25%	-2.20%
R&D intensity	2.59%	2.55%	-1.71%	2.58%	-0.37%	2.31%	-10.76%
average markup	1.404	1.402	-0.14%	1.404	0.01%	1.304	-7.12%
std. dev. markup	0.303	0.299	-1.58%	0.304	0.07%	0.238	-21.63%
labor share	0.599	0.599	-0.01%	0.599	0.00%	0.611	1.98%
entry rate	0.096	0.096	0.00%	0.114	18.65%	0.114	18.65%
β(innov, relative sales)	1.053	0.920	-12.66%	1.054	0.10%	0.894	-15.13%
top point (intra-industry)	0.489	0.485	-0.88%	0.489	-0.02%	0.492	0.69%
avg. profitability	0.224	0.225	0.30%	0.224	0.04%	0.182	-18.50%
avg. leader relative quality	0.678	0.656	-3.33%	0.679	0.19%	0.572	-15.72%
std. dev. leader rel. quality	0.173	0.172	-0.58%	0.173	0.16%	0.151	-12.34%
superstar innovation	0.174	0.186	6.76%	0.174	-0.39%	0.209	19.93%
small firm innovation	0.039	0.042	6.74%	0.039	-0.82%	0.060	53.73%
output share of superstars	0.665	0.672	1.06%	0.665	-0.04%	0.644	-3.22%
avg. superstars per industry	2.321	2.440	5.16%	2.313	-0.32%	2.933	26.41%
mass of small firms	1.000	1.063	6.31%	0.988	-1.21%	1.000	0.00%
initial output	0.966	0.973	0.71%	0.966	-0.03%	0.989	2.37%
CE Welfare change		2.40%		-0.17%		1.37%	

Notes: We estimate the model with Bertrand competition in Section E and carry out the same experiments as in the baseline. The table reports the changes in model moments when setting parameters of interest back to their estimated levels in the early sub-sample while keeping other parameters fixed at their estimated values in the late sub-sample.

TABLE L2: DISENTANGLING THE STRUCTURAL TRANSITION — $\bar{n} = 6$

	Benchmark	Early η	% change	Early ζ	% change	Early ν, ϵ	% change
growth rate	2.31%	1.77%	-23.09%	1.95%	-15.37%	2.62%	13.75%
R&D intensity	2.69%	2.20%	-18.32%	1.89%	-30.00%	3.38%	25.57%
average markup	1.442	1.433	-0.59%	1.341	-6.99%	1.446	0.29%
std. dev. markup	0.446	0.415	-6.86%	0.392	-12.18%	0.432	-3.15%
labor share	0.610	0.608	-0.30%	0.645	5.75%	0.605	-0.79%
entry rate	0.096	0.096	0.00%	0.096	0.00%	0.096	0.00%
β(innov, relative sales)	0.914	0.816	-10.79%	0.911	-0.41%	0.990	8.31%
top point (intra-industry)	0.448	0.436	-2.71%	0.444	-0.81%	0.461	2.88%
avg. profitability	0.209	0.216	3.29%	0.173	-17.29%	0.208	-0.30%
avg. leader relative quality	0.652	0.746	14.42%	0.691	5.93%	0.559	-14.25%
std. dev. leader relative quality	0.160	0.189	18.24%	0.175	9.55%	0.128	-20.14%
superstar innovation	0.222	0.160	-27.80%	0.182	-17.76%	0.286	28.90%
small firm innovation	0.023	0.008	-63.24%	0.015	-35.77%	0.058	153.26%
output share of superstars	0.522	0.542	3.86%	0.449	-14.03%	0.550	5.40%
avg. superstars per industry	2.126	1.726	-18.81%	1.950	-8.29%	2.750	29.32%
mass of small firms	1.000	0.528	-47.18%	0.721	-27.87%	1.359	35.88%
initial output	0.818	0.733	-10.36%	0.839	2.56%	0.835	2.09%
CE Welfare change		-21.12%		-5.35%		9.71%	

	Benchmark	Early χ, ϕ	% change	Early ψ, τ	% change	All	% change
growth rate	2.31%	2.57%	11.25%	2.47%	7.23%	2.17%	-6.04%
R&D intensity	2.69%	2.82%	4.59%	2.99%	11.12%	2.39%	-11.45%
average markup	1.442	1.441	-0.07%	1.444	0.17%	1.316	-8.69%
std. dev. markup	0.446	0.446	3.02E-05	0.440	-1.39%	0.326	-26.88%
labor share	0.610	0.611	0.07%	0.608	-0.40%	0.621	1.76%
entry rate	0.096	0.096	0.00%	0.114	18.65%	0.114	18.65%
β(innov, relative sales)	0.914	0.922	0.80%	0.915	0.01%	0.771	-15.64%
top point (intra-industry)	0.448	0.448	0.05%	0.453	1.21%	0.453	1.25%
avg. profitability	0.209	0.207	-0.77%	0.209	0.06%	0.169	-19.28%
avg. leader relative quality	0.652	0.649	-0.43%	0.613	-6.04%	0.605	-7.25%
std. dev. leader relative quality	0.160	0.161	0.71%	0.143	-10.76%	0.137	-14.13%
superstar innovation	0.222	0.250	12.55%	0.248	11.69%	0.240	8.34%
small firm innovation	0.023	0.027	19.12%	0.032	42.16%	0.025	8.58%
output share of superstars	0.522	0.521	-0.09%	0.535	2.44%	0.509	-2.46%
avg. superstars per industry	2.126	2.160	1.59%	2.339	10.01%	2.230	4.88%
mass of small firms	1.000	1.129	12.90%	1.672	67.17%	1.000	0.00%
initial output	0.818	0.817	-0.08%	0.826	0.96%	0.770	-5.82%
CE Welfare change		6.47%		4.92%		-8.74%	

Notes: We estimate the baseline model with the maximum number of productivity steps between any two superstar firms $\bar{n}$ set to 6 and carry out the same experiments as in the baseline. The table reports the changes in model moments when setting parameters of interest back to their estimated levels in the early sub-sample while keeping other parameters fixed at their estimated values in the late sub-sample.

Table L3: Disentangling the Structural Transition — $\bar{N} = 5$

	Benchmark	Early η	% change	Early ζ	% change	Early ν, ϵ	% change
growth rate	2.31%	2.07%	-10.44%	1.83%	-21.04%	2.76%	19.24%
R&D intensity	2.50%	2.37%	-5.19%	1.55%	-37.76%	3.40%	36.12%
average markup	1.444	1.448	0.24%	1.320	-8.56%	1.450	0.39%
std. dev. markup	0.452	0.425	-6.00%	0.381	-15.73%	0.437	-3.50%
labor share	0.610	0.604	-1.05%	0.653	6.98%	0.605	-0.96%
entry rate	0.096	0.096	0.00%	0.096	0.00%	0.096	0.00%
β(innov, relative sales)	0.786	0.706	-10.07%	0.796	1.37%	0.866	10.22%
top point (intra-industry)	0.447	0.434	-2.89%	0.443	-0.98%	0.469	4.77%
avg. profitability	0.210	0.219	4.35%	0.166	-21.03%	0.209	-0.68%
avg. leader relative quality	0.678	0.720	6.23%	0.728	7.47%	0.560	-17.35%
std. dev. leader rel. quality	0.166	0.177	6.61%	0.181	9.23%	0.136	-18.03%
superstar innovation	0.169	0.145	-13.99%	0.129	-23.59%	0.237	40.55%
small firm innovation	0.019	0.011	-41.98%	0.011	-42.26%	0.062	221.81%
output share of superstars	0.516	0.549	6.24%	0.429	-16.90%	0.550	6.48%
avg. superstars per industry	2.093	1.874	-10.50%	1.868	-10.74%	2.954	41.12%
mass of small firms	1.000	0.717	-28.26%	0.666	-33.39%	1.434	43.36%
initial output	0.793	0.733	-7.50%	0.819	3.31%	0.812	2.39%
CE Welfare change		-12.79%		-7.63%		13.35%	

	Benchmark	Early χ, ϕ	% change	Early ψ, τ	% change	All	% change
growth rate	2.31%	2.39%	3.17%	2.46%	6.28%	2.19%	-5.23%
R&D intensity	2.50%	2.47%	-0.90%	2.72%	8.97%	2.08%	-16.63%
average markup	1.444	1.444	0.02%	1.446	0.14%	1.301	-9.87%
std. dev. markup	0.452	0.451	-0.32%	0.448	-0.90%	0.325	-28.07%
labor share	0.610	0.610	-0.07%	0.609	-0.28%	0.628	2.86%
entry rate	0.096	0.096	0.00%	0.114	18.65%	0.114	18.65%
β(innov, relative sales)	0.786	0.750	-4.47%	0.778	-0.95%	0.688	-12.42%
top point (intra-industry)	0.447	0.446	-0.29%	0.452	0.99%	0.463	3.46%
avg. profitability	0.210	0.211	0.39%	0.210	0.00%	0.162	-22.75%
avg. leader relative quality	0.678	0.668	-1.51%	0.649	-4.27%	0.605	-10.70%
std. dev. leader rel. quality	0.166	0.165	-0.62%	0.155	-6.28%	0.142	-14.13%
superstar innovation	0.169	0.177	4.95%	0.184	9.21%	0.181	7.48%
small firm innovation	0.019	0.022	12.23%	0.025	28.45%	0.030	53.68%
output share of superstars	0.516	0.519	0.56%	0.525	1.72%	0.483	-6.40%
avg. superstars per industry	2.093	2.154	2.92%	2.245	7.25%	2.441	16.61%
mass of small firms	1.000	1.075	7.47%	1.427	42.69%	1.000	0.00%
initial output	0.793	0.794	0.19%	0.798	0.64%	0.769	-3.02%
CE Welfare change		2.06%		4.12%		-5.50%	

Notes: Using the model with the maximum number of superstar firms in an industry $\bar{N}$ set to 5, we carry out the same experiments as in the baseline. The table reports the changes in model moments when setting parameters of interest back to their estimated levels in the early sub-sample while keeping other parameters fixed at their estimated values in the late sub-sample.

TABLE L4: DISENTANGLING THE STRUCTURAL TRANSITION — TOTAL INTANGIBLE INVESTMENT

	Benchmark	Early η	% change	Early ζ	% change	Early ν, ϵ	% change
growth rate	2.34%	2.23%	-4.62%	1.90%	-18.54%	2.54%	8.60%
intangible inv. intensity	3.01%	3.07%	2.23%	1.93%	-35.81%	3.46%	15.15%
average markup	1.493	1.523	1.97%	1.359	-9.01%	1.496	0.17%
std. dev. markup	0.464	0.446	-3.91%	0.397	-14.54%	0.455	-2.03%
labor share	0.592	0.577	-2.50%	0.638	7.71%	0.589	-0.53%
entry rate	0.096	0.096	0.00%	0.096	0.00%	0.096	0.00%
β(intangible inv., relative sales)	7.556	5.163	-31.68%	6.801	-10.00%	8.305	9.91%
top point (intra-industry)	0.442	0.415	-6.14%	0.437	-1.05%	0.446	0.81%
avg. profitability	0.228	0.246	7.72%	0.182	-20.49%	0.228	-0.22%
avg. leader relative quality	0.643	0.672	4.53%	0.693	7.71%	0.590	-8.27%
std. dev. leader rel. quality	0.159	0.164	3.30%	0.177	11.08%	0.140	-11.95%
superstar innovation	0.191	0.174	-8.92%	0.149	-21.79%	0.221	16.01%
small firm innovation	0.024	0.014	-41.42%	0.014	-40.86%	0.038	60.16%
output share of superstars	0.563	0.610	8.30%	0.470	-16.61%	0.581	3.17%
avg. superstars per industry	2.198	1.994	-9.30%	1.965	-10.61%	2.524	14.81%
mass of small firms	1.000	0.743	-25.67%	0.674	-32.57%	1.149	14.92%
initial output	0.789	0.739	-6.32%	0.807	2.37%	0.799	1.28%
CE Welfare change		-8.86%		-7.11%		5.99%	

	Benchmark	Early χ, ϕ	% change	Early ψ, τ	% change	All	% change
growth rate	2.34%	2.74%	17.11%	2.42%	3.52%	2.25%	-3.90%
intangible inv. intensity	3.01%	3.21%	6.62%	3.17%	5.51%	2.53%	-15.74%
average markup	1.493	1.492	-0.09%	1.494	0.08%	1.357	-9.16%
std. dev. markup	0.464	0.463	-0.35%	0.461	-0.75%	0.351	-24.43%
labor share	0.592	0.592	0.02%	0.591	-0.21%	0.607	2.49%
entry rate	0.096	0.096	0.00%	0.114	18.65%	0.114	18.65%
β(intangible inv., relative sales)	7.556	7.982	5.64%	7.817	3.45%	5.862	-22.43%
top point (intra-industry)	0.442	0.446	0.95%	0.443	0.33%	0.430	-2.67%
avg. profitability	0.228	0.227	-0.80%	0.228	-0.02%	0.186	-18.75%
avg. leader relative quality	0.643	0.630	-2.03%	0.624	-3.01%	0.603	-6.21%
std. dev. leader rel. quality	0.159	0.160	0.87%	0.151	-4.81%	0.149	-6.54%
superstar innovation	0.191	0.231	20.99%	0.202	5.67%	0.214	12.07%
small firm innovation	0.024	0.032	35.85%	0.028	17.45%	0.026	9.73%
output share of superstars	0.563	0.566	0.47%	0.570	1.19%	0.536	-4.89%
avg. superstars per industry	2.198	2.303	4.77%	2.303	4.80%	2.289	4.14%
mass of small firms	1.000	1.230	22.96%	1.273	27.34%	1.000	0.00%
initial output	0.789	0.790	0.11%	0.792	0.48%	0.765	-3.05%
CE Welfare change		10.40%		2.39%		-4.76%	

Notes: We examine the robustness of our quantitative results by conducting an alternative estimation for the early and late subsamples in which we target (1) a measure of aggregate intangible investment to GDP rather than R&D to GDP alone, and (2) use a firm-year-level measure of total intangible investment to measure innovation, as opposed to relying on patent data, when obtaining the data moments that let us replicate the inverted-U relationship between innovation and relative sales. Using the re-estimated model, we carry out the same experiments as in the baseline. The table reports the changes in model moments when setting parameters of interest back to their estimated levels in the early sub-sample while keeping other parameters fixed at their estimated values in the late sub-sample.

Table L5: Disentangling the Structural Transition — Linear Small Firm Entry Cost

	Benchmark	Early η	% change	Early ζ	% change	Early ν, ϵ	% change
growth rate	2.31%	1.71%	-26.26%	1.32%	-42.73%	2.81%	21.36%
R&D intensity	2.50%	1.92%	-23.13%	1.10%	-55.93%	3.53%	41.62%
average markup	1.444	1.440	-0.31%	1.314	-9.01%	1.451	0.45%
std. dev. markup	0.452	0.430	-4.85%	0.387	-14.52%	0.434	-4.11%
labor share	0.610	0.609	-0.31%	0.657	7.69%	0.604	-1.13%
entry rate	0.096	0.096	0.00%	0.096	0.00%	0.096	0.00%
β(innov, relative sales)	0.783	0.717	-8.50%	0.818	4.47%	0.875	11.64%
top point (intra-industry)	0.448	0.432	-3.60%	0.440	-1.63%	0.468	4.51%
avg. profitability	0.210	0.218	3.77%	0.165	-21.49%	0.209	-0.72%
avg. leader relative quality	0.678	0.773	14.09%	0.808	19.18%	0.541	-20.26%
std. dev. leader rel. quality	0.165	0.189	14.21%	0.190	15.03%	0.119	-28.19%
superstar innovation	0.169	0.118	-30.20%	0.091	-45.89%	0.247	46.30%
small firm innovation	0.019	0.008	-58.85%	0.006	-67.05%	0.065	242.79%
output share of superstars	0.516	0.529	2.55%	0.408	-20.92%	0.555	7.57%
avg. superstars per industry	2.090	1.688	-19.22%	1.588	-24.00%	3.068	46.81%
mass of small firms	1.000	0.406	-59.37%	0.295	-70.47%	2.183	118.26%
initial output	0.793	0.726	-8.43%	0.808	1.93%	0.815	2.80%
CE Welfare change		-20.84%		-19.22%		15.01%	

	Benchmark	Early χ, ϕ	% change	Early ψ, τ	% change	All	% change
growth rate	2.31%	2.43%	5.25%	2.63%	13.65%	2.19%	-5.40%
R&D intensity	2.50%	2.54%	1.92%	3.04%	21.86%	2.07%	-17.01%
average markup	1.444	1.445	0.06%	1.449	0.31%	1.301	-9.88%
std. dev. markup	0.452	0.450	-0.61%	0.442	-2.19%	0.325	-28.06%
labor share	0.610	0.609	-0.16%	0.606	-0.67%	0.628	2.87%
entry rate	0.096	0.096	0.00%	0.114	18.65%	0.114	18.65%
β(innov, relative sales)	0.783	0.744	-4.99%	0.785	0.15%	0.683	-12.82%
top point (intra-industry)	0.448	0.448	0.04%	0.459	2.57%	0.462	3.17%
avg. profitability	0.210	0.211	0.41%	0.210	-0.09%	0.162	-22.74%
avg. leader relative quality	0.678	0.658	-2.90%	0.607	-10.41%	0.607	-10.44%
std. dev. leader rel. quality	0.165	0.161	-2.77%	0.138	-16.29%	0.140	-15.27%
superstar innovation	0.169	0.182	7.99%	0.207	22.69%	0.180	6.75%
small firm innovation	0.019	0.023	20.67%	0.035	82.27%	0.028	46.31%
output share of superstars	0.516	0.522	1.12%	0.538	4.14%	0.483	-6.46%
avg. superstars per industry	2.090	2.198	5.20%	2.495	19.40%	2.412	15.41%
mass of small firms	1.000	1.206	20.58%	2.530	152.96%	1.000	0.00%
initial output	0.793	0.796	0.40%	0.805	1.54%	0.769	-3.03%
CE Welfare change		3.43%		9.24%		-5.58%	

Notes: Using the alternative model with linear small firm entry cost, we carry out the same experiments as in the baseline. The table reports the changes in model moments when setting parameters of interest back to their estimated levels in the early sub-sample while keeping other parameters fixed at their estimated values in the late sub-sample.

TABLE L6: DISENTANGLING THE STRUCTURAL TRANSITION — ENTREPRENEUR COST CONVEXITY = 3

	Benchmark	Early η	% change	Early ζ	% change	Early ν, ϵ	% change
growth rate	2.31%	2.14%	-7.35%	1.91%	-17.37%	2.71%	17.25%
R&D intensity	2.50%	2.46%	-1.42%	1.64%	-34.38%	3.26%	30.48%
average markup	1.444	1.449	0.35%	1.322	-8.48%	1.449	0.36%
std. dev. markup	0.452	0.424	-6.25%	0.380	-15.96%	0.439	-3.00%
labor share	0.610	0.603	-1.20%	0.652	6.86%	0.605	-0.86%
entry rate	0.096	0.096	0.00%	0.096	0.00%	0.096	0.00%
β(innov, relative sales)	0.783	0.701	-10.50%	0.788	0.60%	0.830	5.91%
top point (intra-industry)	0.448	0.436	-2.66%	0.444	-0.76%	0.466	3.97%
avg. profitability	0.210	0.220	4.46%	0.166	-20.97%	0.209	-0.43%
avg. leader relative quality	0.678	0.709	4.57%	0.714	5.30%	0.578	-14.78%
std. dev. leader rel. quality	0.165	0.173	4.46%	0.177	6.94%	0.134	-19.12%
superstar innovation	0.169	0.151	-10.55%	0.136	-19.41%	0.225	33.25%
small firm innovation	0.019	0.012	-36.57%	0.012	-34.91%	0.049	157.90%
output share of superstars	0.516	0.552	7.00%	0.433	-16.17%	0.545	5.59%
avg. superstars per industry	2.090	1.915	-8.38%	1.925	-7.87%	2.751	31.65%
mass of small firms	1.000	0.815	-18.52%	0.781	-21.86%	1.228	22.80%
initial output	0.793	0.735	-7.30%	0.821	3.57%	0.809	2.07%
CE Welfare change		-11.12%		-5.50%		11.88%	

	Benchmark	Early χ, ϕ	% change	Early ψ, τ	% change	All	% change
growth rate	2.31%	2.37%	2.62%	2.41%	4.12%	2.19%	-5.40%
R&D intensity	2.50%	2.45%	-1.66%	2.64%	5.69%	2.07%	-17.01%
average markup	1.444	1.444	0.01%	1.445	0.09%	1.301	-9.88%
std. dev. markup	0.452	0.451	-0.24%	0.450	-0.57%	0.325	-28.06%
labor share	0.610	0.610	-0.05%	0.609	-0.18%	0.628	2.87%
entry rate	0.096	0.096	0.00%	0.114	18.65%	0.114	18.65%
β(innov, relative sales)	0.783	0.749	-4.43%	0.777	-0.80%	0.683	-12.82%
top point (intra-industry)	0.448	0.446	-0.36%	0.450	0.59%	0.462	3.17%
avg. profitability	0.210	0.211	0.39%	0.210	0.01%	0.162	-22.74%
avg. leader relative quality	0.678	0.670	-1.14%	0.660	-2.71%	0.607	-10.44%
std. dev. leader rel. quality	0.165	0.165	-0.21%	0.158	-4.19%	0.140	-15.27%
superstar innovation	0.169	0.176	4.11%	0.178	5.79%	0.180	6.75%
small firm innovation	0.019	0.021	9.60%	0.022	16.22%	0.028	46.31%
output share of superstars	0.516	0.518	0.42%	0.522	1.10%	0.483	-6.46%
avg. superstars per industry	2.090	2.137	2.27%	2.181	4.38%	2.412	15.41%
mass of small firms	1.000	1.045	4.49%	1.253	25.26%	1.000	0.00%
initial output	0.793	0.794	0.14%	0.796	0.41%	0.769	-3.03%
CE Welfare change		1.71%		2.68%		-5.60%	

Notes: Using the alternative model with entrepreneur cost convexity set to 3, we carry out the same experiments as in the baseline. The table reports the changes in model moments when setting parameters of interest back to their estimated levels in the early sub-sample while keeping other parameters fixed at their estimated values in the late sub-sample.

TABLE L7: DISENTANGLING THE STRUCTURAL TRANSITION — DECREASING RETURNS TO SCALE

	Benchmark	Early η	% change	Early ζ	% change	Early ν, ϵ	% change
growth rate	2.31%	2.02%	-12.67%	1.88%	-18.76%	2.66%	15.14%
R&D intensity	2.78%	2.39%	-14.19%	1.72%	-38.10%	3.48%	25.24%
average markup	1.469	1.455	-0.96%	1.310	-10.83%	1.468	-0.02%
std. dev. markup	0.467	0.458	-2.02%	0.370	-20.83%	0.452	-3.26%
labor share	0.542	0.546	0.71%	0.590	8.85%	0.539	-0.54%
entry rate	0.096	0.096	0.00%	0.096	0.00%	0.096	0.00%
β(innov, relative sales)	0.782	0.804	2.79%	0.742	-5.16%	0.799	2.17%
top point (intra-industry)	0.436	0.442	1.54%	0.429	-1.43%	0.442	1.51%
avg. profitability	0.294	0.292	-0.47%	0.243	-17.12%	0.290	-1.05%
avg. leader relative quality	0.703	0.749	6.52%	0.731	4.04%	0.632	-10.16%
std. dev. leader rel. quality	0.171	0.181	6.25%	0.181	6.18%	0.157	-7.72%
superstar innovation	0.166	0.141	-15.08%	0.133	-19.74%	0.211	26.71%
small firm innovation	0.014	0.011	-24.66%	0.009	-34.45%	0.032	119.26%
output share of superstars	0.534	0.526	-1.46%	0.430	-19.39%	0.558	4.65%
avg. superstars per industry	1.944	1.776	-8.61%	1.823	-6.21%	2.358	21.31%
mass of small firms	1.000	1.005	0.51%	1.100	9.96%	0.983	-1.68%
initial output	0.901	0.858	-4.81%	0.963	6.85%	0.919	1.97%
CE Welfare change		-11.18%		-3.41%		10.52%	

	Benchmark	Early χ, ϕ	% change	Early ψ, τ	% change	All	% change
growth rate	2.31%	2.38%	3.14%	2.28%	-1.08%	2.19%	-5.01%
R&D intensity	2.78%	2.72%	-2.18%	2.77%	-0.55%	2.11%	-24.08%
average markup	1.469	1.468	-0.03%	1.475	0.44%	1.296	-11.75%
std. dev. markup	0.467	0.467	-0.13%	0.472	1.08%	0.339	-27.38%
labor share	0.542	0.542	0.01%	0.541	-0.28%	0.570	4.99%
entry rate	0.096	0.096	0.00%	0.114	18.65%	0.114	18.65%
β(innov, relative sales)	0.782	0.760	-2.85%	0.788	0.69%	0.729	-6.84%
top point (intra-industry)	0.436	0.433	-0.48%	0.436	0.06%	0.438	0.60%
avg. profitability	0.294	0.294	0.24%	0.296	0.71%	0.239	-18.73%
avg. leader relative quality	0.703	0.700	-0.45%	0.708	0.76%	0.653	-7.06%
std. dev. leader rel. quality	0.171	0.171	0.21%	0.172	1.08%	0.166	-2.73%
superstar innovation	0.166	0.173	3.91%	0.164	-1.40%	0.177	6.34%
small firm innovation	0.014	0.015	7.73%	0.014	-2.52%	0.021	44.89%
output share of superstars	0.534	0.534	0.15%	0.535	0.29%	0.458	-14.16%
avg. superstars per industry	1.944	1.965	1.11%	1.923	-1.04%	2.174	11.87%
mass of small firms	1.000	1.000	0.01%	0.928	-7.20%	1.000	0.00%
initial output	0.901	0.901	0.05%	0.897	-0.39%	0.936	3.95%
CE Welfare change		1.95%		-1.07%		1.32%	

Notes: We estimate model with decreasing returns to scale as shown in Section C and carry out the same experiments as in the baseline. The table reports the changes in model moments when setting parameters of interest back to their estimated levels in the early sub-sample while keeping other parameters fixed at their estimated values in the late sub-sample.

TABLE L8: DISENTANGLING THE STRUCTURAL TRANSITION — REVERSE COUNTERFACTUALS

	Benchmark	Late η	% change	Late ζ	% change	Late ν, ϵ	% change
growth rate	2.19%	2.42%	10.51%	2.60%	19.00%	1.60%	-26.66%
R&D intensity	2.07%	2.51%	21.39%	3.47%	67.39%	1.40%	-32.53%
average markup	1.301	1.333	2.45%	1.468	12.83%	1.290	-0.88%
std. dev. markup	0.325	0.365	12.09%	0.406	24.89%	0.330	1.36%
labor share	0.628	0.620	-1.30%	0.570	-9.31%	0.634	1.03%
entry rate	0.114	0.114	0.00%	0.114	0.00%	0.114	0.00%
β(innov, relative sales)	0.683	0.775	13.45%	0.692	1.31%	0.703	2.89%
top point (intra-industry)	0.462	0.460	-0.37%	0.469	1.49%	0.435	-5.85%
avg. profitability	0.162	0.169	4.00%	0.225	38.36%	0.161	-1.08%
avg. leader relative quality	0.607	0.536	-11.75%	0.558	-8.17%	0.737	21.31%
std. dev. leader rel. quality	0.140	0.126	-10.11%	0.121	-13.50%	0.186	32.63%
superstar innovation	0.180	0.229	27.04%	0.230	27.71%	0.120	-33.22%
small firm innovation	0.028	0.054	95.54%	0.046	66.39%	0.009	-68.57%
output share of superstars	0.483	0.482	-0.24%	0.607	25.79%	0.450	-6.73%
avg. superstars per industry	2.412	2.992	24.07%	2.741	13.67%	1.798	-25.46%
mass of small firms	1.000	1.709	70.89%	1.580	57.98%	0.691	-30.86%
initial output	0.769	0.847	10.17%	0.756	-1.61%	0.758	-1.39%
CE Welfare change		16.13%		7.59%		-14.18%	

	Benchmark	Late χ, ϕ	% change	Late ψ, τ	% change	All	% change
growth rate	2.19%	2.12%	-3.30%	2.09%	-4.24%	2.31%	5.71%
R&D intensity	2.07%	2.09%	0.89%	1.94%	-6.48%	2.50%	20.49%
average markup	1.301	1.301	-0.02%	1.299	-0.16%	1.444	10.96%
std. dev. markup	0.325	0.326	0.13%	0.326	0.28%	0.452	39.00%
labor share	0.628	0.628	0.04%	0.629	0.19%	0.610	-2.79%
entry rate	0.114	0.114	0.00%	0.096	-15.72%	0.096	-15.72%
β(innov, relative sales)	0.683	0.707	3.51%	0.681	-0.30%	0.783	14.71%
top point (intra-industry)	0.462	0.460	-0.47%	0.454	-1.69%	0.448	-3.07%
avg. profitability	0.162	0.162	-0.31%	0.162	-0.15%	0.210	29.44%
avg. leader relative quality	0.607	0.619	1.93%	0.633	4.27%	0.678	11.66%
std. dev. leader rel. quality	0.140	0.141	0.69%	0.151	7.88%	0.165	18.03%
superstar innovation	0.180	0.170	-5.57%	0.168	-6.78%	0.169	-6.33%
small firm innovation	0.028	0.024	-14.06%	0.022	-20.92%	0.019	-31.65%
output share of superstars	0.483	0.481	-0.39%	0.477	-1.29%	0.516	6.90%
avg. superstars per industry	2.412	2.325	-3.58%	2.258	-6.36%	2.090	-13.36%
mass of small firms	1.000	0.905	-9.50%	0.700	-30.04%	1.000	0.00%
initial output	0.769	0.768	-0.06%	0.767	-0.27%	0.793	3.13%
CE Welfare change		-1.87%		-2.42%		5.92%	

Notes: The table reports the change in model moments when setting the parameter of interest to its estimated level in late sub-sample while keeping other parameters fixed at their estimated values in the early sub-sample.

TABLE L9: DISENTANGLING THE STRUCTURAL TRANSITION — CONVEXITY = 2

	Benchmark	Early η	% change	Early ζ	% change	Early ν, ϵ	% change
growth rate	2.01%	1.46%	-27.65%	1.33%	-34.08%	2.31%	14.79%
R&D intensity	4.29%	3.38%	-21.20%	2.26%	-47.18%	5.04%	17.64%
average markup	1.551	1.564	0.83%	1.343	-13.43%	1.560	0.60%
std. dev. markup	0.503	0.486	-3.52%	0.379	-24.64%	0.487	-3.32%
labor share	0.578	0.570	-1.38%	0.642	11.20%	0.571	-1.20%
entry rate	0.096	0.096	0.00%	0.096	0.00%	0.096	0.00%
β(innov, relative sales)	1.115	0.763	-31.56%	0.598	-46.34%	1.023	-8.24%
top point (intra-industry)	0.463	0.415	-10.33%	0.374	-19.15%	0.456	-1.48%
avg. profitability	0.233	0.252	8.09%	0.172	-26.15%	0.235	0.49%
avg. leader relative quality	0.684	0.769	12.41%	0.693	1.25%	0.627	-8.40%
std. dev. leader rel. quality	0.168	0.206	22.97%	0.190	13.12%	0.133	-20.70%
superstar innovation	0.142	0.103	-27.66%	0.095	-32.80%	0.166	16.80%
small firm innovation	0.013	0.004	-66.42%	0.003	-80.37%	0.018	39.39%
output share of superstars	0.569	0.594	4.35%	0.461	-18.95%	0.597	4.84%
avg. superstars per industry	1.949	1.614	-17.19%	1.796	-7.88%	2.140	9.79%
mass of small firms	1.000	0.483	-51.69%	0.339	-66.07%	1.091	9.14%
initial output	0.802	0.746	-7.01%	0.865	7.84%	0.822	2.55%
CE Welfare change		-18.32%		-7.24%		9.60%	

	Benchmark	Early χ, ϕ	% change	Early ψ, τ	% change	All	% change
growth rate	2.01%	0.64%	-68.16%	2.42%	20.36%	1.86%	-7.59%
R&D intensity	4.29%	0.59%	-86.21%	5.36%	25.11%	3.06%	-28.57%
average markup	1.551	1.509	-2.70%	1.565	0.90%	1.319	-14.98%
std. dev. markup	0.503	0.567	12.70%	0.478	-5.04%	0.327	-34.94%
labor share	0.578	0.608	5.25%	0.567	-1.80%	0.620	7.40%
entry rate	0.096	0.096	0.00%	0.114	18.65%	0.114	18.65%
β(innov, relative sales)	1.115	6.391	473.04%	1.005	-9.90%	0.500	-55.13%
top point (intra-industry)	0.463	0.507	9.54%	0.452	-2.27%	0.347	-25.02%
avg. profitability	0.233	0.232	-0.49%	0.236	1.00%	0.162	-30.43%
avg. leader relative quality	0.684	0.879	28.40%	0.600	-12.27%	0.664	-3.00%
std. dev. leader rel. quality	0.168	0.132	-21.07%	0.113	-32.59%	0.179	6.69%
superstar innovation	0.142	0.038	-73.21%	0.176	23.79%	0.142	0.00%
small firm innovation	0.013	0.015	13.07%	0.022	71.70%	0.005	-60.68%
output share of superstars	0.569	0.452	-20.59%	0.610	7.26%	0.500	-12.08%
avg. superstars per industry	1.949	1.590	-18.45%	2.243	15.05%	1.858	-4.70%
mass of small firms	1.000	0.894	-10.60%	2.966	196.65%	1.000	0.00%
initial output	0.802	0.720	-10.18%	0.833	3.86%	0.811	1.14%
CE Welfare change		-33.81%		13.78%		-1.40%	

Notes: Using the re-estimated baseline model with R&D cost convexity parameters ϵ and ϕ set to 2, we carry out the same experiments as in the baseline. The table reports the changes in model moments when setting parameters of interest back to their estimated levels in the early sub-sample while keeping other parameters fixed at their estimated values in the late sub-sample.

Table L10: Counter-Example with Flipped Dynamic Impact of Reducing ζ – Experiment

	Counter-example Baseline	Reducing ζ by 25%	% change
growth rate	0.71%	0.25%	-65.01%
R&D intensity	2.87%	1.74%	-39.13%
average markup	1.883	2.347	24.64%
std. dev. markup	0.478	0.940	96.81%
labor share	0.456	0.426	-6.56%
entry rate	0.115	0.115	0.00%
β(innov, relative sales)	-8.645	-18.366	112.45%
top point (intra-industry)	0.599	0.503	-16.09%
avg. profitability	0.389	0.439	13.06%
avg. leader relative quality	0.674	0.937	39.06%
std. dev leader rel. quality	0.214	0.155	-27.37%
superstar innovation	0.120	0.039	-67.42%
small firm innovation	0.001	0.003	139.16%
output share of superstars	0.803	0.699	-12.88%
avg. superstars per industry	1.703	1.144	-32.83%
mass of small firms	1.000	1.450	44.98%
initial output	0.821	0.635	-22.56%
CE Welfare change		-30.20%	

Notes: This table presents the changes in the relevant macroeconomic aggregates when the relative productivity of the competitive fringe ζ is reduced to 75% of its value in the counter-example economy.

Table L11: Counter-Example with Flipped Dynamic Impact of Reducing ζ – Parameters

Parameter	*Description*	*Values*
λ	innovation step size	0.1244
η	elasticity within industry	13.2030
χ	superstar cost scale	4.5125
ν	small firm cost scale	2.8312
ζ	competitive fringe ratio	0.4075
ϕ	superstar cost convexity	2.2666
ϵ	small firm cost convexity	1.2340
τ	exit rate	0.1151
ψ	entry cost scale	0.0075

Notes: This table presents parameters that are used in the counter-example shown in Table L10.

TABLE L12: CHANGES IN STATIC EFFICIENCY FOR THE MODEL WITH COLLUDING FRINGE

	ΔW	CEWC
competitive fringe productivity	3.297	14.10%
relative wage	-0.427	-1.69%
output of superstar firms	-1.998	-7.68%
consumption/output	0.000	0.00%
output growth	0.000	0.00%
total	0.872	3.55%

Notes: The table reports the changes in static efficiency for the model with colluding fringe as shown in Section C when setting ζ back to its estimated level in the early subsample while keeping other parameters fixed at their estimated values in the late subsample.

TABLE L13: MODEL PARAMETERS AND TARGET MOMENTS — BERTRAND COMPETITION

A. Parameter estimates

Parameter	*Description*	*Early sub-sample*	*Late sub-sample*
λ	innovation step size	0.2964	0.3235
η	elasticity within industry	5.0481	4.4352
χ	superstar cost scale	226.3518	101.5290
ν	small firm cost scale	1.2287	2.0047
ζ	competitive fringe ratio	0.6049	0.5230
ϕ	superstar cost convexity	4.0661	3.7156
ϵ	small firm cost convexity	3.0752	2.5098
τ	exit rate	0.1144	0.0964
ψ	entry cost scale	0.0402	0.0530

B. Moments

	Early sub-sample		Late sub-sample	
Target moments	Data	Model	Data	Model
growth rate	2.19%	2.25%	2.31%	2.30%
R&D intensity	2.40%	2.31%	2.50%	2.59%
average markup	1.3014	1.3043	1.4442	1.4043
std. dev. markup	0.306	0.238	0.421	0.303
labor share	0.656	0.611	0.644	0.599
firm entry rate	0.114	0.114	0.096	0.096
β(innovation, relative sales)	0.449	0.894	0.631	1.053
top point (intra-industry)	0.443	0.492	0.515	0.489
average profitability	0.136	0.182	0.152	0.224
average leader relative quality	0.751	0.572	0.746	0.678
std. dev. leader relative quality	0.224	0.151	0.222	0.173

Notes: We estimate the model with Bertrand competition in Section E using the simulated method of moments. Panel A reports the estimated parameters. Panel B reports the simulated and actual moments.

TABLE L14: MODEL PARAMETERS AND TARGET MOMENTS — $\bar{n} = 6$

A. Parameter estimates

Parameter	*Description*	*Early sub-sample*	*Late sub-sample*
λ	innovation step size	0.2074	0.2415
η	elasticity within industry	21.8696	5.9872
χ	superstar cost scale	35.4096	32.2514
ν	small firm cost scale	1.2204	1.9232
ζ	competitive fringe ratio	0.6003	0.5370
ϕ	superstar cost convexity	3.8410	3.6361
ϵ	small firm cost convexity	2.7888	2.3619
τ	exit rate	0.1144	0.0964
ψ	entry cost scale	0.0079	0.0244

B. Moments

	Early sub-sample		Late sub-sample	
Target moments	Data	Model	Data	Model
growth rate	2.19%	2.17%	2.31%	2.31%
R&D intensity	2.40%	2.39%	2.50%	2.69%
average markup	1.3014	1.3163	1.4442	1.4416
std. dev. markup	0.306	0.326	0.421	0.446
labor share	0.656	0.621	0.644	0.610
firm entry rate	0.114	0.114	0.096	0.096
β(innovation, relative sales)	0.449	0.771	0.631	0.914
top point (intra-industry)	0.443	0.453	0.515	0.448
average profitability	0.136	0.169	0.152	0.209
average leader relative quality	0.751	0.605	0.746	0.652
std. dev. leader relative quality	0.224	0.137	0.222	0.160

Notes: The baseline model is re-estimated using the simulated method of moments, with the maximum number of productivity steps between any two superstar firms, $\bar{n}$, set to 6. Panel A reports the estimated parameters. Panel B reports the simulated and actual moments.

Table L15: Model Parameters and Target Moments — Total Intangible Investment

A. Parameter estimates

Parameter	*Description*	*Early sub-sample*	*Late sub-sample*
λ	innovation step size	0.2534	0.2943
η	elasticity within industry	20.9179	7.0532
χ	superstar cost scale	97.8103	65.5492
ν	small firm cost scale	1.3744	2.3948
ζ	competitive fringe ratio	0.5792	0.5072
ϕ	superstar cost convexity	4.0096	3.6158
ϵ	small firm cost convexity	2.4831	2.3904
τ	exit rate	0.1144	0.0964
ψ	entry cost scale	0.0164	0.0326

B. Moments

	Early sub-sample		Late sub-sample	
Target moments	Data	Model	Data	Model
growth rate	2.19%	2.25%	2.31%	2.34%
intangible investment intensity	3.98%	2.53%	4.28%	3.01%
average markup	1.3014	1.3565	1.4442	1.4933
std. dev. markup	0.306	0.351	0.421	0.464
labor share	0.656	0.607	0.644	0.592
firm entry rate	0.114	0.114	0.096	0.096
β(innovation, relative sales)	3.403	5.862	5.343	7.556
top point (intra-industry)	0.562	0.430	0.509	0.442
average profitability	0.136	0.186	0.152	0.228
average leader relative quality	0.751	0.603	0.746	0.643
std. dev. leader relative quality	0.224	0.149	0.222	0.159

Notes: We examine the robustness of our quantitative results by conducting an alternative estimation for the early and late subsamples in which we target (1) a measure of aggregate intangible investment to GDP rather than R&D to GDP alone, and (2) use a firm-year-level measure of total intangible investment to measure innovation, as opposed to relying on patent data, when obtaining the data moments that let us replicate the inverted-U relationship between innovation and relative sales. Panel A reports the estimated parameters. Panel B reports the simulated and actual moments.

TABLE L16: MODEL PARAMETERS AND TARGET MOMENTS —DECREASING RETURNS TO SCALE

A. Parameter estimates

Parameter	*Description*	*Early sub-sample*	*Late sub-sample*
λ	innovation step size	0.3009	0.3202
η	elasticity within industry	8.7117	6.5279
χ	superstar cost scale	106.5888	71.0248
ν	small firm cost scale	1.3808	0.8403
ζ	competitive fringe ratio	0.6806	0.5615
ϕ	superstar cost convexity	3.8689	3.6394
ϵ	small firm cost convexity	2.1112	1.6198
τ	exit rate	0.1144	0.0964
ψ	entry cost scale	1.5545	1.8038

B. Moments

	Early sub-sample		Late sub-sample	
Target moments	Data	Model	Data	Model
growth rate	2.19%	2.19%	2.31%	2.31%
R&D intensity	2.40%	2.11%	2.50%	2.78%
average markup	1.3014	1.2961	1.4442	1.4687
std. dev. markup	0.306	0.339	0.421	0.467
labor share	0.656	0.570	0.644	0.542
firm entry rate	0.114	0.114	0.096	0.096
β(innovation, relative sales)	0.449	0.729	0.631	0.782
top point (intra-industry)	0.443	0.438	0.515	0.436
average profitability	0.136	0.239	0.152	0.294
average leader relative quality	0.751	0.653	0.746	0.703
std. dev. leader relative quality	0.224	0.166	0.222	0.171

Notes: We estimate the model with decreasing return to scale in Section C using the simulated method of moments. Panel A reports the estimated parameters. Panel B reports the simulated and actual moments. Following notations in Section C.1, the degree of return to scale is determined by α which is set to 0.9 in our estimation.

Table L17: Model Parameters and Target Moments — Convexity=2

A. Parameter estimates

Parameter	*Description*	*Early sub-sample*	*Late sub-sample*
λ	innovation step size	0.2924	0.3159
η	elasticity within industry	12.0424	6.0021
χ	superstar cost scale	1.8798	3.0228
ν	small firm cost scale	0.4033	0.7179
ζ	competitive fringe ratio	0.6128	0.4866
ϕ	superstar cost convexity	2.0000	2.0000
ϵ	small firm cost convexity	2.0000	2.0000
τ	exit rate	0.1144	0.0964
ψ	entry cost scale	0.0010	0.0089

B. Moments

	Early sub-sample		Late sub-sample	
Target moments	Data	Model	Data	Model
growth rate	2.19%	1.86%	2.31%	2.01%
R&D intensity	2.40%	3.06%	2.50%	4.29%
average markup	1.3014	1.3185	1.4442	1.5509
std. dev. markup	0.306	0.327	0.421	0.503
labor share	0.656	0.620	0.644	0.578
firm entry rate	0.114	0.114	0.096	0.096
β(innovation, relative sales)	0.449	0.500	0.631	1.115
top point (intra-industry)	0.443	0.347	0.515	0.463
average profitability	0.136	0.162	0.152	0.233
average leader relative quality	0.751	0.664	0.746	0.684
std. dev. leader relative quality	0.224	0.179	0.222	0.168

Notes: The baseline model is re-estimated using the simulated method of moments, with the R&D cost convexity parameters, ϕ and ϵ, set to 2. Panel A reports the estimated parameters. Panel B reports the simulated and actual moments.

TABLE L18: THE DIMENSIONALITY OF THE FIRM STATE VARIABLE

	$\bar{N}=2$	$\bar{N}=3$	$\bar{N}=4$	$\bar{N}=5$	$\bar{N}=6$	$\bar{N}=7$	$\bar{N}=8$
$\bar{n}=1$	4	11	26	57	120	247	502
$\bar{n}=2$	6	25	90	301	966	3,025	9,330
$\bar{n}=3$	8	45	220	1,001	4,368	18,565	77,540
$\bar{n}=4$	10	71	440	2,541	14,070	75,811	400,900
$\bar{n}=5$	12	103	774	5,425	36,456	238,267	1,527,258
$\bar{n}=6$	14	141	1,246	10,277	81,270	624,877	4,710,062
$\bar{n}=7$	16	185	1,880	17,841	162,336	1,435,945	12,448,360
$\bar{n}=8$	18	235	2,700	28,981	298,278	2,984,095	29,253,600

Notes: This table reports the dimensionality of the firm state variable for each specific value of the maximum number of superstars in an industry, $\bar{N}$, and the maximum number of productivity steps between any two superstar firms, $\bar{n}$.

TABLE L19: THE DIMENSIONALITY OF THE INDUSTRY STATE VARIABLE

	$\bar{N}=2$	$\bar{N}=3$	$\bar{N}=4$	$\bar{N}=5$	$\bar{N}=6$	$\bar{N}=7$	$\bar{N}=8$
$\bar{n}=1$	3	6	10	15	20	25	30
$\bar{n}=2$	4	10	20	35	65	125	245
$\bar{n}=3$	5	15	35	70	175	490	1,435
$\bar{n}=4$	6	21	56	126	406	1,526	6,006
$\bar{n}=5$	7	28	84	210	840	3,990	19,740
$\bar{n}=6$	8	36	120	330	1,590	9,150	54,510
$\bar{n}=7$	9	45	165	495	2,805	18,975	132,165
$\bar{n}=8$	10	55	220	715	4,675	36,355	289,795

Notes: This table reports the dimensionality of the industry state variable for each specific value of the maximum number of superstars in an industry, $\bar{N}$, and the maximum number of productivity steps between any two superstar firms, $\bar{n}$.

TABLE L20: FIRM INNOVATION AND RELATIVE SALES – SINGLE-INDUSTRY FIRMS

Panel A

	avg. citations	tail innov. (10%)	avg. originality	avg. generality
relative sales	14.491	15.476	9.738	23.569
	(2.801)***	(3.351)***	(3.354)***	(3.569)***
relative sales sq.	-19.213	-21.820	-10.539	-26.210
	(4.199)***	(5.326)***	(4.680)**	(5.083)***
R^2	0.16	0.13	0.26	0.22
N	21,540	21,540	21,540	21,540

Panel B

	log total patents	log total citations	log R&D spending	log R&D spending 2
relative sales	1.896	3.721	1.507	0.870
	(0.304)***	(0.489)***	(0.190)***	(0.118)***
relative sales sq.	-1.826	-3.952	-1.767	-0.955
	(0.395)***	(0.627)***	(0.253)***	(0.149)***
R^2	0.49	0.42	0.95	0.95
N	21,540	21,540	12,700	21,540

Panel C

	log(xad)	log(capx)	sales growth	employment growth	asset growth
relative sales	12.097	14.566	0.451	0.328	0.500
	(0.784)***	(0.526)***	(0.066)***	(0.050)***	(0.065)***
relative sales sq.	-12.803	-15.448	-0.520	-0.353	-0.526
	(1.122)***	(0.830)***	(0.091)***	(0.071)***	(0.093)***
R^2	0.63	0.58	0.15	0.16	0.17
N	7,806	21,321	20,280	18,705	20,907

Notes: Robust asymptotic standard errors reported in parentheses are clustered at the firm level. The sample period is from 1976 to 2004 at the annual frequency, restricted to observations that only operate in a single 4-digit SIC industry. All regressions control for profitability, leverage, market-to-book ratio, log R&D stock, firm age, the coefficient of variation of the firm's stock price, year dummies, and a full set of four-digit SIC industry dummies. $***p < 0.01, **p < 0.05, *p < 0.1$.

Table L21: Firm Innovation and Relative Sales – Multiple-Industry Firms

Panel A

	avg. citations	tail innov. (10%)	avg. originality	avg. generality
relative sales	6.751	5.465	8.345	16.093
	(1.293)***	(1.484)***	(1.959)***	(2.054)***
relative sales sq.	-6.442	-5.081	-6.016	-13.162
	(1.524)***	(1.924)***	(2.372)**	(2.502)***
R^2	0.15	0.10	0.26	0.26
N	83,371	83,371	83,371	83,371

Panel B

	log total patents	log total citations	log R&D spending	log R&D spending 2
relative sales	2.167	3.502	1.358	0.996
	(0.213)***	(0.332)***	(0.101)***	(0.087)***
relative sales sq.	-1.450	-2.550	-1.184	-0.921
	(0.286)***	(0.429)***	(0.128)***	(0.113)***
R^2	0.59	0.52	0.96	0.94
N	83,371	83,371	48,486	83,371

Panel C

	log(xad)	log(capx)	sales growth	employment growth	asset growth
relative sales	10.446	11.560	0.202	0.146	0.174
	(0.351)***	(0.236)***	(0.021)***	(0.017)***	(0.022)***
relative sales sq.	-9.690	-10.525	-0.178	-0.132	-0.151
	(0.455)***	(0.300)***	(0.025)***	(0.020)***	(0.026)***
R^2	0.75	0.71	0.13	0.13	0.15
N	29,973	82,237	82,446	78,013	82,691

Notes: Robust asymptotic standard errors reported in parentheses are clustered at the firm level. The sample period is from 1976 to 2004 at the annual frequency, restricted to observations that operate in more than one 4-digit SIC industry. All regressions control for profitability, leverage, market-to-book ratio, log R&D stock, firm age, the coefficient of variation of the firm's stock price, year dummies, and a full set of four-digit SIC industry dummies. $***p < 0.01, **p < 0.05, *p < 0.1$.

Table L22: CR4 Distributions: Data vs. Model

CR4	Data	Model
Mean	48.41%	46.67%
25th percentile	35.20%	37.64%
50th percentile	46.86%	45.86%
75th percentile	54.22%	54.55%

Notes: This table reports the distribution of four-firm concentration ratio (CR4), representing the market share of the four largest firms in each industry, both in the data and the model. The CR4 in the data is calculated based on all 3-digit BEA industries between 1976-2004 using Compustat data for top firms. The CR4 in the model is calculated based on the parameter estimates of the whole sample (1976-2004). All statistics are calculated using total industry sales as weights.

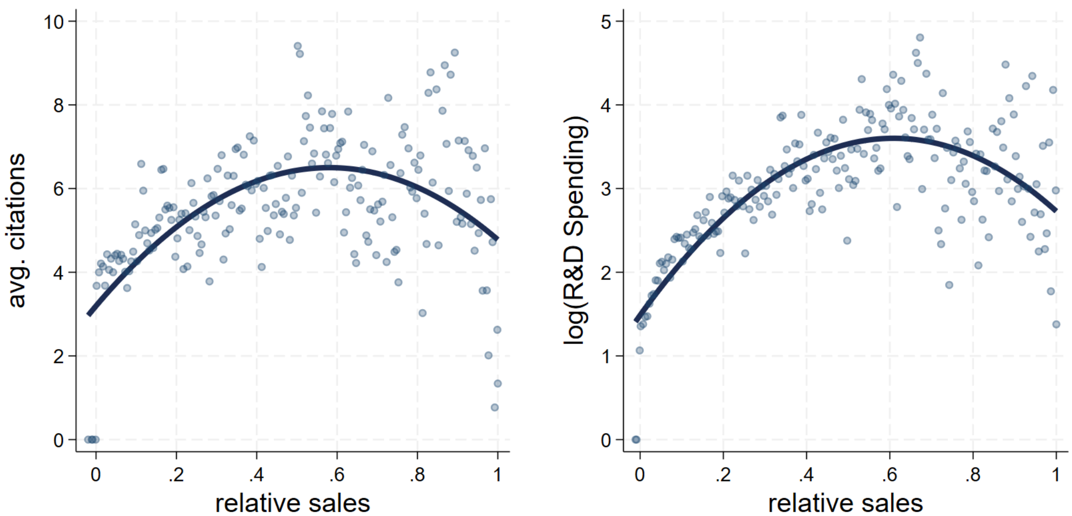

FIGURE L1: INNOVATION, R&D EXPENSES, AND FIRM RELATIVE SALES

Notes: This figure displays the relationship between the raw values of average patent citations and firm relative sales in the left panel, and the relationship between the log of R&D expenses and firm relative sales in the right panel. We divide the relative sales into 200 quantiles and calculate the average value of innovation and R&D expenses for each quantile. The blue curve illustrates the quadratic fit of innovation and R&D expense against firm relative sales.

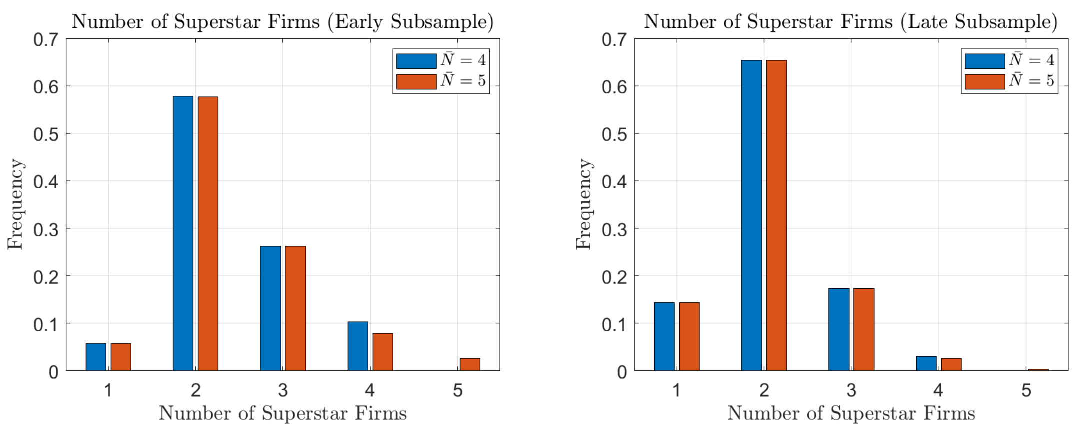

FIGURE L2: NUMBER OF SUPERSTAR FIRM DISTRIBUTION COMPARISON

Notes: This figure illustrates the distribution of industries over states with a different number of superstar firms in the early subsample (left panel) and late subsample (right panel) for the baseline model where $\bar{N}$ is set to 4 and an alternative model where $\bar{N}$ is set to 5.

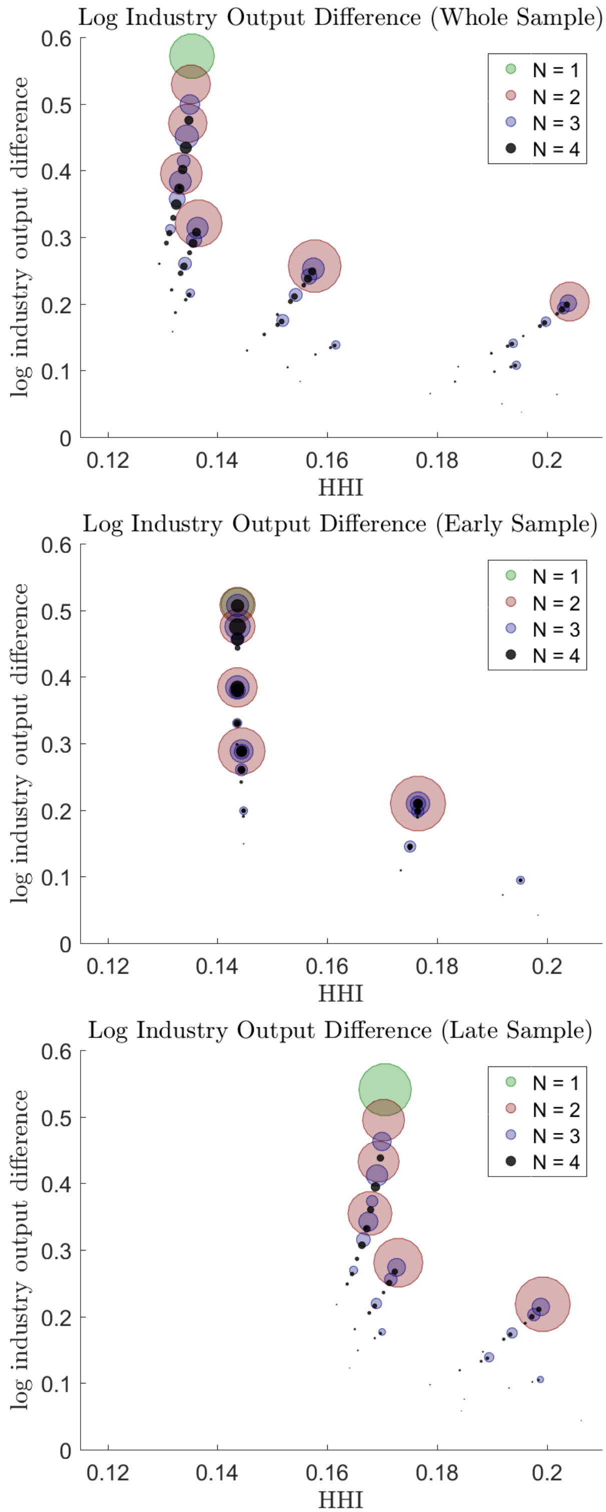

FIGURE L3: DIFFERENCES IN LOG INDUSTRY OUTPUT BETWEEN BASELINE AND COLLUDING FRINGE MODEL

Notes: We calculate log industry output in each industry state using the baseline model that was estimated using the whole sample, as well as the early and late period subsamples. Then, using the same parameter values, we calculate the log industry output in each industry state using the alternative model with the colluding fringe as shown in Section C. The three subfigures of this figure depict the log industry output in the baseline model minus that in the alternative model for each industry state for the three estimations, where the size of a dot indicates the fraction of the industry state $\mu(\Theta)$ in the baseline stationary equilibrium. N denotes the number of superstar firms in the industry the observation is coming from.